Electron Cooling in Traps

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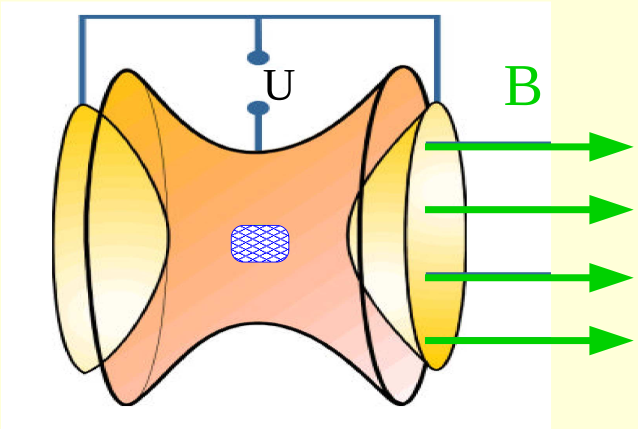
- Energy loss of ions in magnetized plasmas[1]
  - Theoretical challenges and methods
  - Energy loss and cooling times
  - Cooling of antiprotons and positron cooling
- Future tasks and open problems

[1] supported by BMBF and GSI

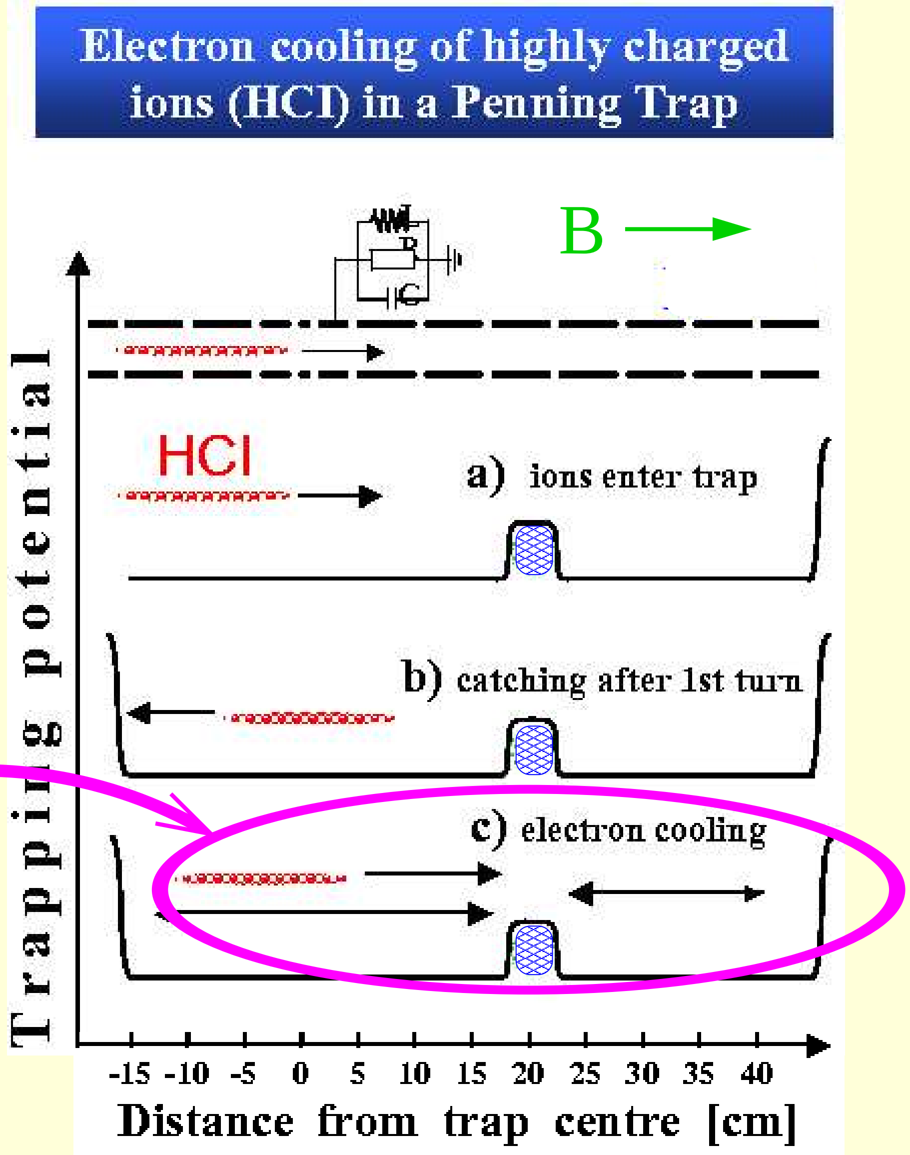
Electron Cooling in a Penning Trap

Principle of a Penning Trap:



radial confinement by a strong homogeneous magnetic field

axial confinement by an electrostatic field



Theoretical description of cooling:
Energy loss of ions in an electron plasma

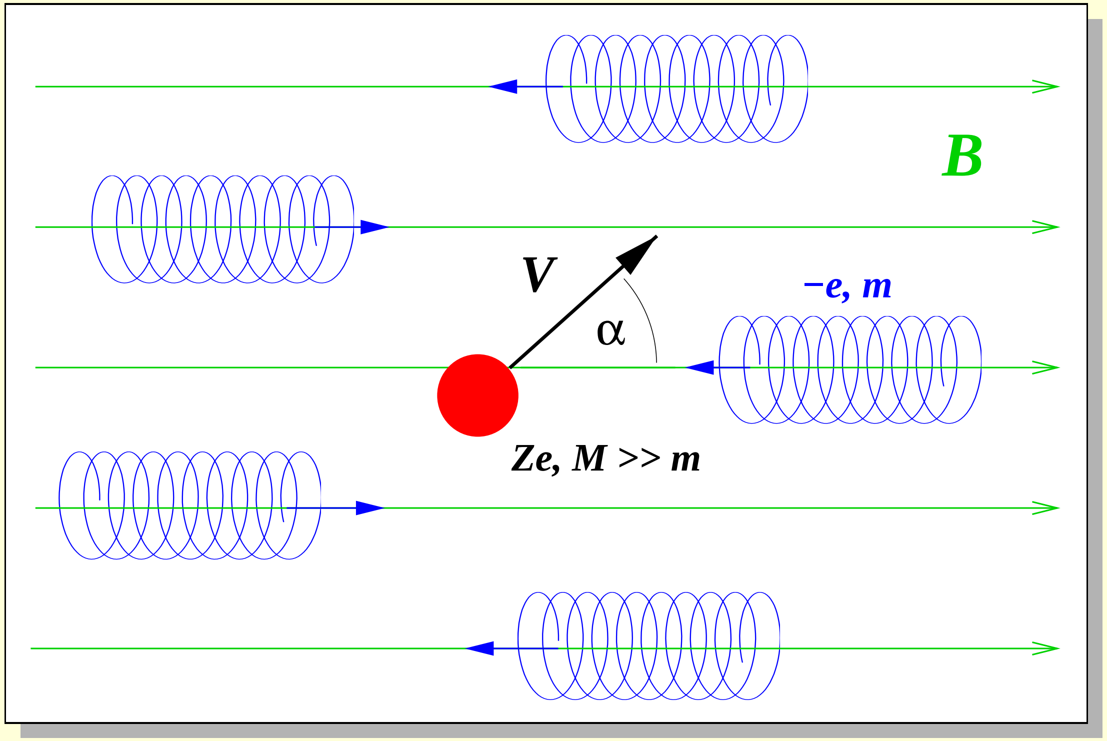
Typical parameters:

- $Z = -1, 1 \ldots 92$
- $B \lesssim 6$ T
- $n_e \approx 10^7$ cm^{-3}
- $T_e \approx 4$ K

Energy loss of ions by collisions with magnetized electrons

- Goal: $\frac{dE}{dt} = \vec{F}(\vec{V}, Z, n_e, T_e, B) \cdot \vec{V} = \boxed{\frac{dE}{ds}} |\vec{V}| \quad \rightarrow \; f_i(\vec{V}, t) \quad \rightarrow \; T_i(t)$

- Challenges:
  - Two-body problem is chaotic
  - High charge states of ions
  - Strong magnetic field
  - Electron-electron-interaction (collective effects)



▶ Requires different, complementary theoretical approaches:
  - Analytical: perturbation theory, linear response
  - Numerical simulations: CTMC,PIC,MD

- Models: binary collisions $\longleftrightarrow$ dielectric theory (stopping by medium polarization)

Hierarchy of Methods

Complete Dynamics, Ion-e^- and e^--e^-
Moleculardynamics Simulations (MD)

→ mean-field approach for e^--e^- interaction →

Vlasov-Poisson, Dielectric Theory
Particle-in-Cell/ Testparticles (PIC)

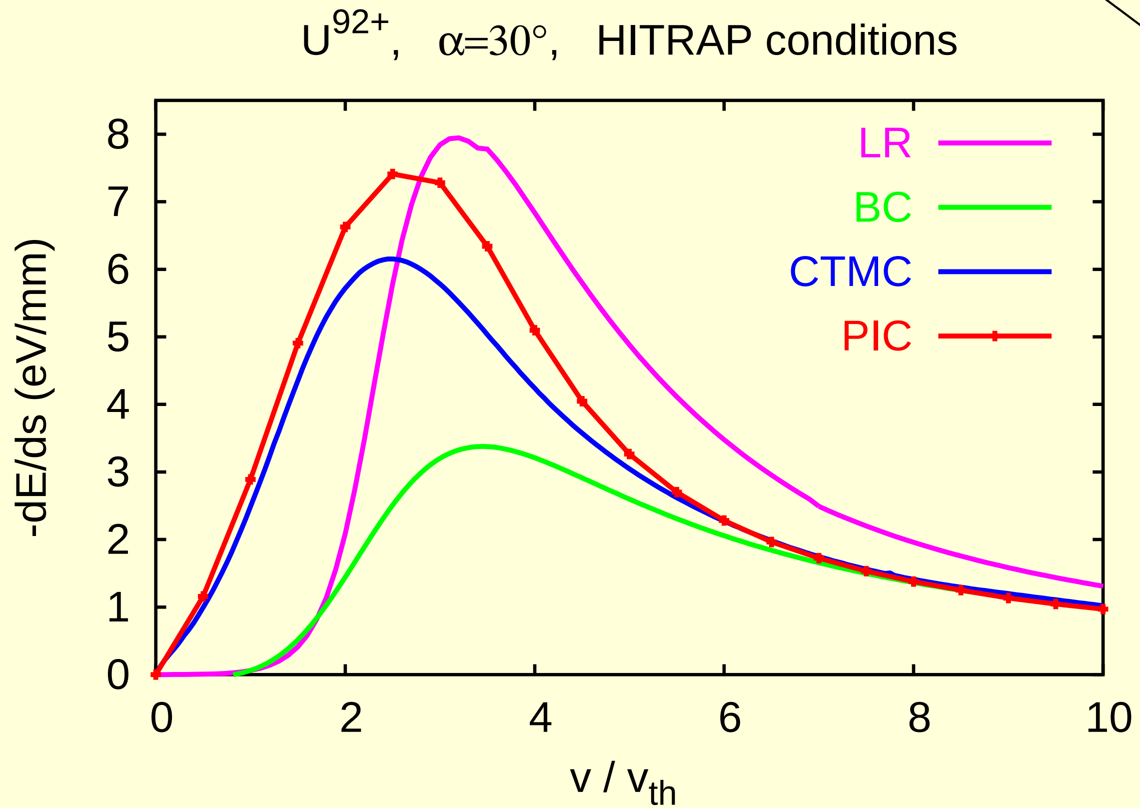
→ effective ϕ_{ie} for $-Ze^2/r$ →

Effective 2-body-dynamics
Classical trajectory Monte-Carlo (CTMC)

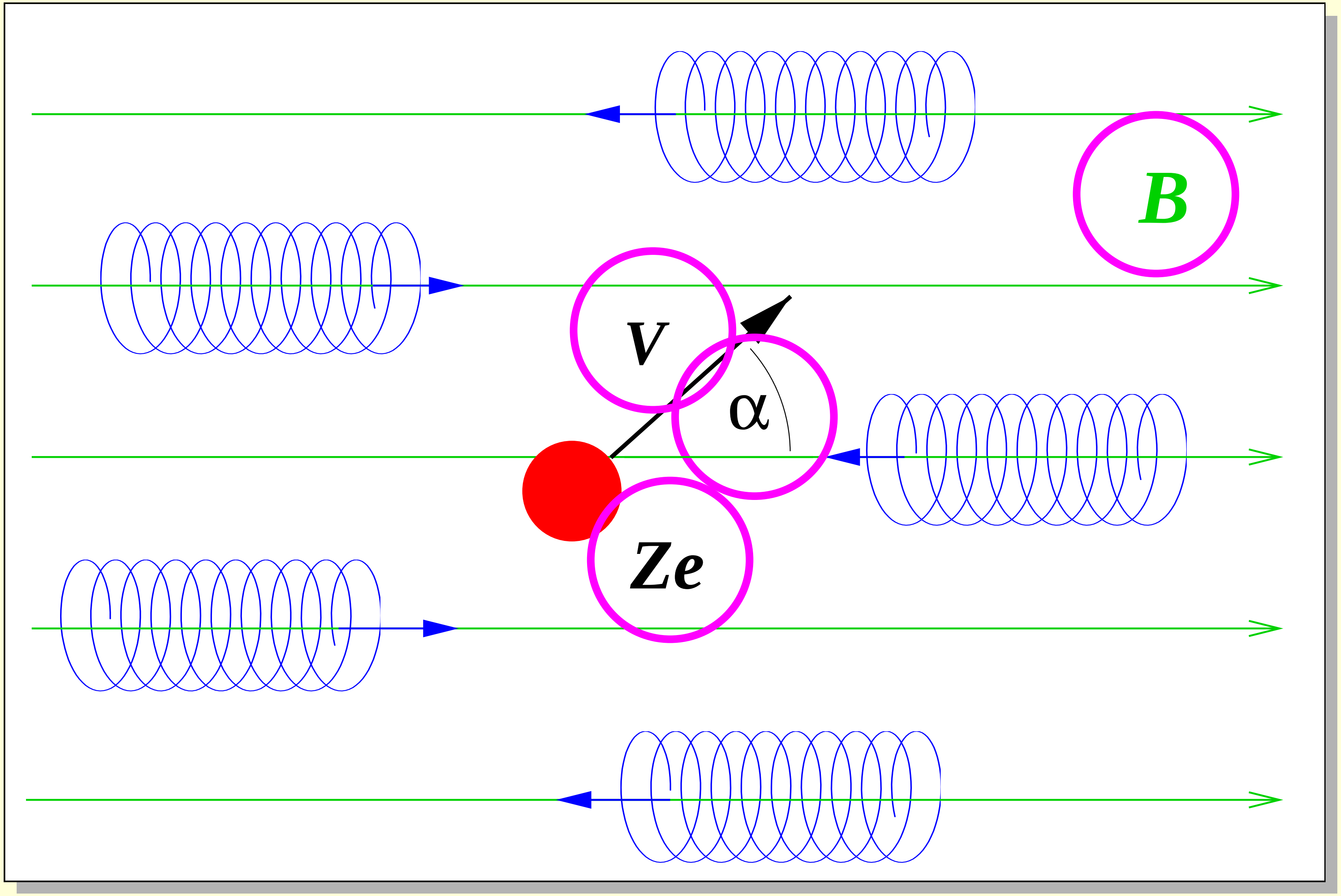
ϕ_{ie} as small perturbation ↓

Binary collisions in $O(Z^2)$ (BC)

