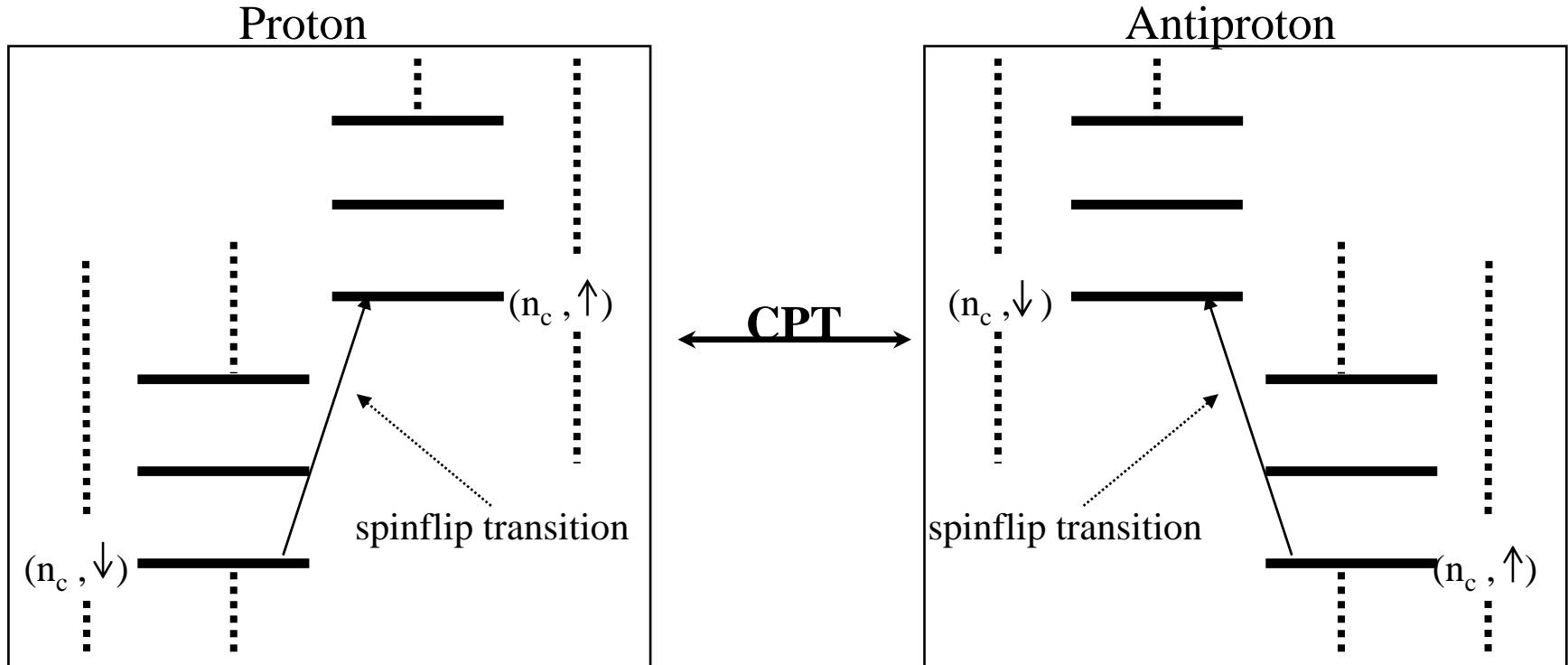


Analytical field calculations for the proton g-factor trap.

J. Verdú
and the g-factor collaboration
GSI / University of Mainz

Motivation: test of CPT Invariance



Blum, Kostelecky, Russell; Phys. Rev. D, 57, 3932

Figure of merit for CPT violation:

$$\left| \hbar(\omega_a^+ - \omega_a^-) \right| / 2m_p c^2 \leq 1 \cdot 10^{-25}$$

Introduction



The magnetic moment of the proton/antiproton is defined by:

$$\frac{|\bar{\mu}_p|}{\mu_N} = g_p \cdot \frac{|\bar{s}|}{h}$$

with:

μ_p : magnetic moment
 g_p : g-factor = 5.585 694 67 (3)
 s: spin
 μ_N : nuclear magneton = $e h / 4\pi m_p$

Comparing with electron: $\mu_N = 5.050\,786 \times 10^{-27} \text{ Am}^2$
 $\mu_B = 9.274\,015 \times 10^{-24} \text{ Am}^2$ \longrightarrow Three orders of magnitude smaller !!!

The magnetic dipole moment of the proton μ_p is 658 times smaller than that of the electron !!

Definition of the goals



We have to measure the spin state of the proton

In a magnetic Bottle, spinflip jump:

$$Dn_z = \frac{B_2}{4 p^2 m_p n_z} g_p m_N$$

Axial Frequency:

