CHANNELLING OF CHARGED PARTICLES THROUGH PERIODICALLY BENT CRYSTALS

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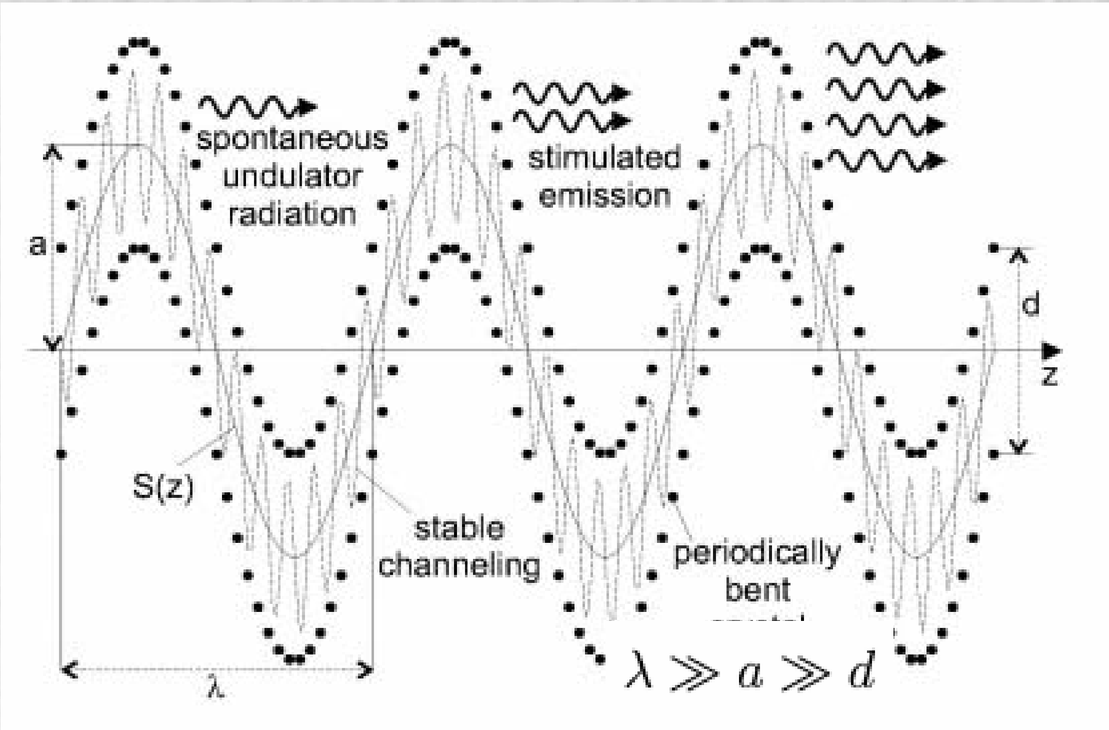
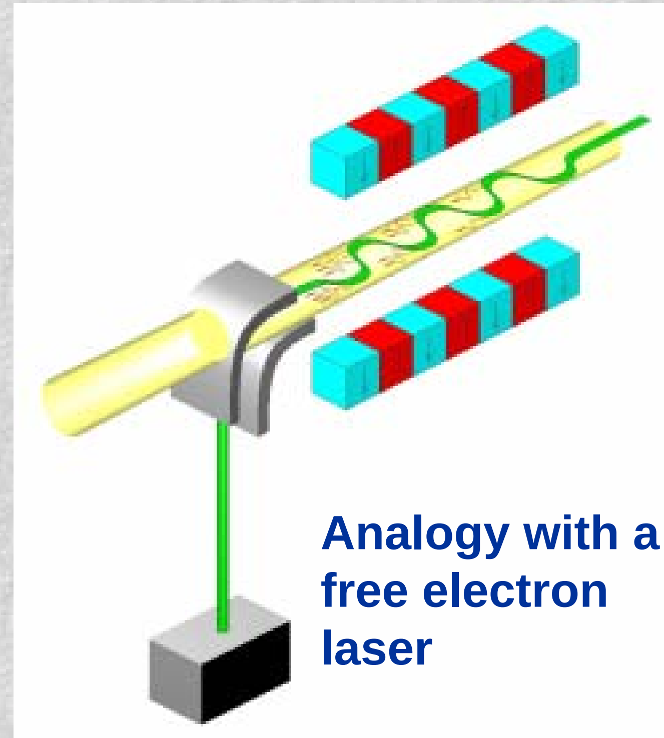
Basic idea





