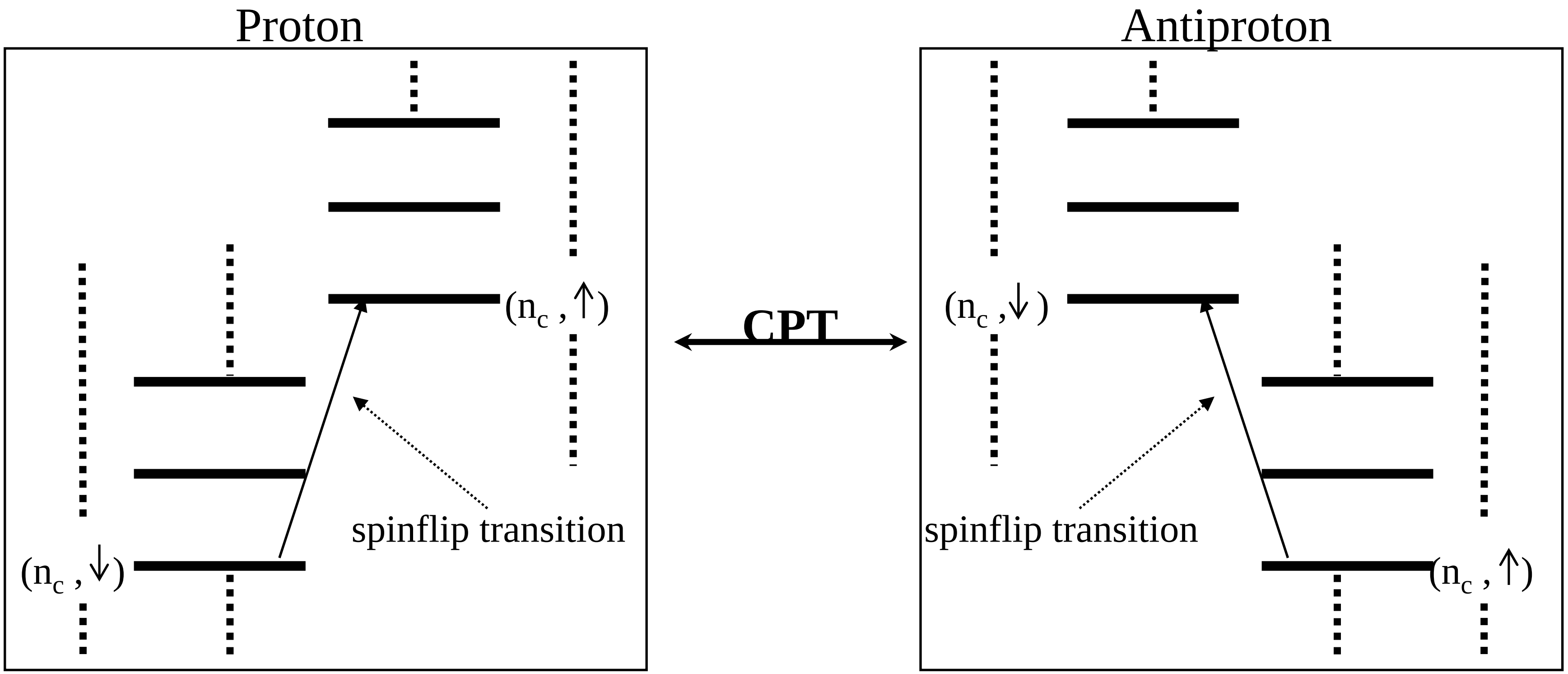
Analytical field calculations for the proton g-factor trap.

J. Verdú
and the g-factor collaboration
GSI / University of Mainz

GSI Darmstadt 12/11/ 2004

Motivation: test of CPT Invariance



Bluhm, Kostelecky, Russell; Phys. Rev. D, 57, 3932
Figure of merit for CPT violation:

$$\left|\hbar(\omega_a^+ - \omega_a^-)\right| / 2m_p c^2 \leq 1 \cdot 10^{-25}$$

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Introduction

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The magnetic moment of the proton/antiproton is defined by:

$$\frac{|\overline{\mu}_p|}{\mu_N} = g_p \cdot \frac{|\overline{s}|}{h}$$

with:

μ_p: magnetic moment
g_p: g-factor = 5. 585 694 67 (3)
s: spin
μ_N: nuclear magneton = e h / 4πm$_p$

Comparing with electron: μ_N = 5. 050 786 x 10^{-27} Am^2
μ_B = 9. 274 015 x 10^{-24} Am^2 → Three orders of magnitude smaller !!!

The magnetic dipole moment of the proton μ_P is 658 times smaller than that of the electron !!

Definition of the goals

We have to measure the spin state of the proton

In a magnetic Bottle, spinflip jump:

$$\Delta\nu_z = \frac{B_2}{4\pi^2 m_p \nu_z} g_p \mu_N$$

Axial Frequency:

$$\nu_z = \frac{[\text{illegible}]\ \frac{2 c_2 U_0}{m_p}}{2\pi}$$

U_0 Ring Voltage; should be >1 Volt

We will set U_0 = 1 Volt for all following simulations

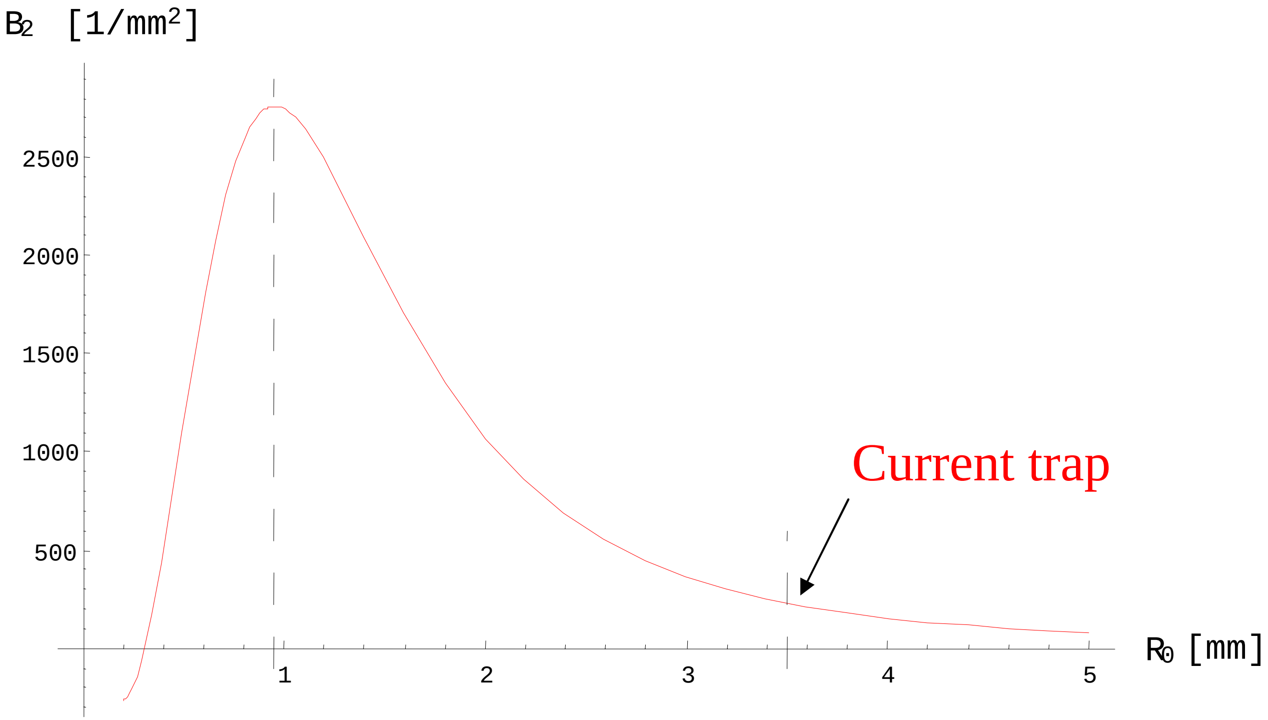
$$c_2 = \frac{1}{2 U_0} \frac{\partial^2 \phi}{\partial z^2}$$

Curvature of Potential [1/mm²]

It is a function only of the geometry of the trap

How to increase B_2

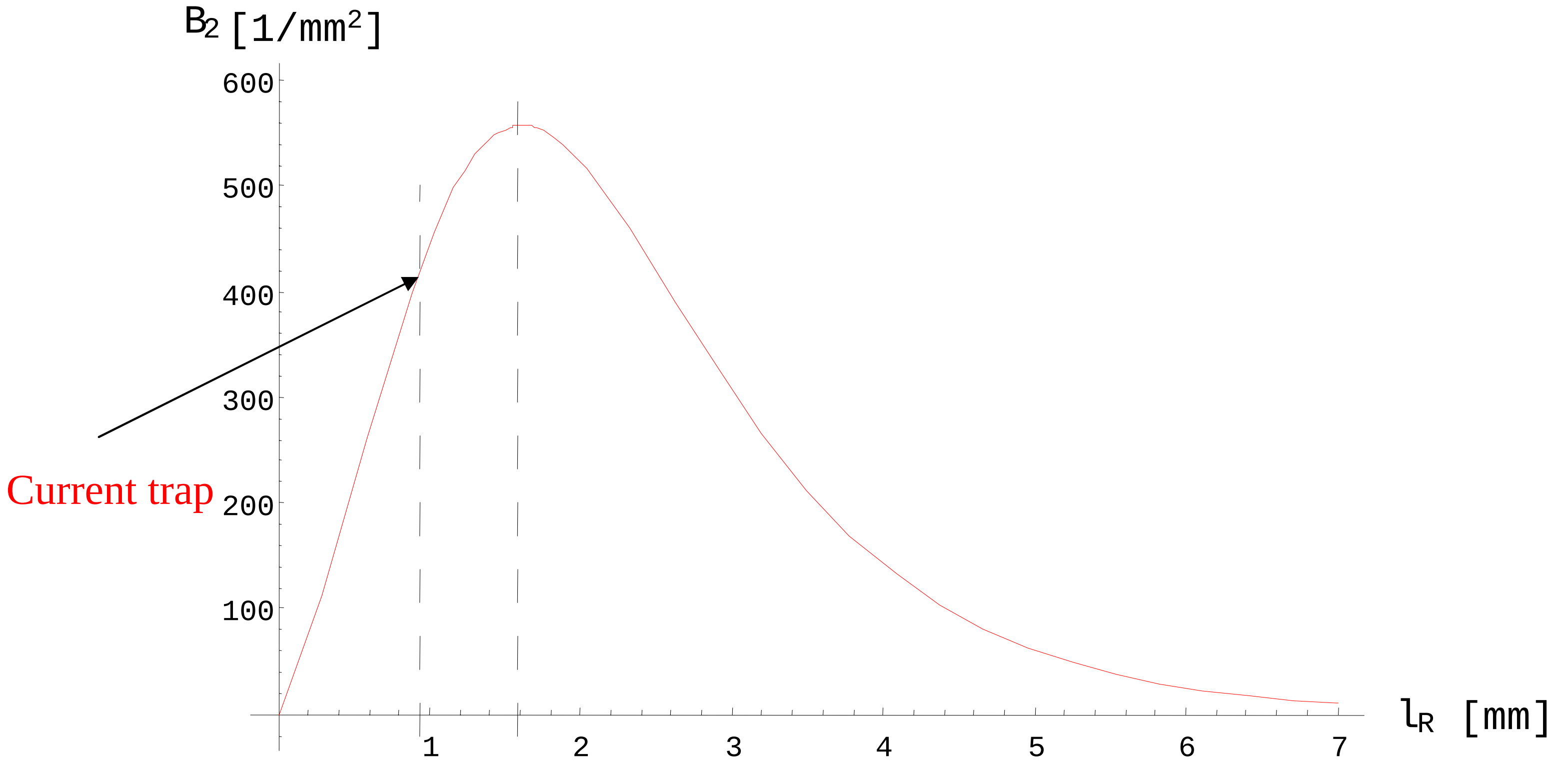
First possibility : Decrease the trap radius