$$n_z = \frac{m_p \omega_z^2}{2 p} \longrightarrow$$

U_0 Ring Voltage; should be >1 Volt

We will set $U_0 = 1$ Volt for all following simulations

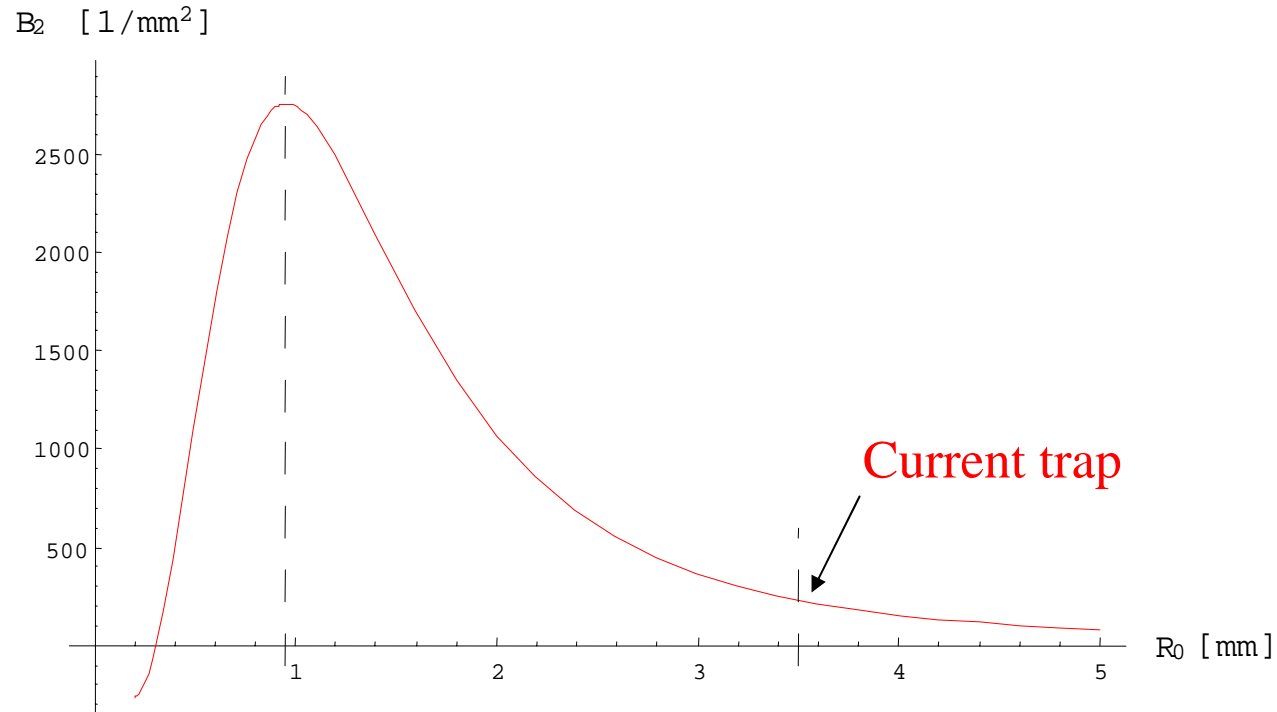
$$c_2 = \frac{1}{2 U_0} \frac{\nabla^2 f}{\nabla z^2} \quad \text{Curvature of Potential [1/mm}^2\text{]}$$