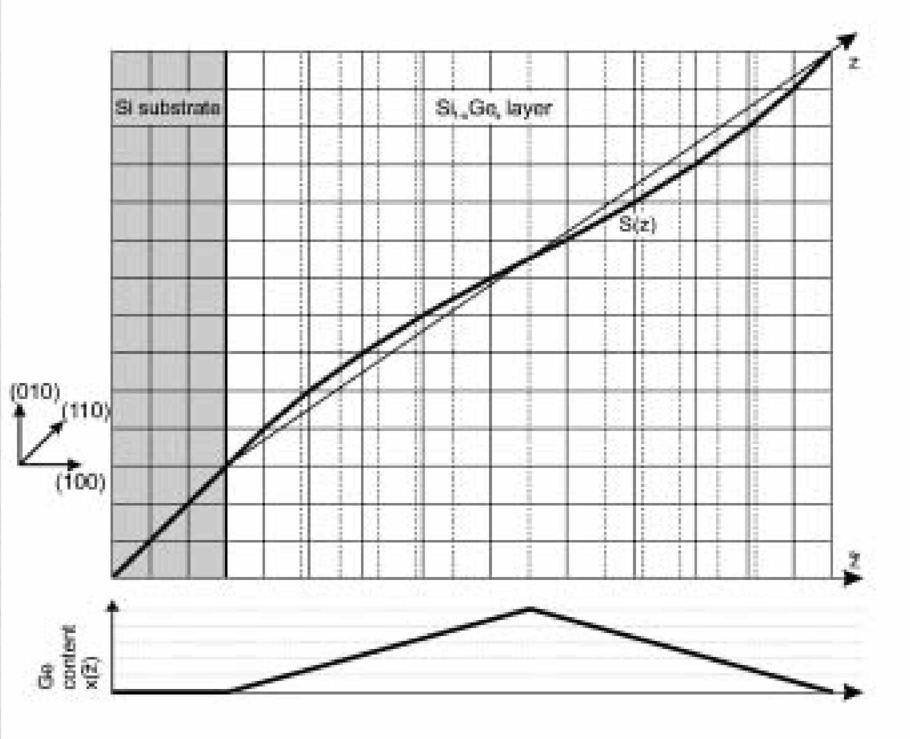
Scheme for the coherent and stimulated photon emission in a bent crystal, A.Korol, A.Solov‘yov, W.Greiner (1997)

Periodical bending of a crystal can be achieved by a transverse acoustic wave

The parameters of the photon spectrum can be varied by changing the beam energy, the type of a crystal, and the parameters of the shape function S(z).
It is meaningful to discuss photon stimulation for the photons ω = 1eV – 10 MeV

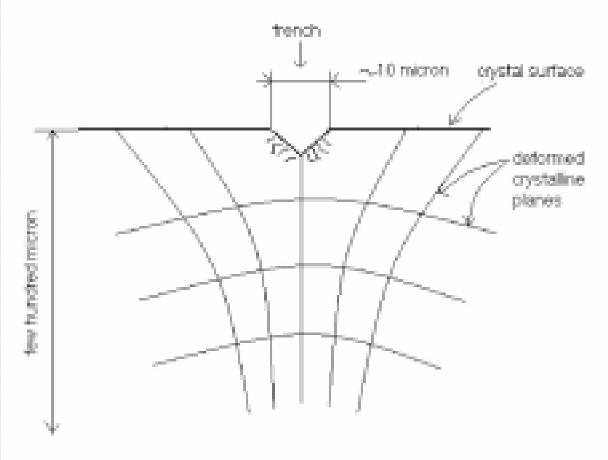
Other methods of making periodically bent crystals