Linear Response (LR) $O(Z^2)$

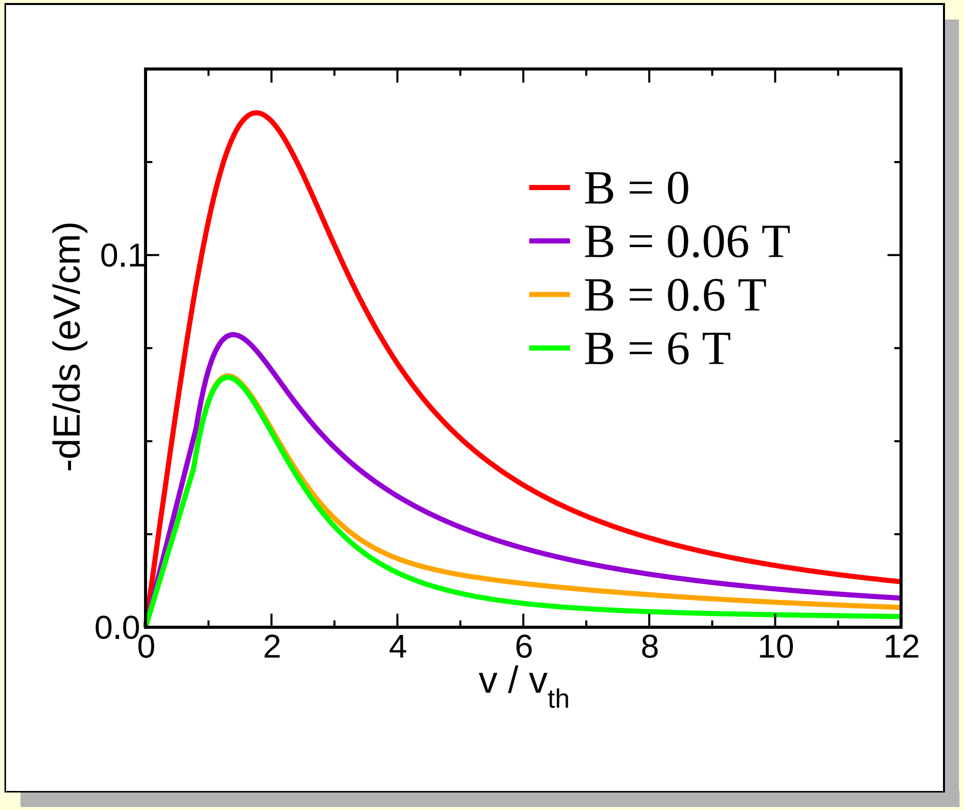


Relevant Parameters

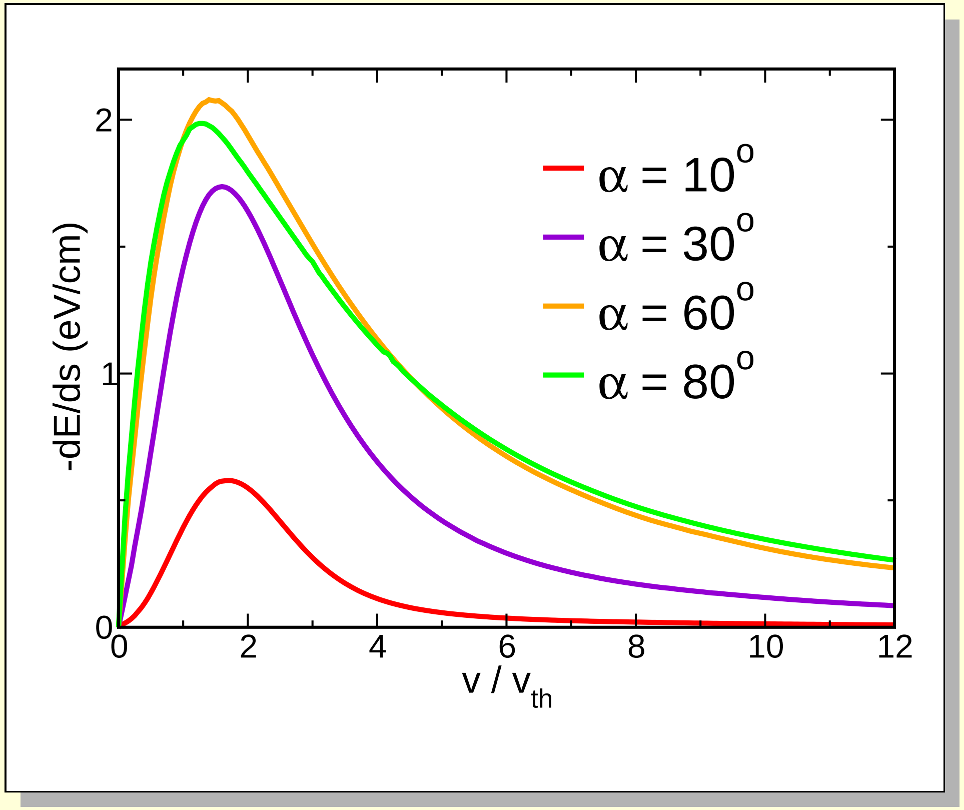


dE/ds: Typical behavior and dependencies

H$^{+}, \alpha = 30^{o}$, HITRAP-conditions



Ne$^{10+}, B = 6$ T, HITRAP-conditions



- Reduction of dE/ds with B
- Increase of cooling times?

- dE/ds, $\vec{F}$ strongly anisotropic
- High sensitivity to beam emittance?

Scaling with ion charge: $dE/ds \propto Z^x$ with $x < 2$ for high Z

- Friction coefficient:

$$R = \lim_{V \to 0} \frac{1}{V} \frac{dE}{ds}(V)$$

- Effective coupling:

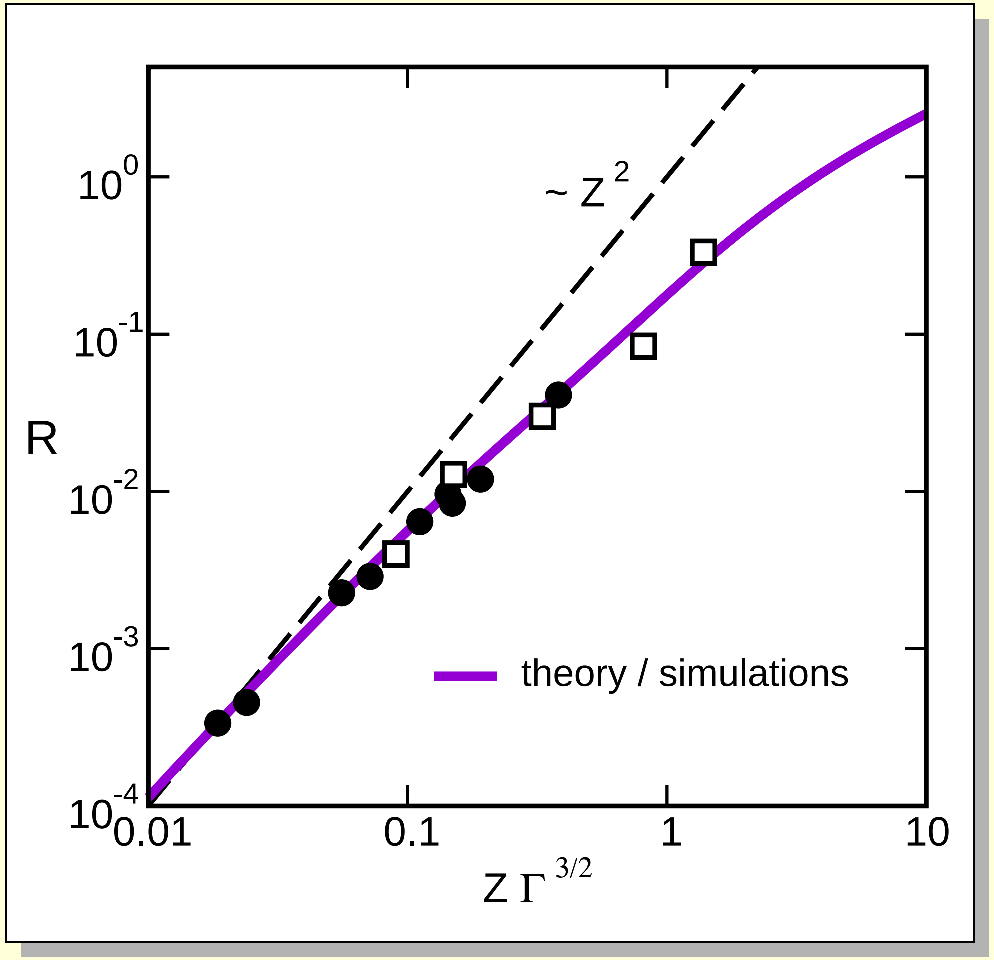
$$Z\Gamma^{3/2}, \qquad \Gamma = C\frac{n_e^{1/3}}{T_e}$$

- Comparison with cooling force measurements [2,3] for

  □ C^{6+}, Ne^{10+}, Ti^{22+}, Xe^{54+}, U^{92+} (ESR)

  • D^{+}, Li^{3+}, C^{6+}, O^{8+}, S^{16+} (TSR)

- More recent measurements at the TSR[4] confirmed this scaling



[2] Th. Winkler et al., Hyp.Int.99 (1996) 277.

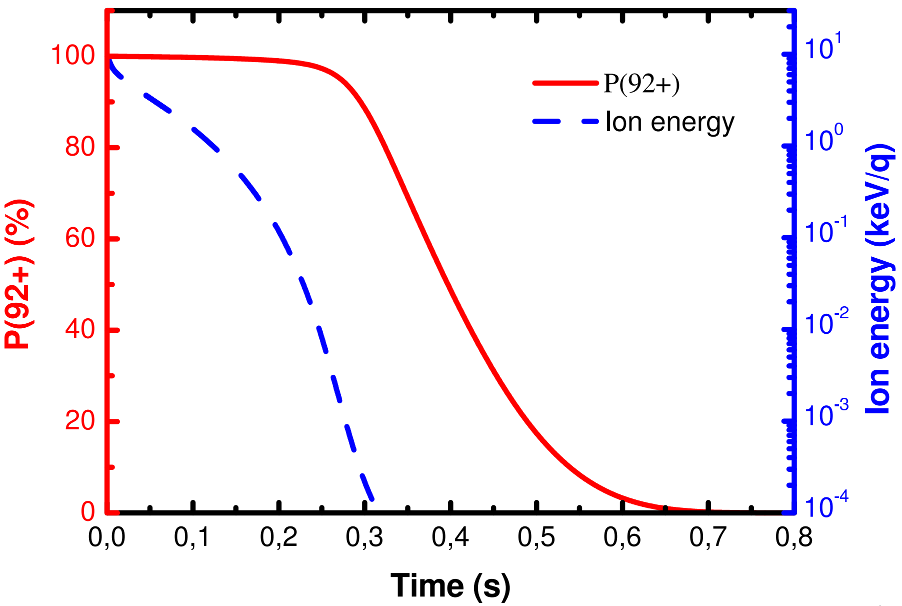
[3] A. Wolf et al., *Beam Cooling and Related Topics*, J.Bosser ed., CERN 94–03, Genf, 1994, p. 416.

[4] M. Beutelspacher, MPI H - V18 - 2000

Cooling times for U^{92+} in HITRAP

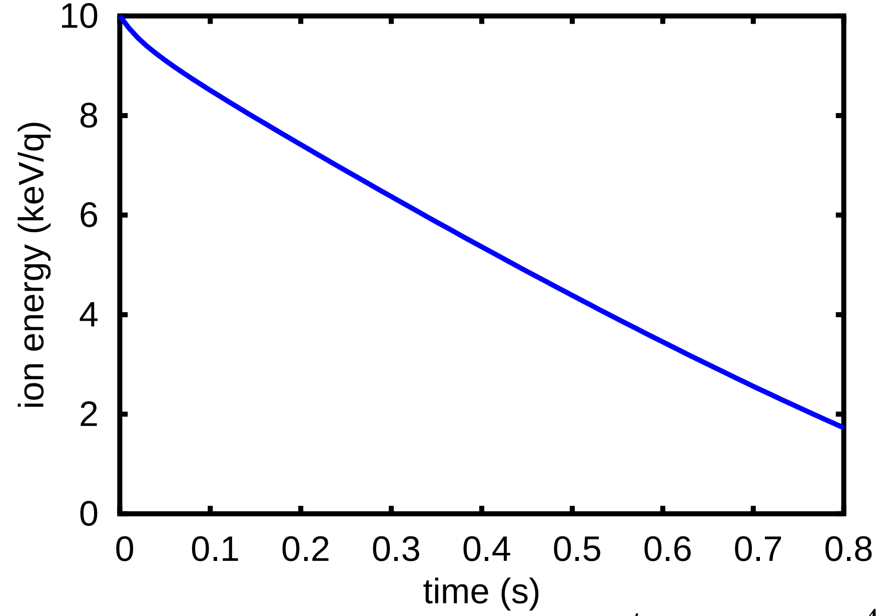
$$\frac{dE}{dt}(t) = M\vec{V}\cdot\frac{d\vec{V}}{dt} = \vec{V}\cdot\vec{F}[\vec{V}(t), n_e, T_e], \qquad \frac{dT_e}{dt} = -\frac{2}{3k_B}\frac{n_i}{n_e}\frac{dE}{dt}(t) - \frac{1}{\tau}(T_e - T_{res})$$

dE/dt from T_i–T_e-relaxation at B=0



J. Bernard et al. (GSI) $n_i/n_e = 10^{-4}$

dE/dt from $\vec{F}[\vec{V}]$ at B=6 T (preliminary)