It is a function only of the geometry of the trap

How to increase B_2



First possibility : Decrease the trap radius

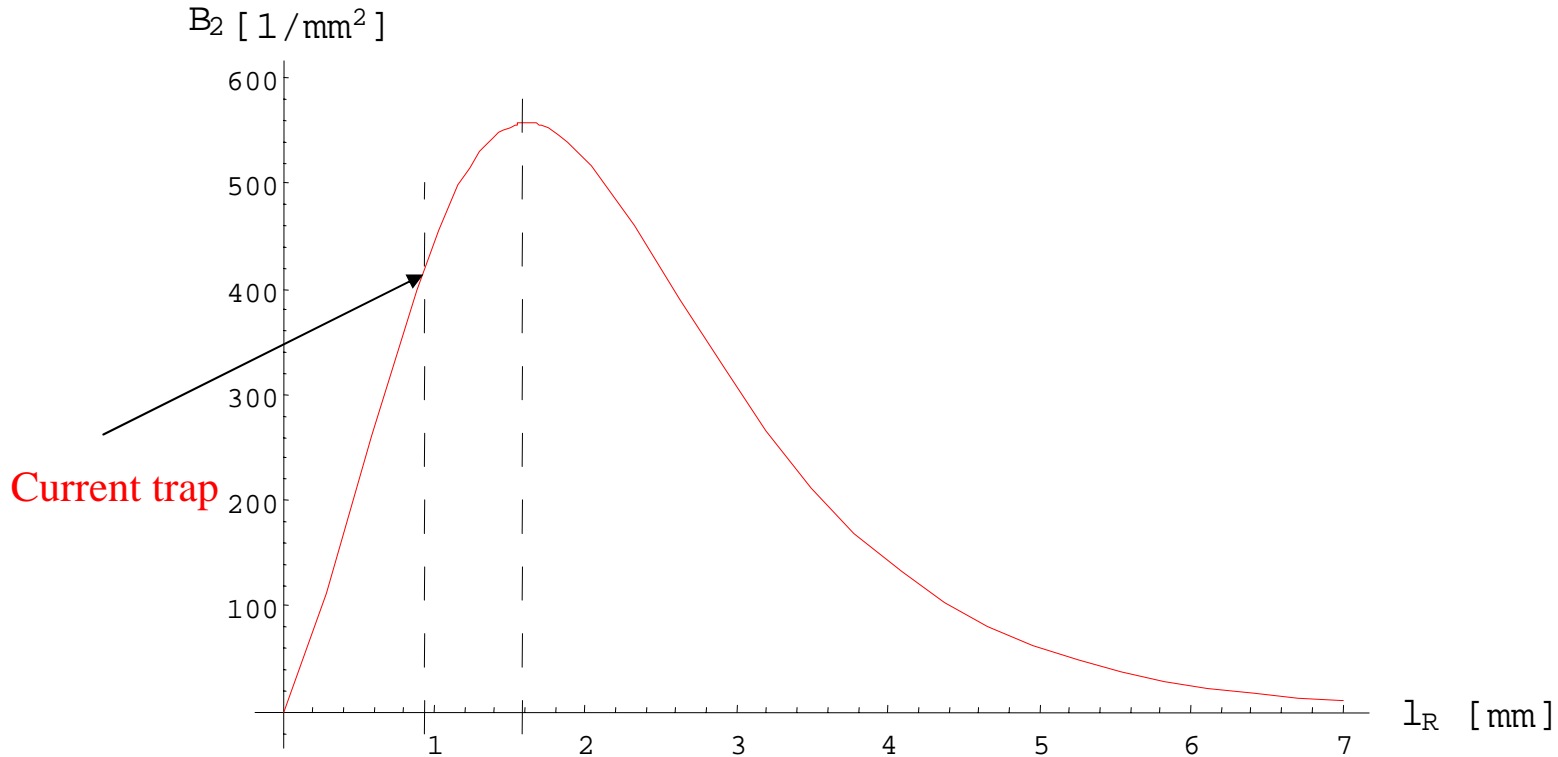


This graph is calculated for $l_R = 2.0$ mm

How to increase B_2



Second possibility : Increase the ring length



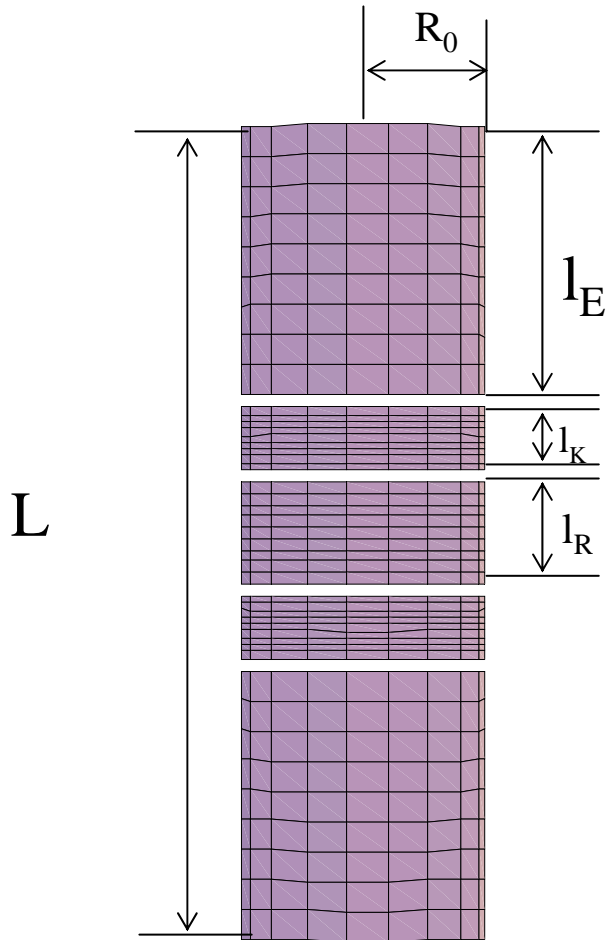
This graph is calculated for $R_0 = 2.8$ mm

We will have to use a combination of both: reduce radius, increase length

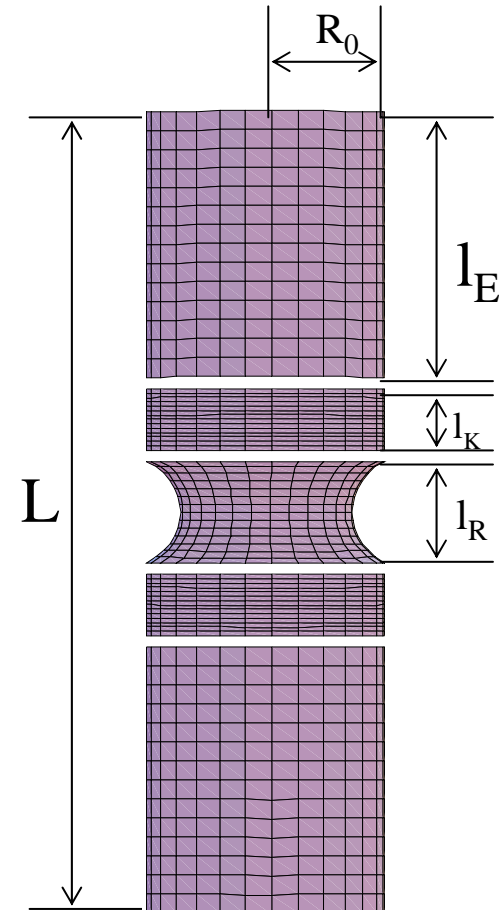
Two Possible Traps



Cylindrical Trap

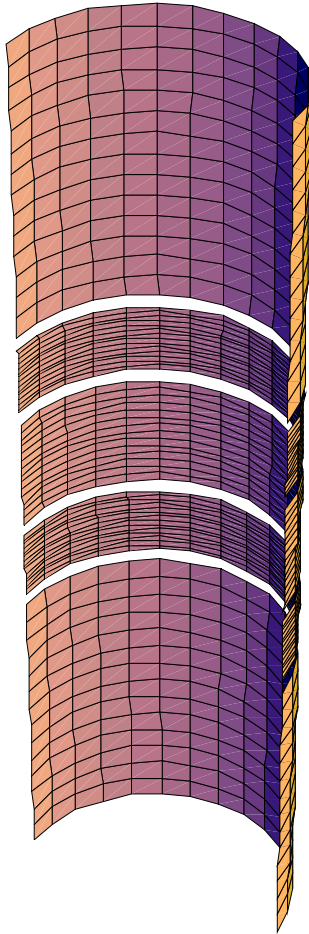


Hybrid Trap (toroidal Ring)

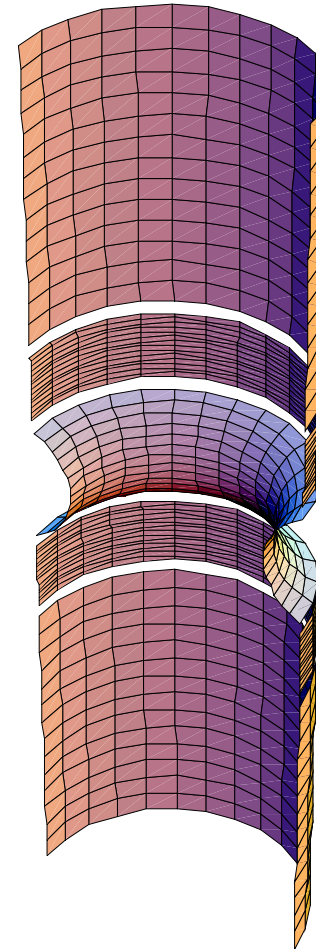


Two Possible Traps

Cylindrical Trap



Hybrid Trap (toroidal Ring)



Ring made out of CoFe
 $M_0 = 2.35$ Tesla

$$Dn_z = \frac{B_2}{4 p^2 m_p n_z} g_p m_N$$

$B_2 \sim M_0$
 Limit reached with CoFe

Using Green's Function Method

$$G_{\text{Zylinder}}(\vec{r}, \vec{r}') = \frac{4}{L} \sum_{n=0}^{\infty} \cos(k_n z) \cos(k_n z') \frac{I_0(k_n r_{<})}{I_0(k_n R_0)} (I_0(k_n R_0) K_0(k_n r_{>}) - I_0(k_n r_{>}) K_0(k_n R_0))$$

$$k_n = \frac{(2n+1)\pi}{L};$$

$$G_{\text{Torus}}(\vec{r}, \vec{r}') = \frac{\sqrt{2}}{a\pi} \sqrt{s-\tau} \cdot \sqrt{s'-\tau'} \sum_{n=0}^{\infty} \varepsilon_n \cos(nu) \cos(nu') \frac{P_{n-1/2}(s_{<})}{P_{n-1/2}(s_0)} \{P_{n-1/2}(s_0) Q_{n-1/2}(s_{>}) - P_{n-1/2}(s_{>}) Q_{n-1/2}(s_0)\}$$

$$s_{>} = \cosh(v_{>}), \tau = \cos(u), \tau' = \cos(u')$$

$$s_{<} = \cosh(v_{<})$$

$$s' = \cosh(v')$$

$$s_0 = \cosh(v_0)$$

The electrical Potential is calculated ANALYTICALLY !