This graph is calculated for l_R = 2.0 mm

How to increase B_2

Second possibility : Increase the ring length

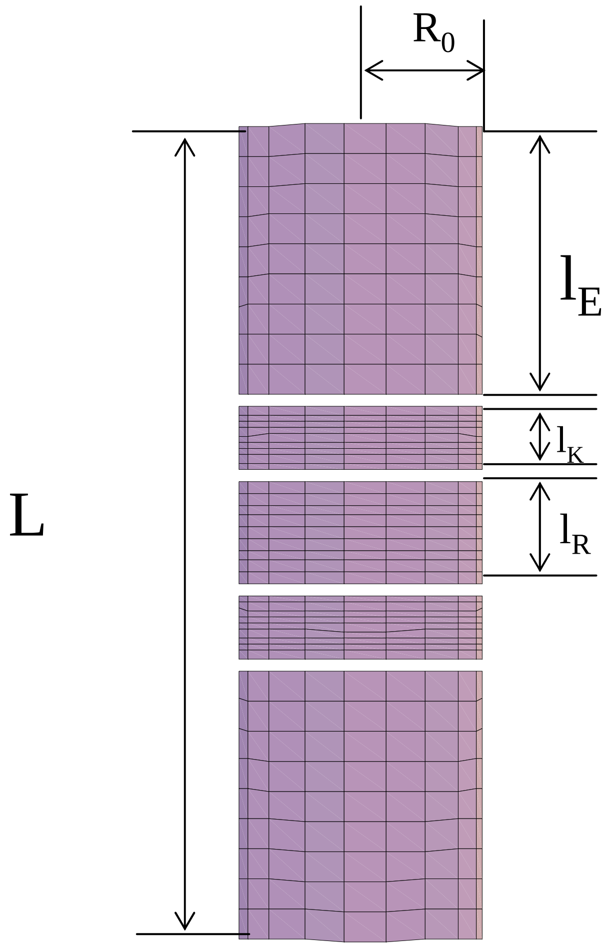


This graph is calculated for R_0 = 2.8 mm

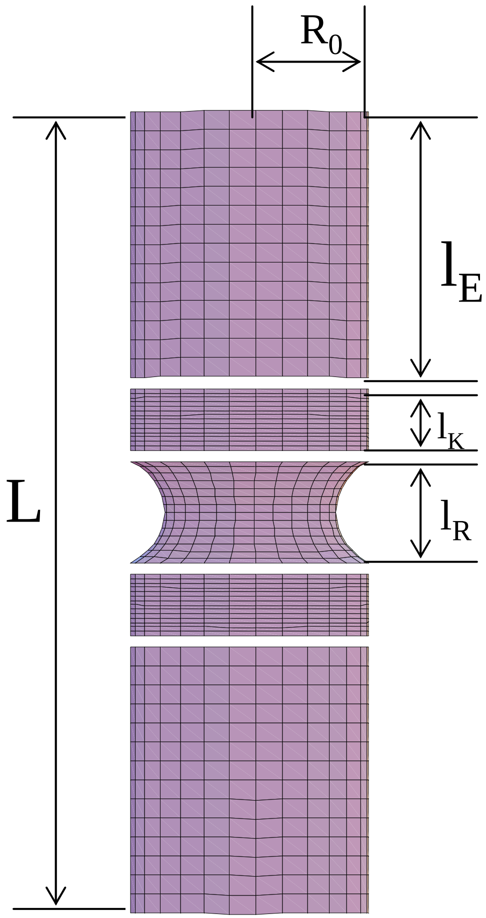
We will have to use a combination of both: reduce radius, increase length

Two Possible Traps

Cylindrical Trap



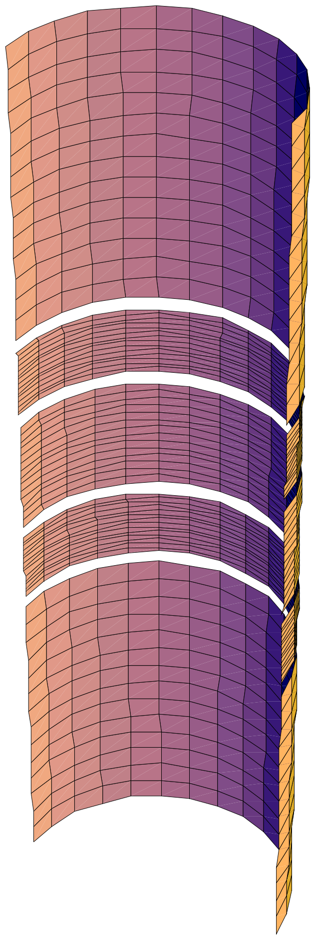
Hybrid Trap (toroidal Ring)



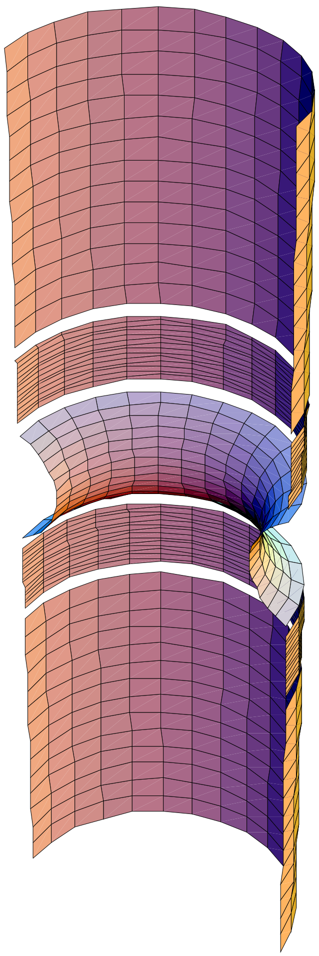
For both traps: gap between electrodes = 0.14 mm

Two Possible Traps

Cylindrical Trap



Hybrid Trap (toroidal Ring)