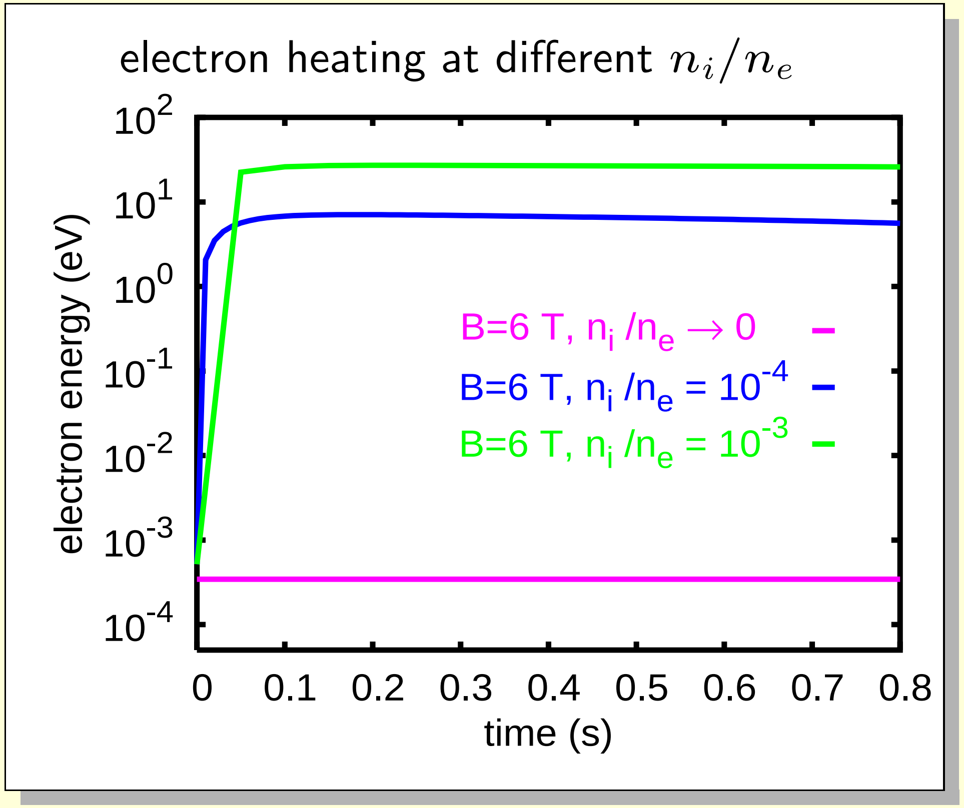
$n_i/n_e = 10^{-4}$

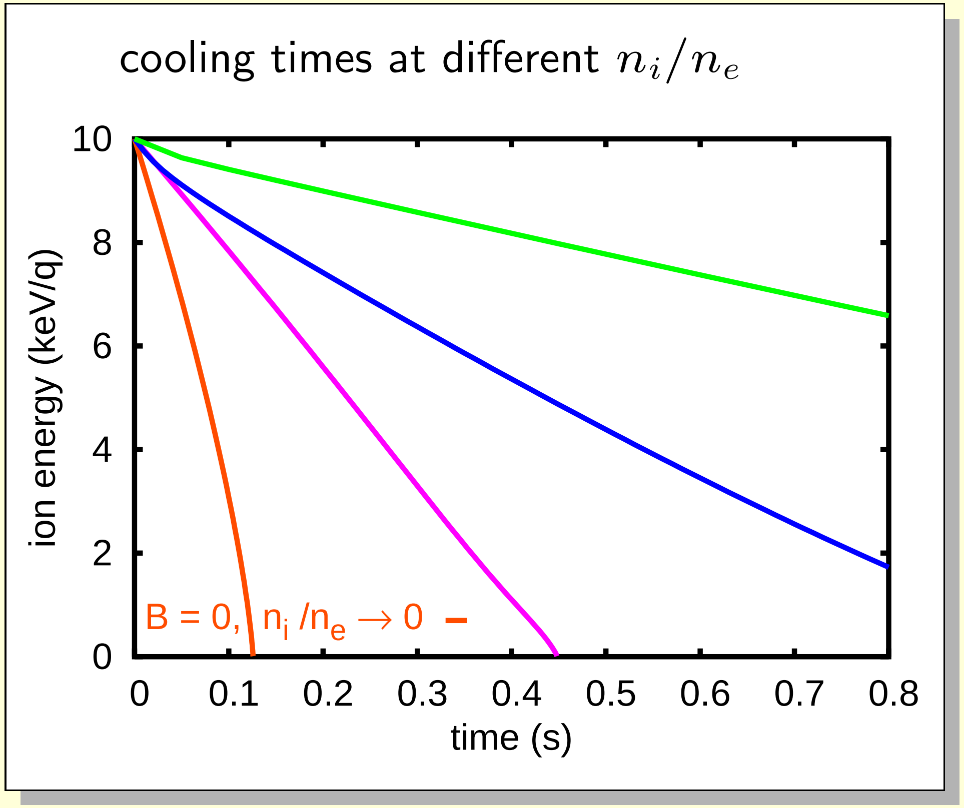
- recombination faster than cooling for large B?

▶ recombination has to be redone

Cooling times for U^{92+} in HITRAP (2)

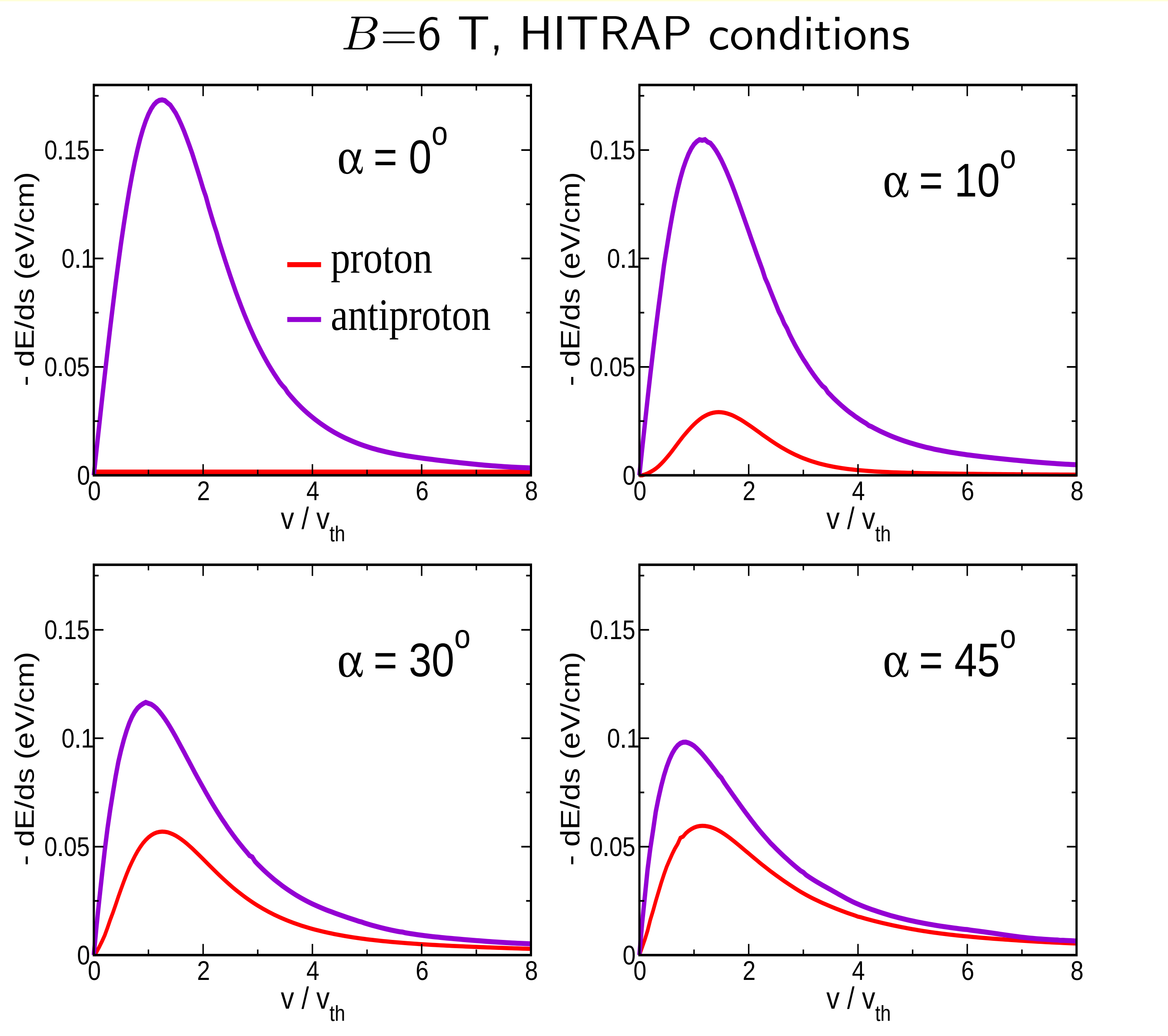
$$\frac{dT_e}{dt} = -\frac{2}{3k_B}\frac{n_i}{n_e}\frac{dE}{dt}(t) \; - \; \frac{1}{\tau}(T_e - T_{res}), \qquad \frac{dE}{dt}(t) = \vec{V}\cdot\vec{F}[\vec{V}(t), T_e]$$