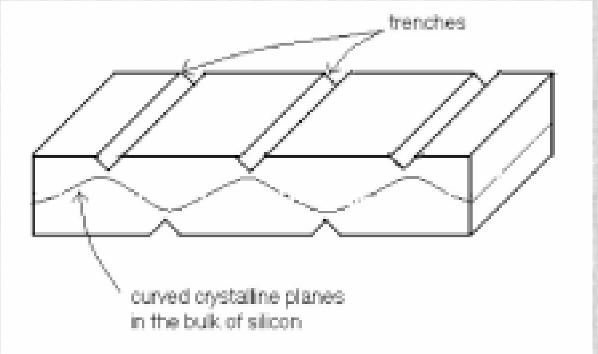
GROWING STRAINED GRYSTAL LAYERS



Strained layers in a superlattice result in a periodical bending of crystal channels, A.Korol, W.Krause, A.Solov‘yov, W.Greiner (2000)

MAKING REGULAR DEFECTS

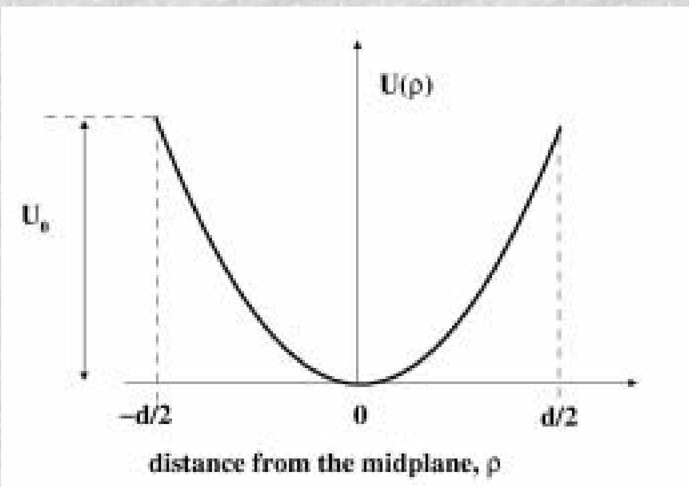




S.Belluci, S.Bini et al (2002)