Ring made out of CoFe

M_0= 2.35 Tesla

$$\Delta\nu_z = \frac{B_2}{4\pi^2 m_p \nu_z} g_p \mu_N$$

$B_2 \sim M_0$

Limit reached with CoFe

Analytical Calculation of the Potential

Using Green's Function Method

$$G_{Zylinder}(\vec{r},\vec{r}\,')=\frac{4}{L}\sum_{n=0}^{\infty}\cos(k_n z)\cos(k_n z')\frac{I_0(k_n r_<)}{I_0(k_n R_0)}\left(I_0(k_n R_0)K_0(k_n r_>)-I_0(k_n r_>)K_0(k_n R_0)\right)$$

$$k_n=\frac{(2n+1)\pi}{L};$$

$$G_{Torus}(\vec{r},\vec{r}\,')=\frac{\sqrt{2}}{a\pi}\sqrt{s-\tau}\cdot\sqrt{s'-\tau'}\sum_{n=0}^{\infty}\varepsilon_n\cos(nu)\cos(nu')\frac{P_{n-1/2}(s_<)}{P_{n-1/2}(s_0)}\left\{P_{n-1/2}(s_0)Q_{n-1/2}(s_>)-P_{n-1/2}(s_>)Q_{n-1/2}(s_0)\right\}$$

$$s_>=\cosh(v_>),\tau=\cos(u),\tau'=\cos(u')$$

$$s_<=\cosh(v_<)$$

$$s'=\cosh(v')$$

$$s_0=\cosh(v_0)$$

The electrical Potential is calculated ANALYTICALLY !

The same can be done for a general elliptic ring (including hyperbolical case)

Analytical Calculation of the Potential

Potential of a torus:

$$\Psi_t(v,u) = U_0 \frac{\sqrt{2\,(\cosh v - \cos u)}}{\pi} \sum_{n=0}^{\infty} \epsilon_n \cdot \frac{Q_{n-1/2}(\cosh v_0)}{P_{n-1/2}(\cosh v_0)} \cdot P_{n-1/2}\,(\cosh v) \cos\,(n\,u)\ .$$

Potential of a cylindrical trap:

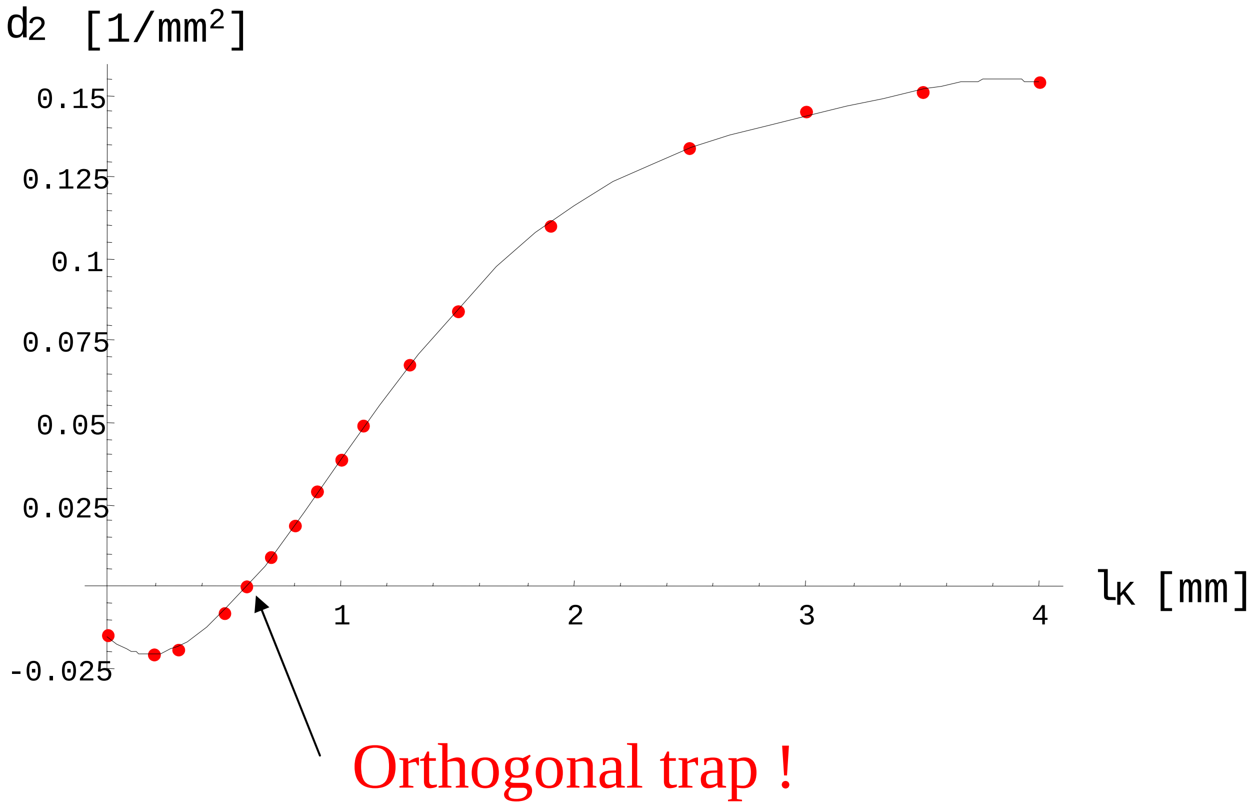
$$\phi(r,z) = \sum_m \left\{ \frac{8\,U_0}{l \cdot d \cdot {k_m}^2 \cdot I_0(k_m a)} \sin\left(\frac{k_m d}{2}\right) \left[\sin\left(\frac{k_m(d+l_r)}{2}\right) + \cdots \right.\right.$$
$$\left.\left. + 2\,T \sin\left(\frac{k_m(d+l_k)}{2}\right) \cos\left(\frac{k_m(2d+l_r+l_k)}{2}\right)\right] I_0(k_m r)\cos(k_m z) \right\}$$

For the hybrid trap the potential is a linear combination of both base functions: Legendre and Bessel.

The coefficients of the linear combination are calculated numerically.