▶ more studies needed; ultimate aim: time evolution of $f(\vec{V}, t)$ and $f(\vec{v}_e, t)$

Protons p versus antiprotons $\overline{p}$



- At large magnetic fields: $dE/ds(\overline{p}) > dE/ds(p)$

▶ Antiprotons are more efficiently cooled by electrons than protons

▶ Cooling of HCI with positrons is more efficient than electron cooling (at $n_{e^-} = n_{e^+}$)

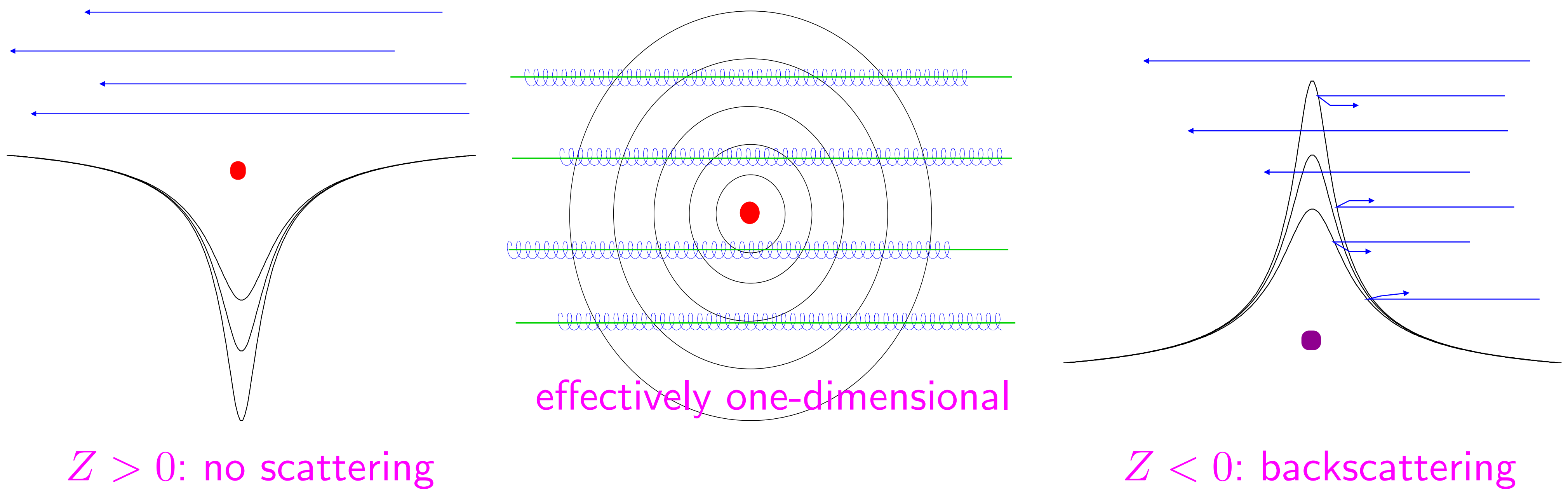
Summary and Outlook

- Energy loss of ions in magnetized electron plasmas
  - Reduction of dE/ds with increasing B, $dE/ds(\vec{V}), \vec{F}(\vec{V})$ highly anisotropic
  - Z-scaling $Z^x, x < 2$, for large Z
  - ▶ Cooling times are longer than expected by extrapolating from low to high Z, B
  - At large B: p/HCI ($\overline{p}$) are more efficiently cooled by positrons (electrons)

- Open questions and future tasks (for theory and experiment)
  - Cooling times and recombination rates at large B
  - Time evolution of ion and electron velocities/energies $[f(\vec{V}, t), f(\vec{v}_e, t)]$
  - ▶ Optimization of the cooling process
  - Positron cooling of HCI?

Protons p versus antiprotons $\overline{p}$ at large B

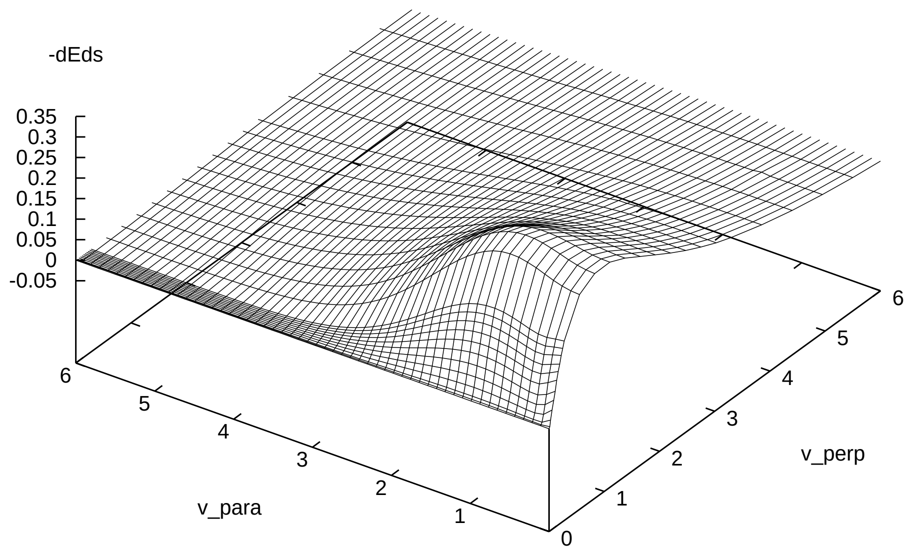
- At large magnetic fields the electrons move along **B** like beads on a wire.
- For positive ions (p) moving along **B** the drag vanishes for symmetry reasons.
- For negatively charged ions ($\overline{p}$) electrons are reflected.