The same can be done for a general elliptic ring (including hyperbolical case)

Potential of a torus:

$$\Psi_t(v, u) = U_0 \frac{\sqrt{2 (\cosh v - \cos u)}}{\pi} \sum_{n=0}^{\infty} \epsilon_n \cdot \frac{Q_{n-1/2}(\cosh v_0)}{P_{n-1/2}(\cosh v_0)} \cdot P_{n-1/2}(\cosh v) \cos(nu) .$$

Potential of a cylindrical trap:

$$\phi(r, z) = \sum_m \left\{ \frac{8U_0}{l \cdot d \cdot k_m^2 \cdot I_0(k_m a)} \sin\left(\frac{k_m d}{2}\right) \left[\sin\left(\frac{k_m(d+l_r)}{2}\right) + \dots \right. \right. \\ \left. \left. + 2T \sin\left(\frac{k_m(d+l_k)}{2}\right) \cos\left(\frac{k_m(2d+l_r+l_k)}{2}\right) \right] I_0(k_m r) \cos(k_m z) \right\}$$

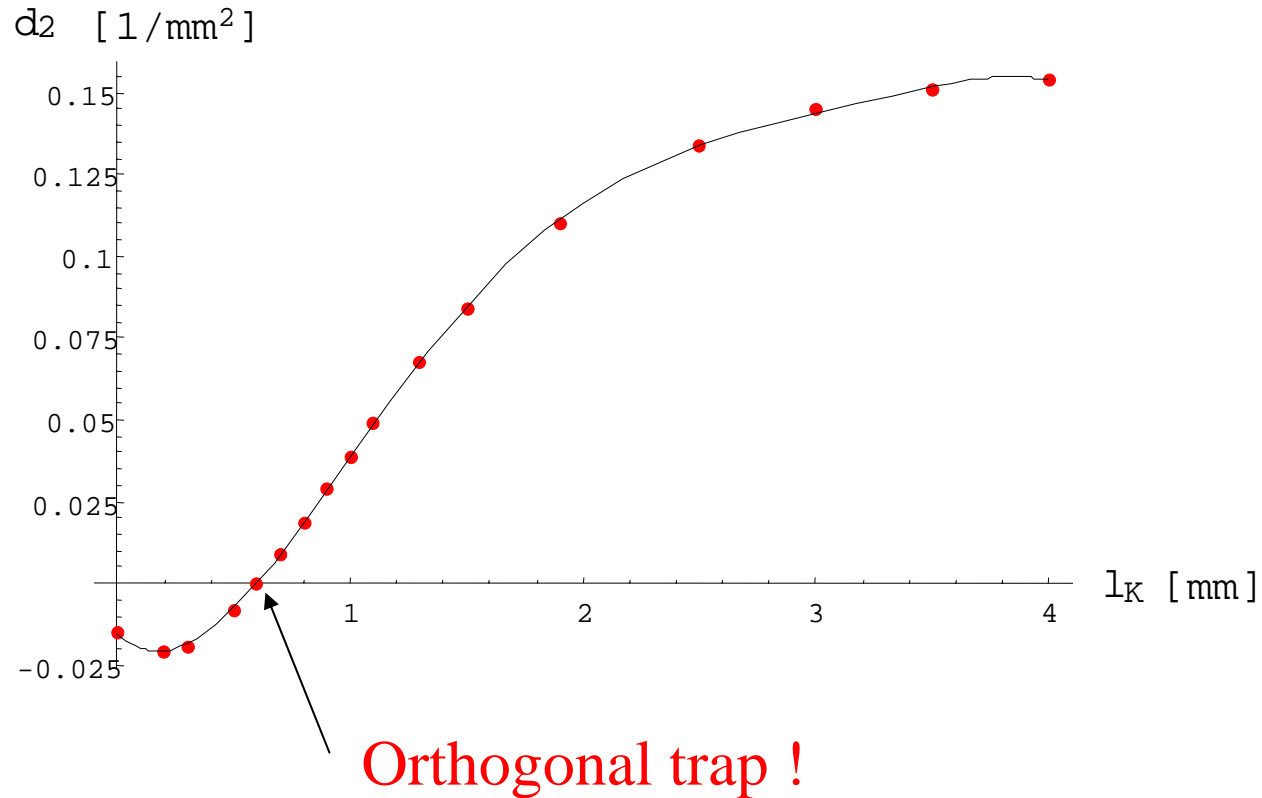
For the hybrid trap the potential is a linear combination of both base functions:
Legendre and Bessel.

The coefficients of the linear combination are calculated numerically.