Particles motion in a periodically bent crystal

Interplanar continuum potential in a linear crystal



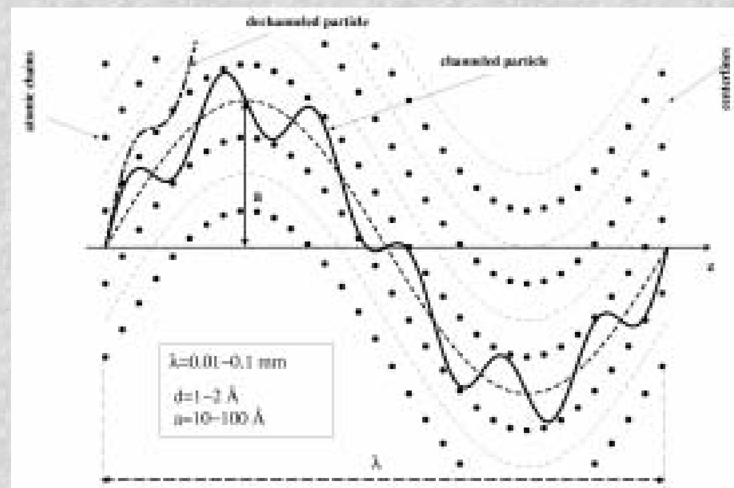
Typical figures: $d=10^{-8}$ cm

U_0=10...100 eV

max. gradient = 1...100 GeV/cm

$\theta < \theta_L = (2U_0/\varepsilon)^{1/2}$ – the Lindhard's angle

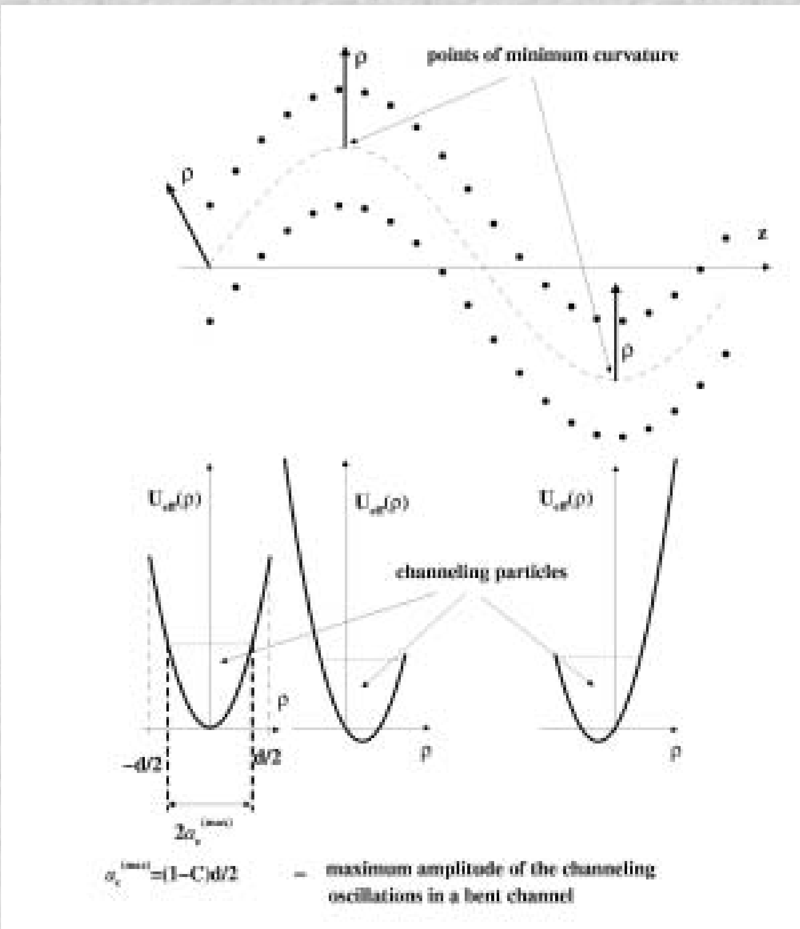
There are two types of the oscillatory motion in a bent crystal



$$\lambda \gg a \gg d$$

In a bent crystal particles experience the action of additional centrifugal force

Condition for channelling in a bent crystal



Additional centrifugal force in a bent channel must be smaller than the interplaner potential force

$$m\gamma v^2/R_{\min} < q\,U'_{\max}$$

Thus, the channelling condition reads:

$$C \equiv \frac{\varepsilon}{U'_{\max} R_{\min}} < 1$$

If the channel centerline is defined as

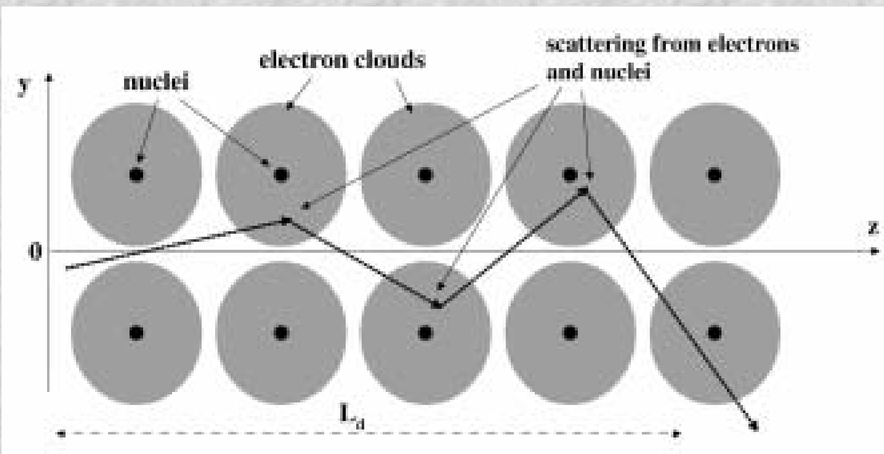
$$y(z) = a \cdot \sin\left(2\pi(z/\lambda)\right)$$