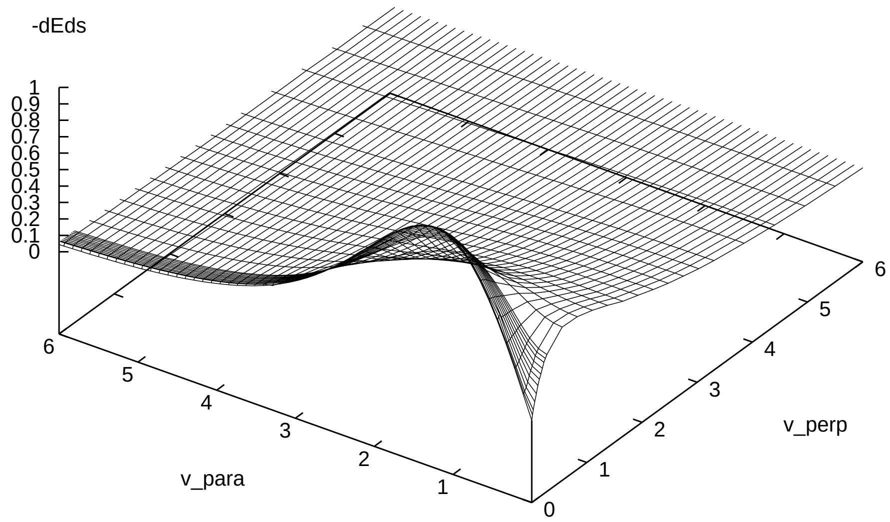
$\Rightarrow$ This cannot be accounted for in a perturbation treatment.

Protons p versus antiprotons $\bar{p}$

Proton



Antiproton



Binary ion-electron collisions in a homogeneous magnetic field

- Lagrangian in cm ($\dot{\mathbf{R}}_{cm} = \mathbf{V}_{cm}$) and relative ($\dot{\mathbf{r}}_r = \mathbf{v}_r = \mathbf{v}_e - \mathbf{v}_i$) coordinates

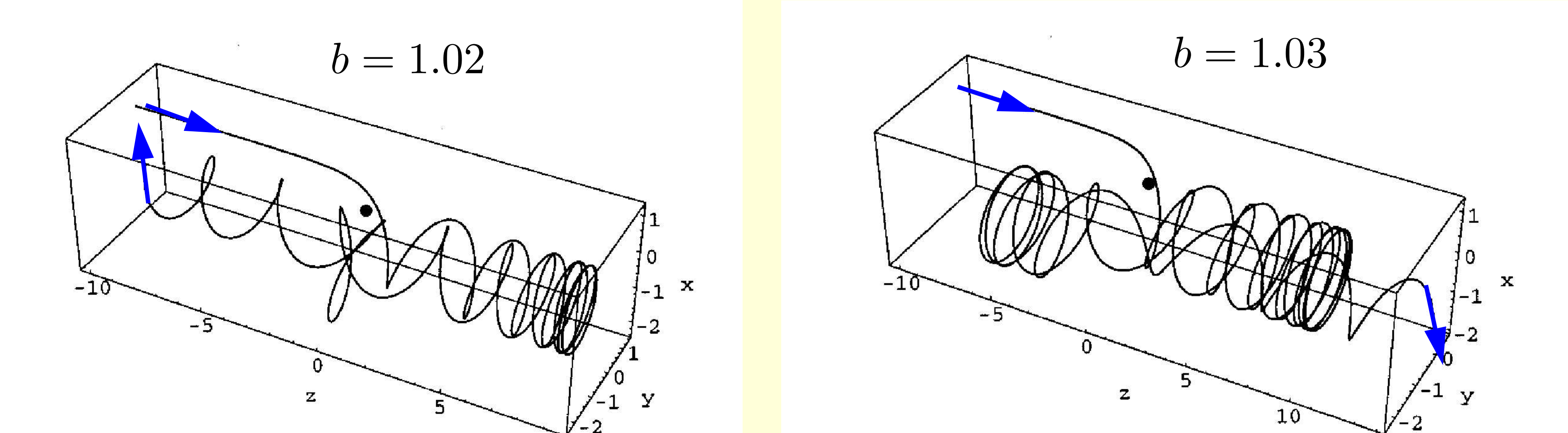
$$
\begin{aligned}
\mathcal{L} &= \frac{m+M}{2} V_{cm}^2 \quad + \frac{(Ze-e)}{2} (\mathbf{B} \times \mathbf{R}_{cm}) \cdot \mathbf{V}_{cm} \\
&+ \frac{\mu}{2} v_r^2 - \Phi(r_r) + \frac{\mu^2}{2} (\frac{Ze}{M^2} - \frac{e}{m^2}) (\mathbf{B} \times \mathbf{r}_r) \cdot \mathbf{v}_r \\
&- \frac{\mu}{2} (\frac{Ze}{M} + \frac{e}{m}) \{(\mathbf{B} \times \mathbf{R}_{cm}) \cdot \mathbf{v}_r + (\mathbf{B} \times \mathbf{r}_r) \cdot \mathbf{V}_{cm}\}
\end{aligned}
$$

- cm and relative motion is coupled, E_{cm} and E_r are not conserved separately

- for comparison: electron-electron scattering ($Z = -1, M = m$): $\mathcal{L}_{ee} = \mathcal{L}_{cm} + \mathcal{L}_r$

Binary collisions

- Ion-plasma interaction → Two-body-Problem with effective Φ_{ei}, e.g. $\frac{Ze^2}{r}\exp(-\frac{r}{\lambda})$
- Energy loss from $\Delta E, \Delta V$ of successive independent binary collisions

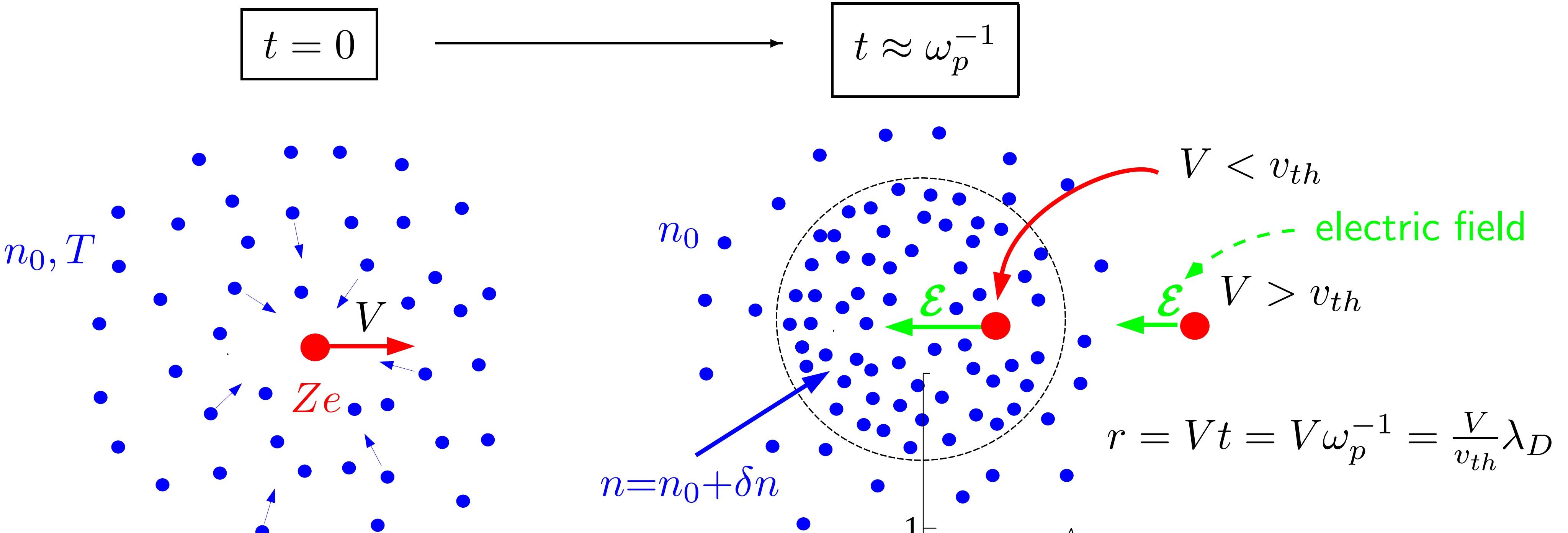


B. Hu et al., Phys.Plasmas **9**, 1116 (2002)

▶ Numerical treatment by Classical Trajectory Monte Carlo (CTMC)