Orthogonality of the trap: d_2



The toroidal trap can be made orthogonal

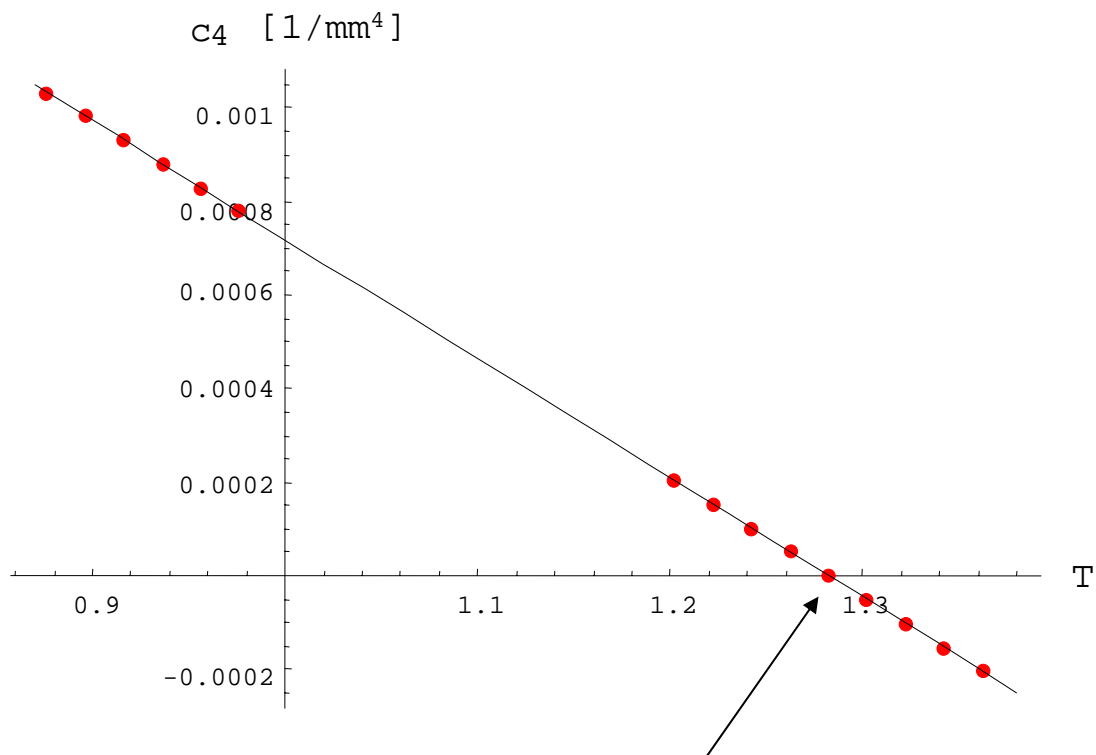


With $L = 20.96$ mm, $l_R = 1.0$ mm, $R_0 = 2.0$ mm

Harmonicity of the trap: c_4

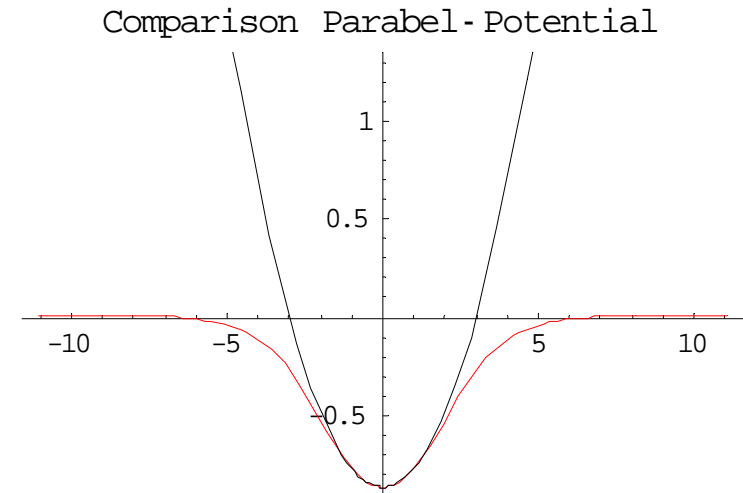
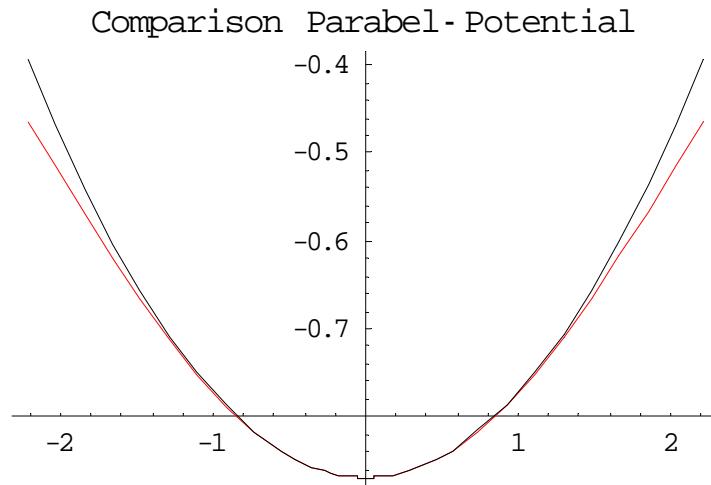


The potential of the hybrid trap can be compensated



Optimal Tuning Ratio = 1.282 013 ..

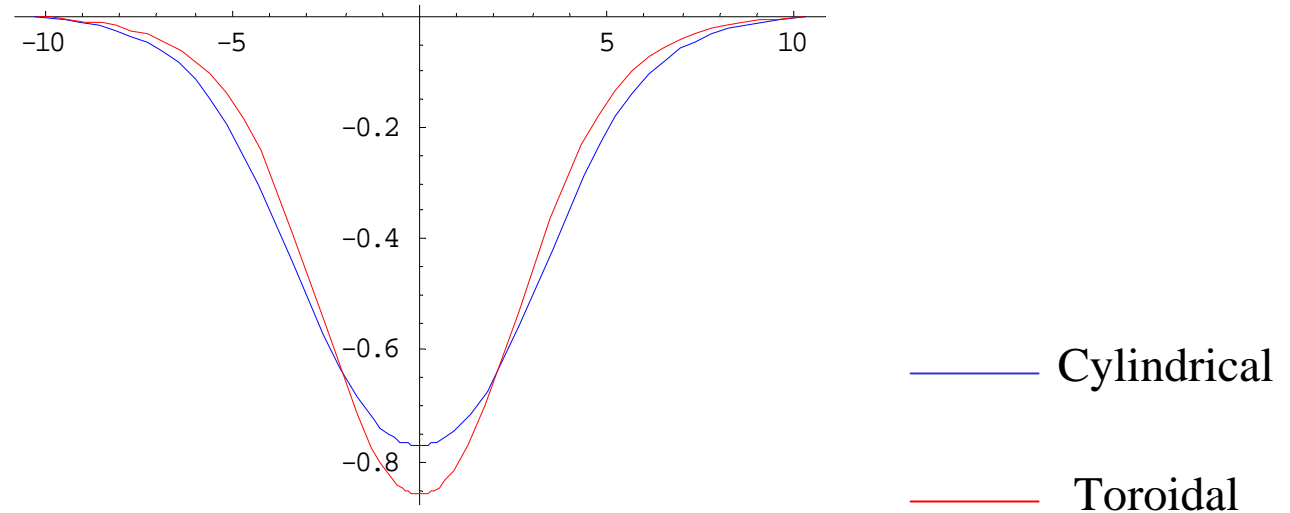
Representation of the Potential on the z-Axis:



The Volume of harmonicity is $\sim 8 \text{ mm}^3$

The Harmonicity Range of the toroidal trap is essentially smaller than the cylindrical one

Comparison hybrid - cylindrical trap



Disadvantages of the toroidal trap:

- 1) Smaller harmonic region
- 2) Mechanically a little more complicated than the cylindrical one

For the rest, both traps are electrically essentially identical

The useful „c, d“ coefficients can be calculated trivially

Curvature

$$c_2 = - \frac{1}{2} \sum_{n=0}^{\infty} \frac{(n+1)^2}{L^2} A_n + \frac{1}{2p} \frac{2}{a_0^2} \sum_{n=0}^{\infty} \frac{1}{n^2+1} \frac{1}{P_{n-1/2}} \frac{1}{\cosh \beta_0} B_n$$

Anharmonicity

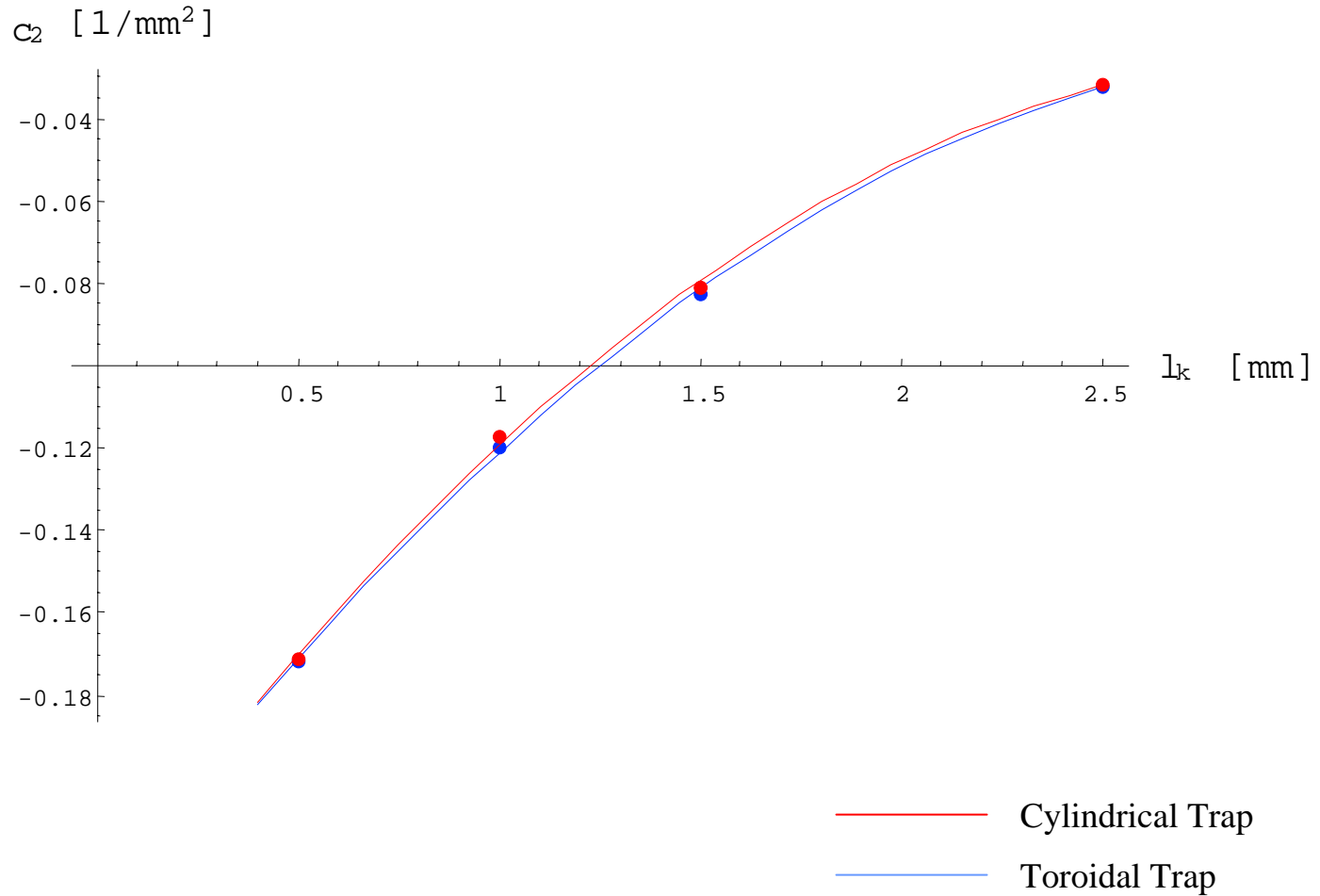
$$c_4 = \frac{1}{24} \sum_{n=0}^{\infty} \frac{(n+1)^4}{L^4} A_n - \frac{1}{2p} \frac{2}{a_0^4} \sum_{n=0}^{\infty} \frac{6n^4 + 56n^2 + 9}{n^2+1} \frac{1}{P_{n-1/2}} \frac{1}{\cosh \beta_0} B_n$$