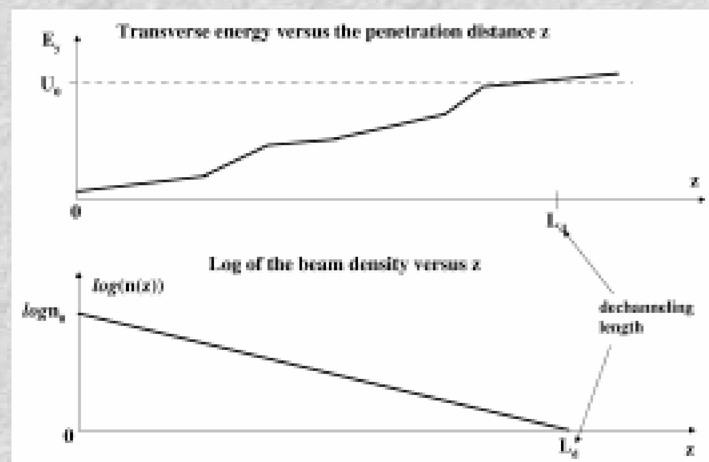
The minimum curvature radius is equal to

$$R_{\min} = (\lambda/2\pi)^2/a$$

Dechannelling process

The length L of the crystalline undulator should be smaller than the dechannelling length L_d (γ, R)





The volume density of channelled particles decreases with the penetration distance and roughly satisfies the exponential decay low

$$n(z) = n_0 \exp\left(-z/L_d(\gamma, R)\right)$$

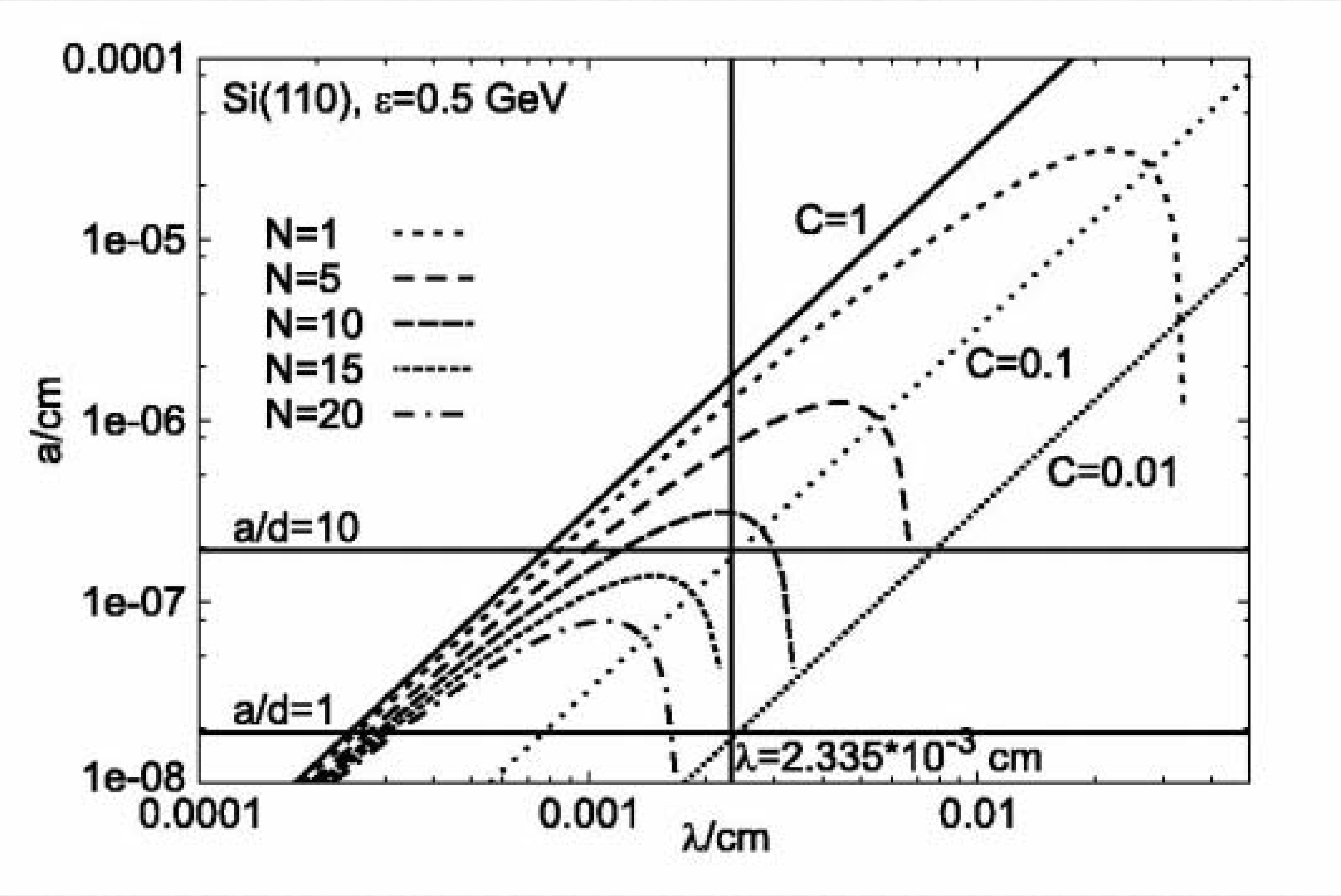
$$L_d^{\varepsilon}(\bar{R}) = \left(1 - R_c/\bar{R}\right)^2 L_d^{\varepsilon}(\infty), \qquad L_d^{\varepsilon}(\infty) = \frac{256}{9\pi^2}\frac{a_{\mathrm{TF}}}{r_0}\frac{d}{L_c}\gamma$$