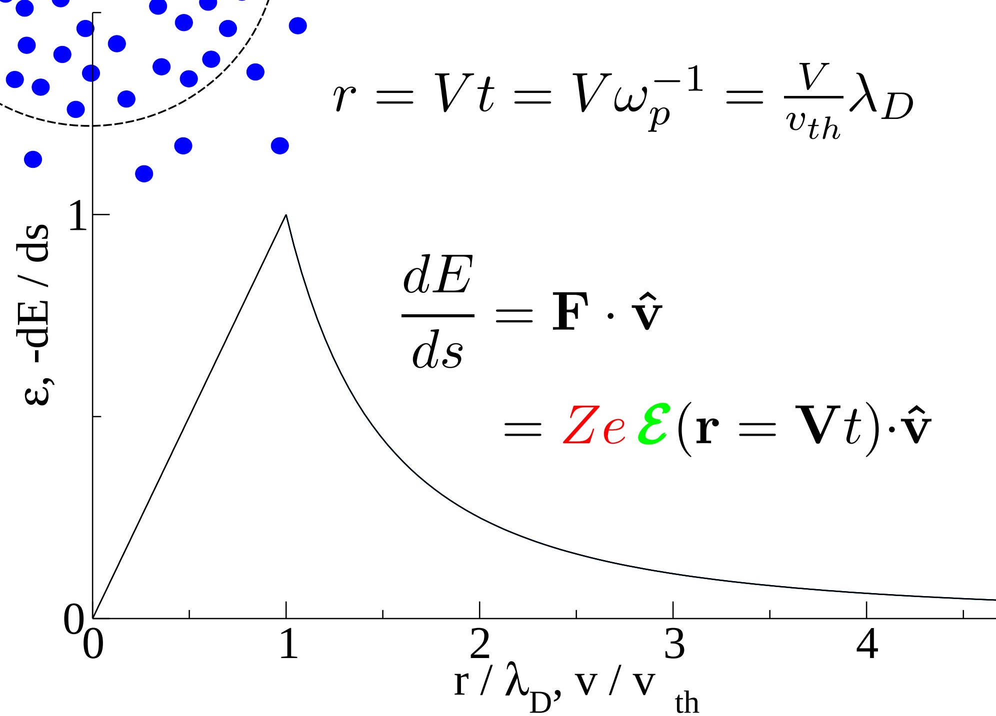
▶ Treating the ion as a small perturbation to the spiral motion of electons in B
→ Analytical expressions for the energy/momentum transfer → $\frac{dE}{ds} \propto Z^2$

Stopping by target polarization (1): simplified model



$$r = Vt = V\omega_p^{-1} = \frac{V}{v_{th}}\lambda_D$$

$$\delta\rho = e\,\delta n \approx \frac{Ze}{\frac{4\pi}{3}\lambda_D^3}\theta(\lambda_D - r)$$



$$\frac{dE}{ds} = \mathbf{F}\cdot\hat{\mathbf{v}}$$
$$= Ze\,\mathcal{E}(\mathbf{r} = \mathbf{V}t)\cdot\hat{\mathbf{v}}$$

typical scales:

- time: ω_p^{-1}
- length: λ_D
- velocity: $v_{th} = \lambda_D\omega_p = \sqrt{\frac{k_B T}{m}}$

dielectric theory, stopping by target polarization (2)

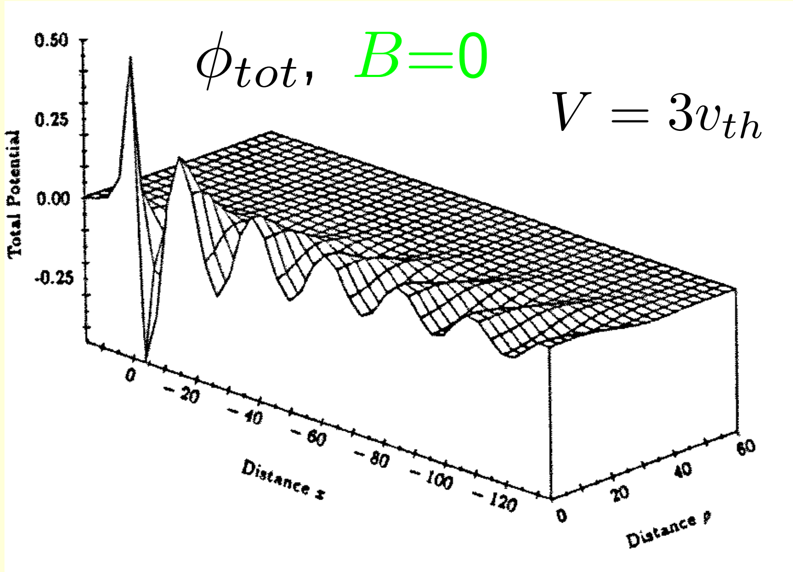
- General description in terms of the phase space distribution (kinetic theory, Vlasov-Poisson-eqs)

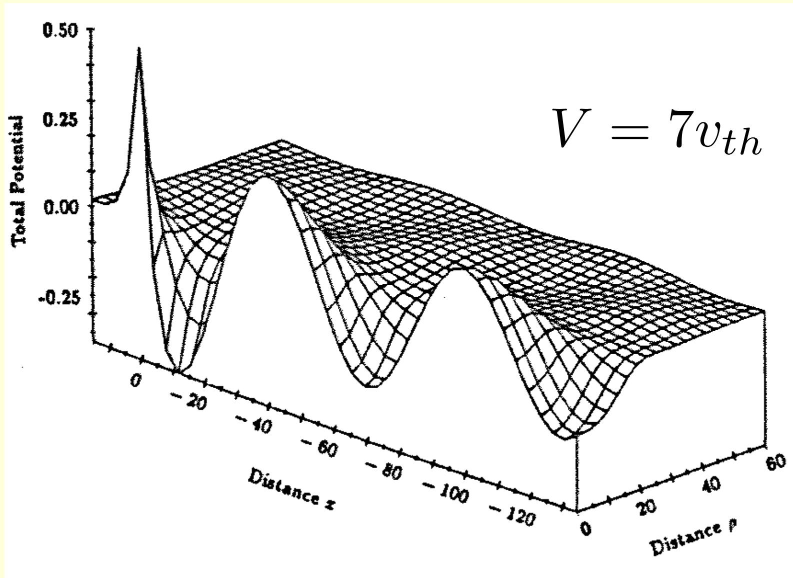
▶ Numerical treatment by
PIC (particle-in-cell)/testparticle simulations

▶ Linear response: $\phi_{\text{ind}} \propto \dfrac{Ze}{|\mathbf{r} - \mathbf{V}t|}$

$$\frac{dE}{ds} \propto Ze\nabla\phi_{\text{ind}} \propto Z^2 \int \frac{d^3k}{Vk^2} \,\mathrm{Im}\left[\frac{1}{\varepsilon(k, \omega = \mathbf{k}\cdot\mathbf{V})}\right]$$

- includes dynamic screening and the excitation of plasma waves
- fails for high Z and strong magnetic fields





T. Peter, MPQ 137 (1988)