Orthogonality

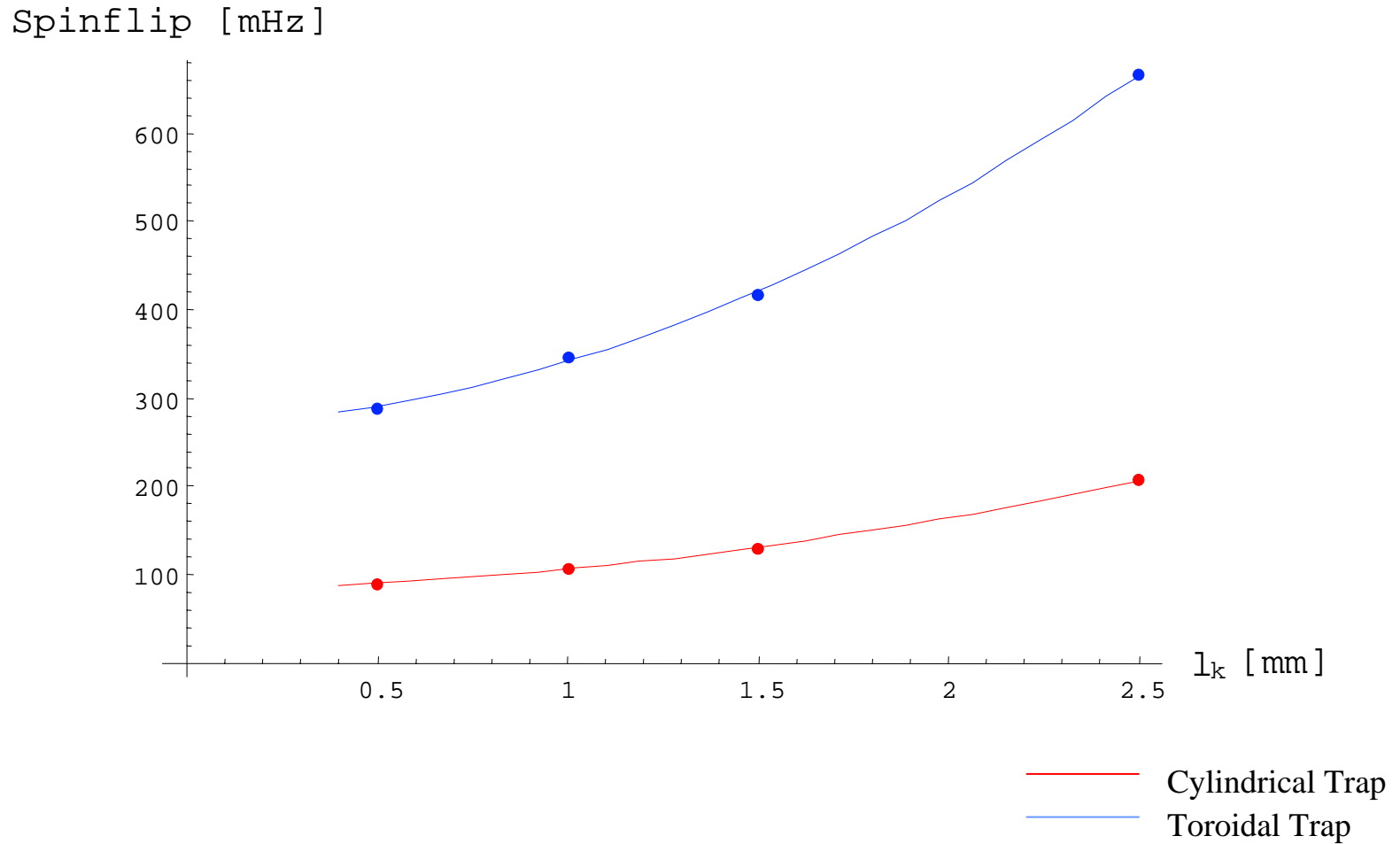
$$d_2 = - \frac{1}{2} \sum_{n=0}^{\infty} \frac{(n+1)^2}{L^2} \Gamma_n \frac{U_0}{2p} \frac{2}{a_0^2} \sum_{n=0}^{\infty} \frac{1}{n^2+1} \frac{1}{P_{n-1/2}} \frac{1}{\cosh \beta_0} B_n$$

All can be calculated in less than 1 second.