Orthogonality of the trap: d_2

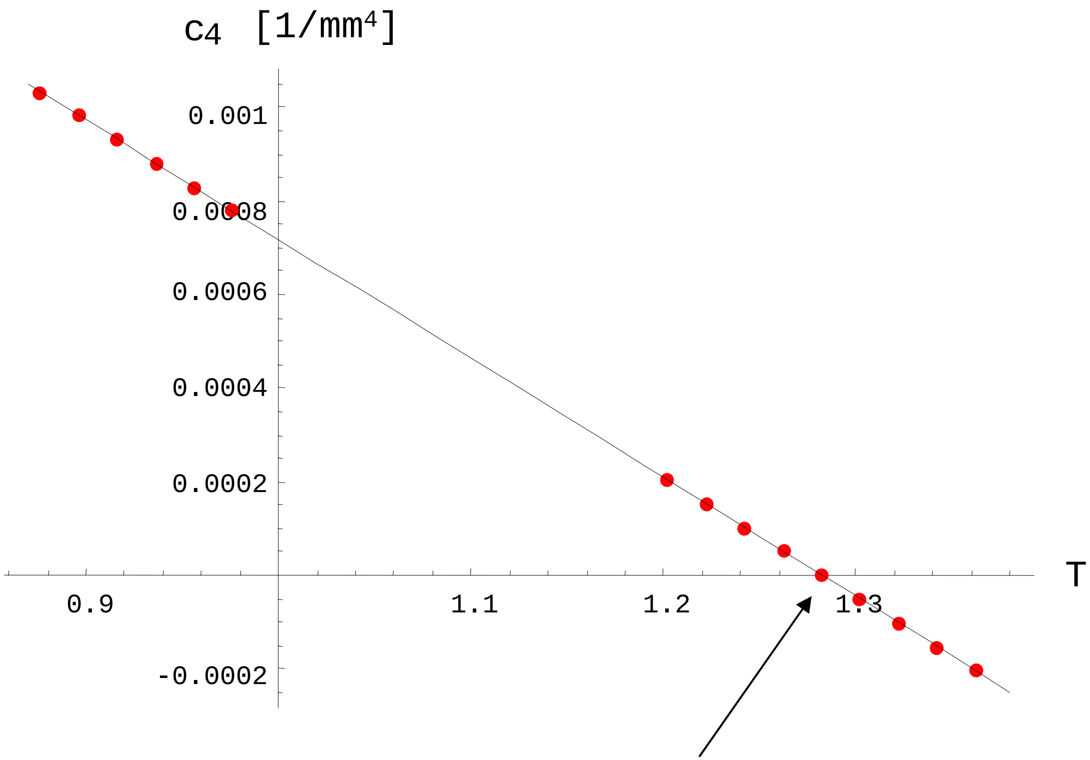
The toroidal trap can be made orthogonal



With $L = 20.96$ mm, $l_R = 1.0$ mm , $R_0 = 2.0$ mm

Harmonicity of the trap: c_4

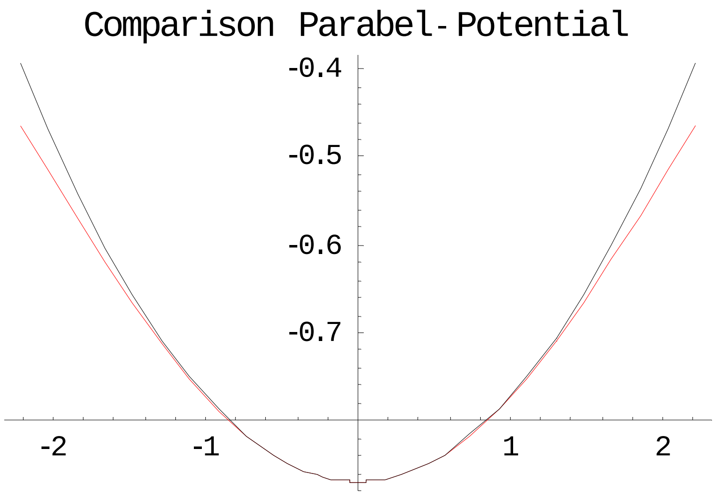
The potential of the hybrid trap can be compensated

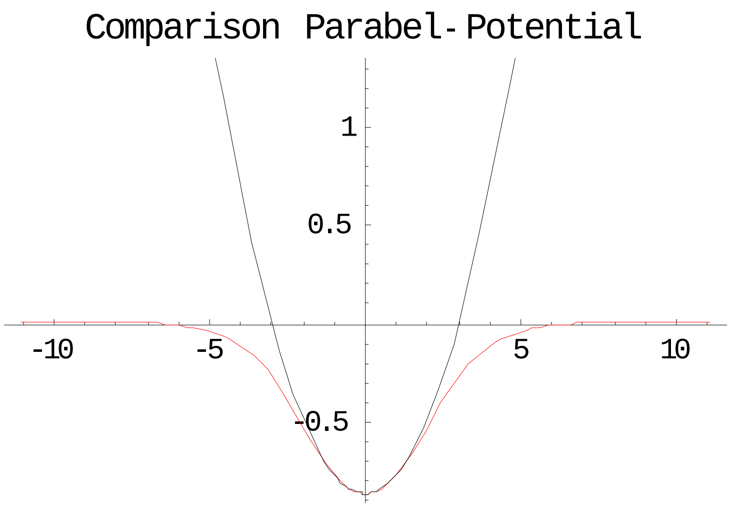


Optimal Tuning Ratio = 1. 282 013 ..

Comparison hybrid - cylindrical trap

Representation of the Potential on the z-Axis:

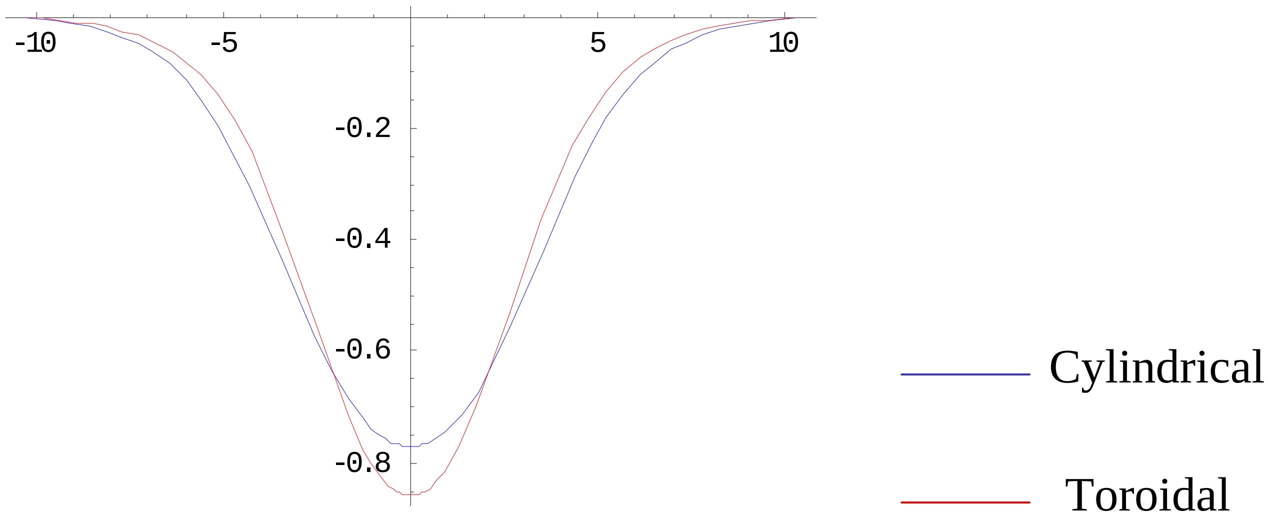




The Volume of harmonicity is ~ 8 mm^3

The Harmonicity Range of the toroidal trap is essentially smaller than the cylindrical one

Comparison hybrid - cylindrical trap



Disadvantages of the toroidal trap:

1) Smaller harmonicity region

2) Mechanically a little more complicated than the cylindrical one

For the rest, both traps are electrically essentially identical

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Some mathematical remarks

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The useful „c, d“ coefficients can be calculated trivially

Curvature

$$c_2 = -\frac{1}{2}\sum_{n=0}^{\infty}\left\{\left[\frac{(n+1)\pi}{L}\right]^2 A_n + \frac{1}{2\pi}\frac{2}{a_0{}^2}\left[4n^2+1\right] \frac{1}{P_{n-1/2}(\cosh\xi_0)} B_n\right\}$$

Anharmonicity

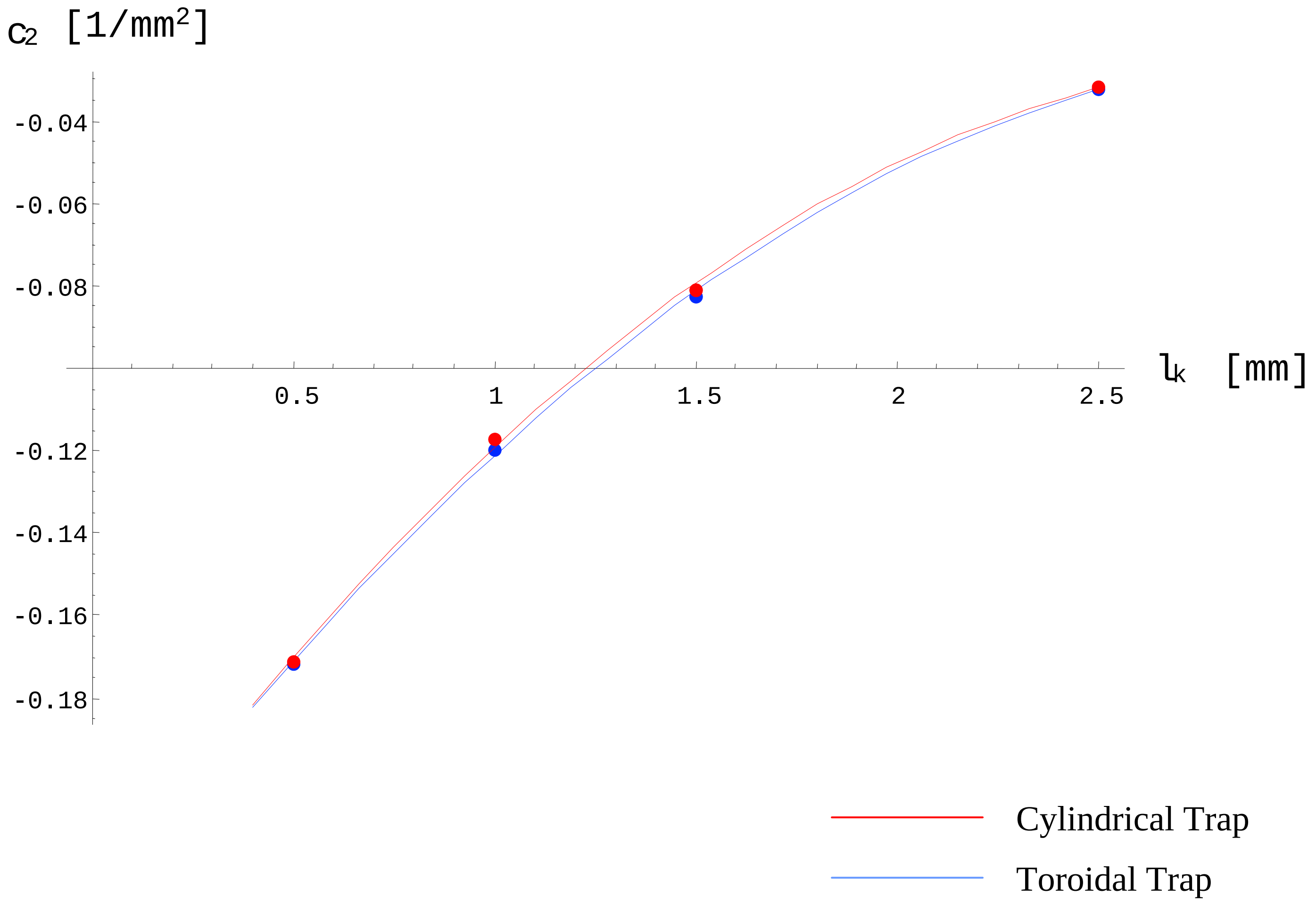
$$c_4 = \frac{1}{24}\sum_{n=0}^{\infty}\left\{\left[\frac{(n+1)\pi}{L}\right]^4 A_n \frac{1}{2\pi}\frac{2}{a_0{}^4}\left[16n^4+56n^2+9\right] \frac{1}{P_{n-1/2}(\cosh\xi_0)} B_n\right\}$$