$$R_c = \varepsilon/U'_{\max}$$

$$L_c = \ln\left(\sqrt{2\gamma}\, mc^2/I\right) - 23/24$$

$$I = 16 Z^{0.9}\ \mathrm{eV}$$

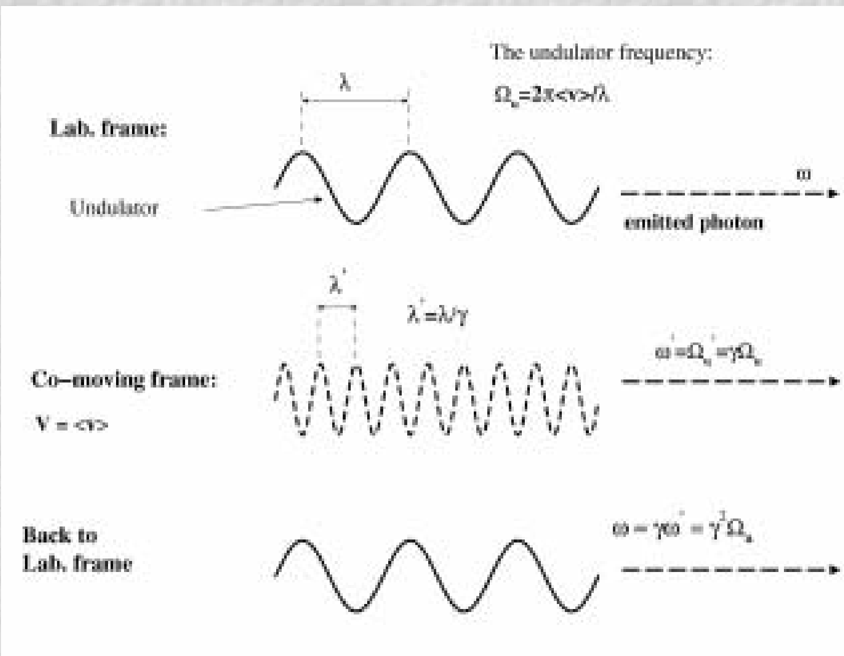
Parameter ranges



The ranges of the allowed parameters of the AW, a versus λ, consistent with the channelling condition and L < L_d (γ, R) for $\varepsilon = 0.5$ Gev ($\gamma = 10^3$) positron channeling in Si along (110) crystallographic plane.