Curvature of the electric Potential c_2

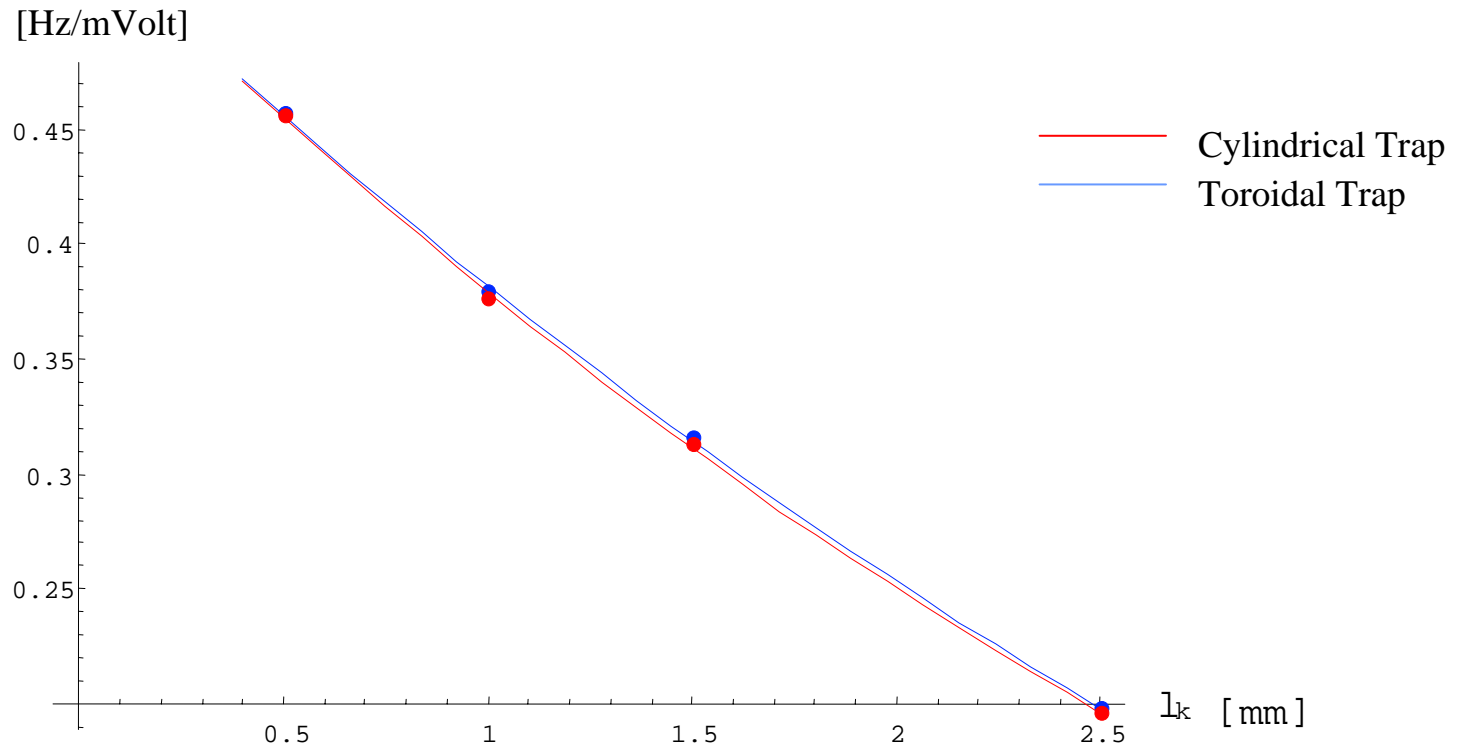


Frequency jump due to a spin-flip



Spin-flip at least 3 times bigger

Our biggest enemy: $\frac{Dn_z}{DU_0} = \frac{1}{2} \frac{n_z}{U_0}$



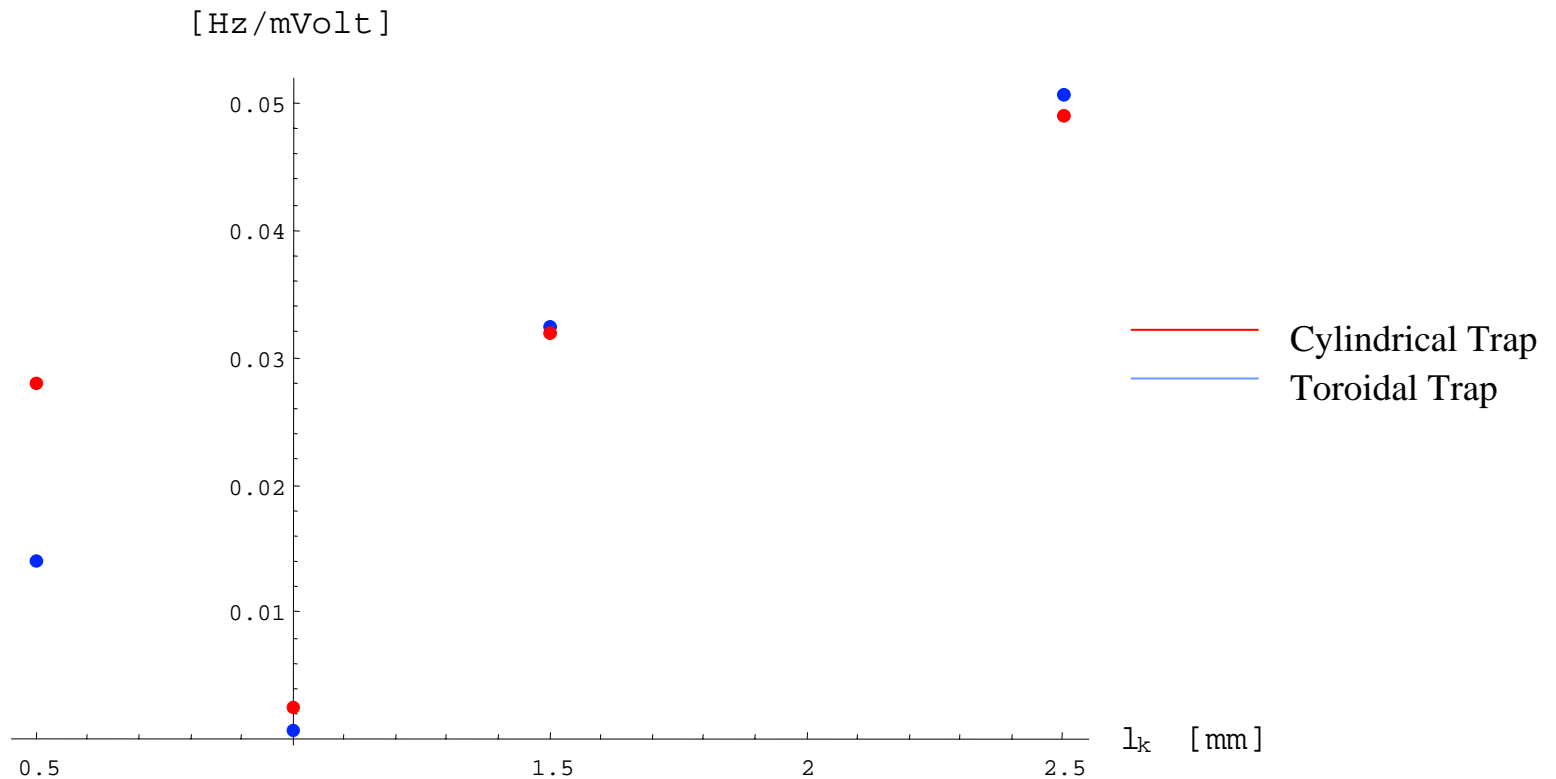
At least a factor 4 bigger fluctuations than with old trap !!

The cost of not orthogonality



Frequency fluctuations due to correction Voltage (U_K) instabilities

$$\frac{Dn_z}{DU_K} = \frac{n_z}{2} \frac{d_2}{c_2}$$



Maybe it is worth to sacrifice orthogonality ?

Conclusion and Outlook



We have presented an extension of the cylindrical trap

The potential within the trap has been calculated analytically

Application to the g -factor of the proton/antiproton

Thank-you very much for your attention !

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Rafael Ferrer
Fernando Galve
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Susanne Kreim
Wolfgang Quint
Stefan Stahl
Tristán Valenzuela
Manuel Vogel
Christine Weber
Günther Werth