Orthogonality

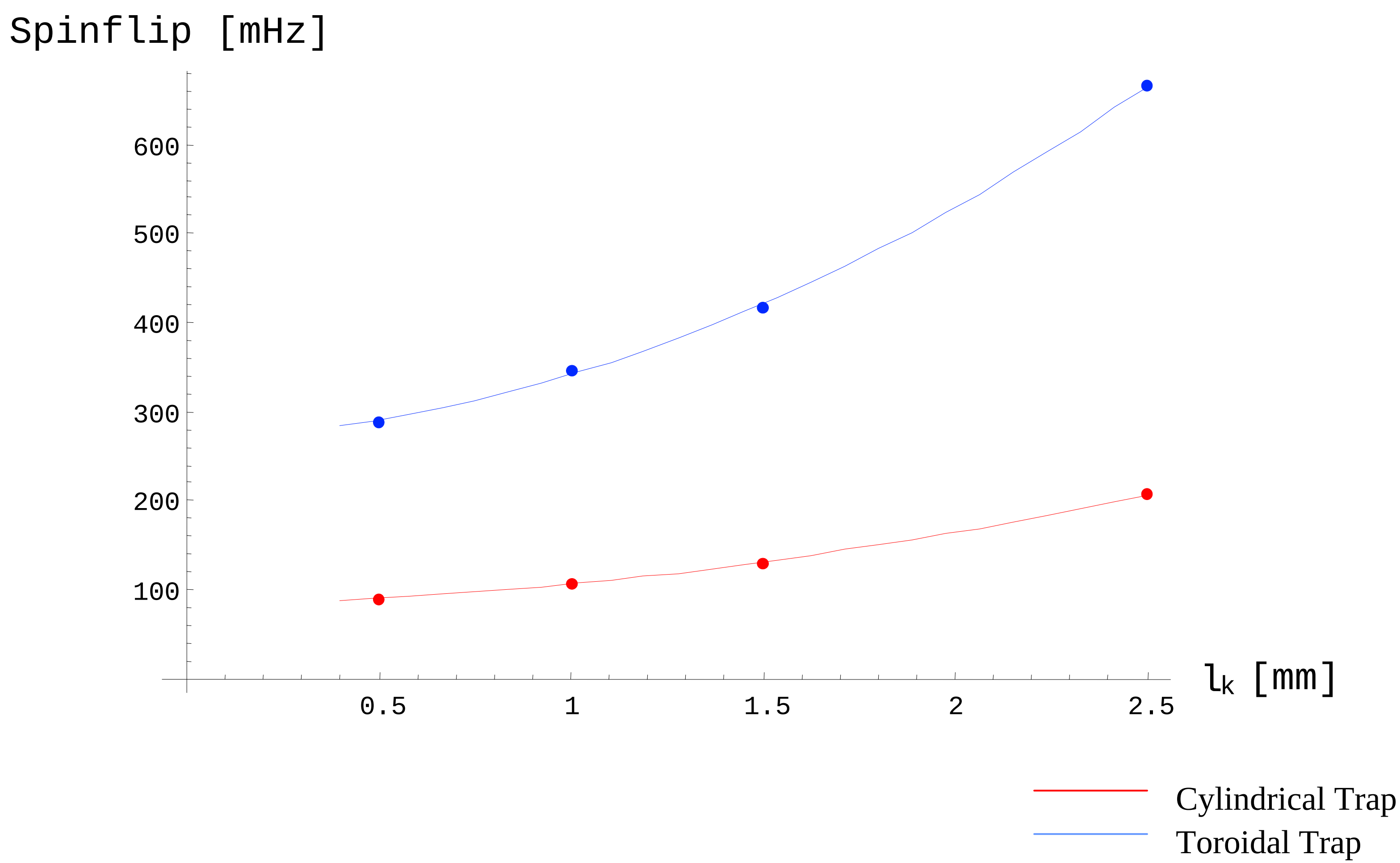
$$d_2 = -\frac{1}{2}\sum_{n=0}^{\infty}\left\{\left[\frac{(n+1)\pi}{L}\right]^2 AT_n \frac{U_0}{2\pi}\frac{2}{a_0{}^2}\left[4n^2+1\right] \frac{1}{P_{n-1/2}(\cosh\xi_0)} BT_n\right\}$$

All can be calculated in less than 1 second.