Characteristic frequencies

Relationship between the undulator frequency and the frequency of emitted photons



Characteristic oscillatory frequencies

$$\Omega_{ch} \sim \sqrt{\frac{U'}{d\, m \gamma}}, \qquad \Omega_u = \frac{2\pi c}{\lambda}$$

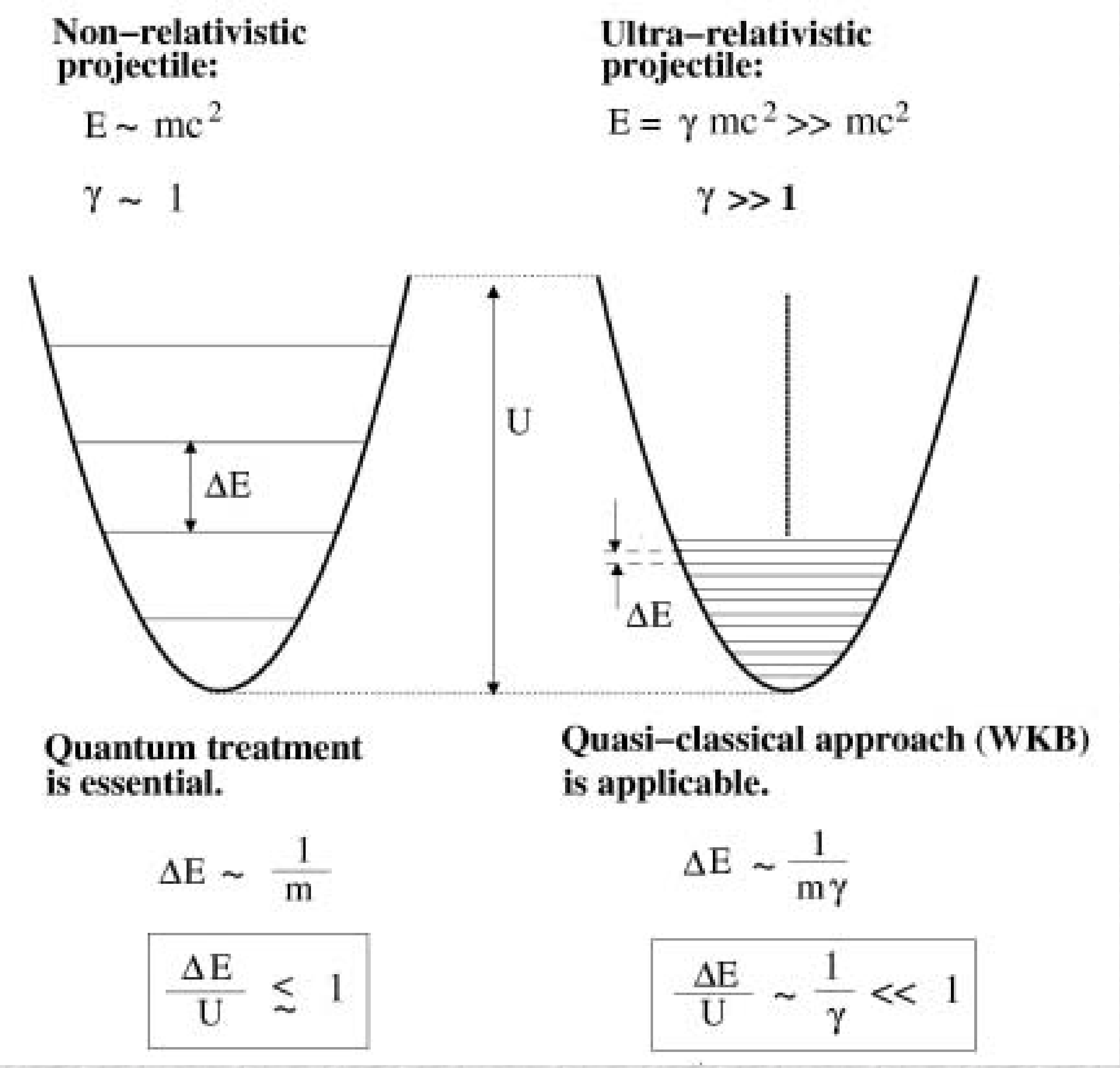
Characteristic frequencies of the emitted channelling and undulator radiation