Curvature of the electric Potential c_2



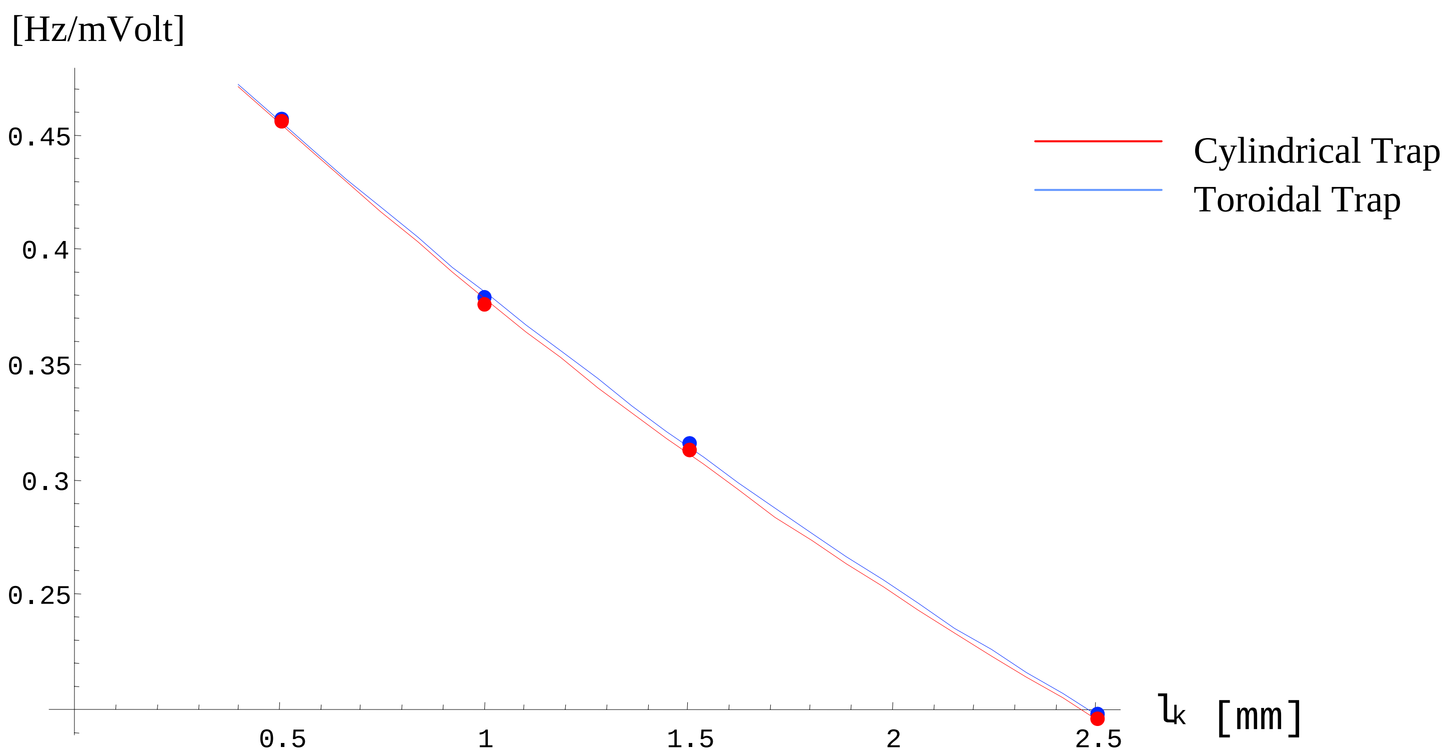
Frequency jump due to a spin-flip



Spin-flip at least 3 times bigger

Frequency-fluctuations due to ring voltage instabilities

Our biggest enemy: $$\frac{\Delta \nu_z}{\Delta U_0} = \frac{1}{2}\frac{\nu_z}{U_0}$$

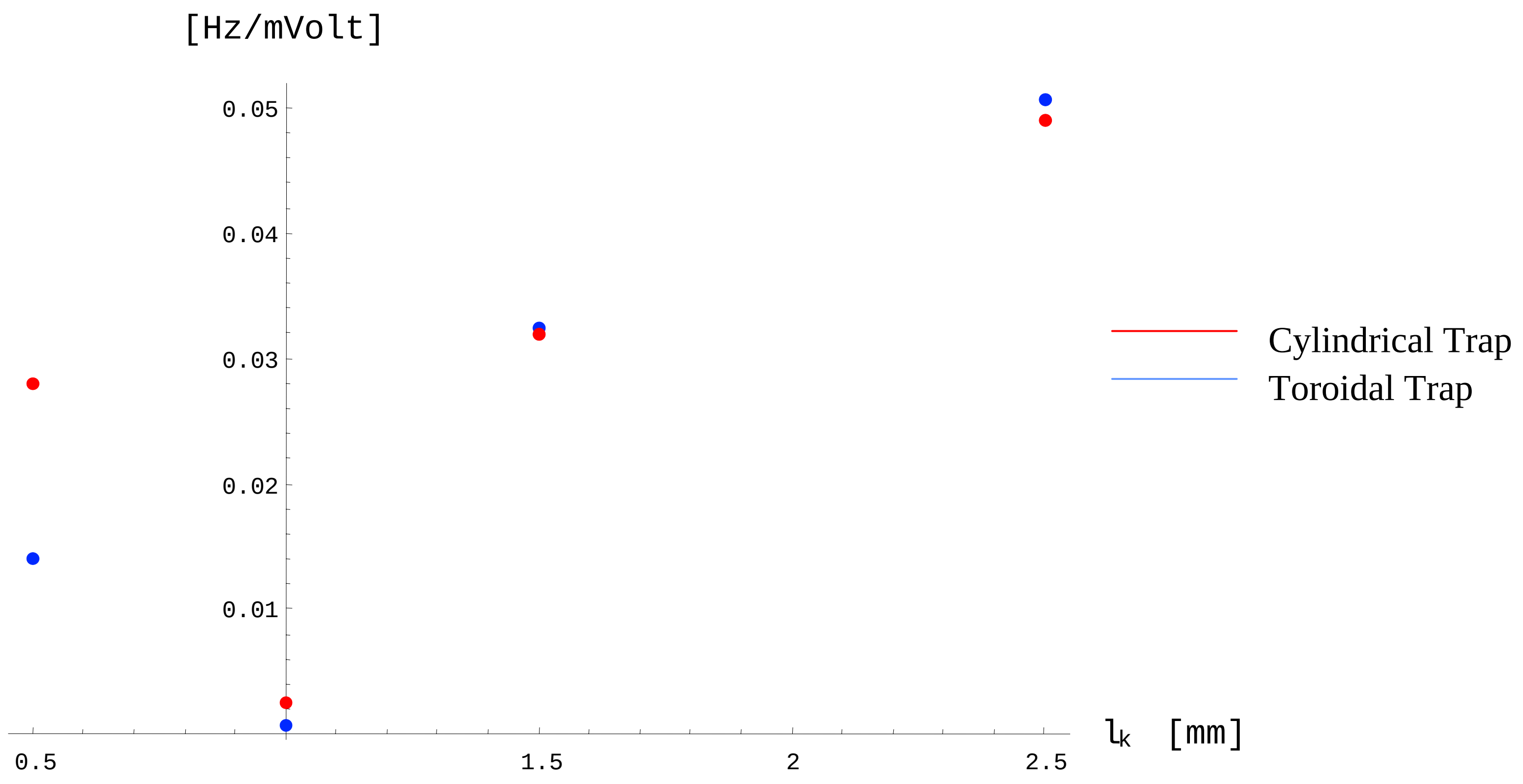


At least a factor 4 bigger fluctuations than with old trap !!

The cost of not orhogonality

Frequency fluctuations due to correction Voltage (U_K) instabilities

$$\frac{D\nu_z}{DU_K} = \frac{\nu_z}{2}\frac{d_2}{c_2}$$



Maybe it is worth to sacrifize orthogonality ?

Conclusion and Outlook

We have presented an extension of the cylindrical trap

The potential wirhin the trap has been calculated analytically

Application to the g-factor of the proton/antiproton

Thank-you very much for your attention !

José Verdú
Joseba Alonso
Klaus Blaum
Slobodan Djekic
Rafael Ferrer
Fernando Galve
Hans-Jürgen Kluge
Susanne Kreim
Wolfgang Quint
Stefan Stahl
Tristán Valenzuela
Manuel Vogel
Christine Weber
Günther Werth