$$\omega_{ch} \sim \gamma^2\, \Omega_{ch}, \qquad \omega_u \sim \gamma^2\, \Omega_u$$

Ratio of the frequencies

$$\frac{\Omega_u^2}{\Omega_{ch}^2} \sim \frac{\omega_u^2}{\omega_{ch}^2} \sim \frac{d}{a} \frac{\varepsilon}{R_{\min}\, U'_{\max}} = \frac{d}{a} C \ll 1$$

Quasiclassical approximation



With increasing γ or m the quasiclassical approach becomes more adequate

Spectral and angular distribution of spontaneous radiation

$$\frac{\mathrm{d}E_\omega(\mathbf{n})}{\mathrm{d}\omega\mathrm{d}\Omega_{\mathbf{n}}} = \hbar\alpha\frac{q^2\,\omega^2}{4\pi^2}\int_0^T \mathrm{d}t_1 \int_0^T \mathrm{d}t_2\; \mathrm{e}^{\mathrm{i}\omega'\varphi(t_1,t_2)}\; f(t_1,t_2)$$

$$\varphi(t_1,t_2) = t_1 - t_2 - \frac{1}{c}\,\mathbf{n}\cdot(\mathbf{r}_1 - \mathbf{r}_2)\ ,$$

$$f(t_1,t_2) = \frac{1}{2}\Big\{\big(1+(1+u)^2\big)\Big(\frac{\mathbf{v}_1\mathbf{v}_2}{c^2} - 1\Big) + \frac{u^2}{\gamma^2}\Big\}$$

$$\omega \longrightarrow \omega' = \frac{\varepsilon}{\varepsilon - \hbar\omega}\,\omega, \qquad u = \frac{\hbar\omega}{\varepsilon - \hbar\omega}$$

$$\frac{\mathrm{d}E}{\mathrm{d}\omega\,\mathrm{d}\Omega_{\mathbf{n}}} = \hbar\,\alpha\,\frac{\omega^2}{4\pi^2}\,\Big\{\Big|\,I_1 - \vartheta\,\cos\varphi\, I_0\Big|^2 + \vartheta^2\,\sin^2\varphi\Big|\,I_0\,\Big|^2\Big\}$$

$$I_0 = \int_0^T \mathrm{d}t\; \exp\left(\mathrm{i}\omega\Phi(t)\right), \qquad I_1 = \int_0^T \mathrm{d}t\,\frac{\dot{y}(t)}{c}\,\exp\left(\mathrm{i}\omega\Phi(t)\right)$$

$$\Phi(t) = t\,\left(\frac{1}{2\gamma^2} + \frac{\vartheta^2}{2} + \frac{p^2}{4\gamma^2}\right) + \frac{\Delta(t)}{2} - \vartheta\,\cos\varphi\,\frac{y(t)}{c}$$

$$\Delta(t) = \int^t \mathrm{d}t'\left(\frac{\dot{y}^2(t')}{c^2} - \frac{p^2}{2\gamma^2}\right),$$

$$p^2 = 2\gamma^2\,\frac{\overline{\dot{y}^2}}{c^2}$$

The quasiclassical formalism originally developed by Baier, Katkov, Strahovenko (1967) has been used for the calculation of spectral and angular distribution of spontaneous radiation.

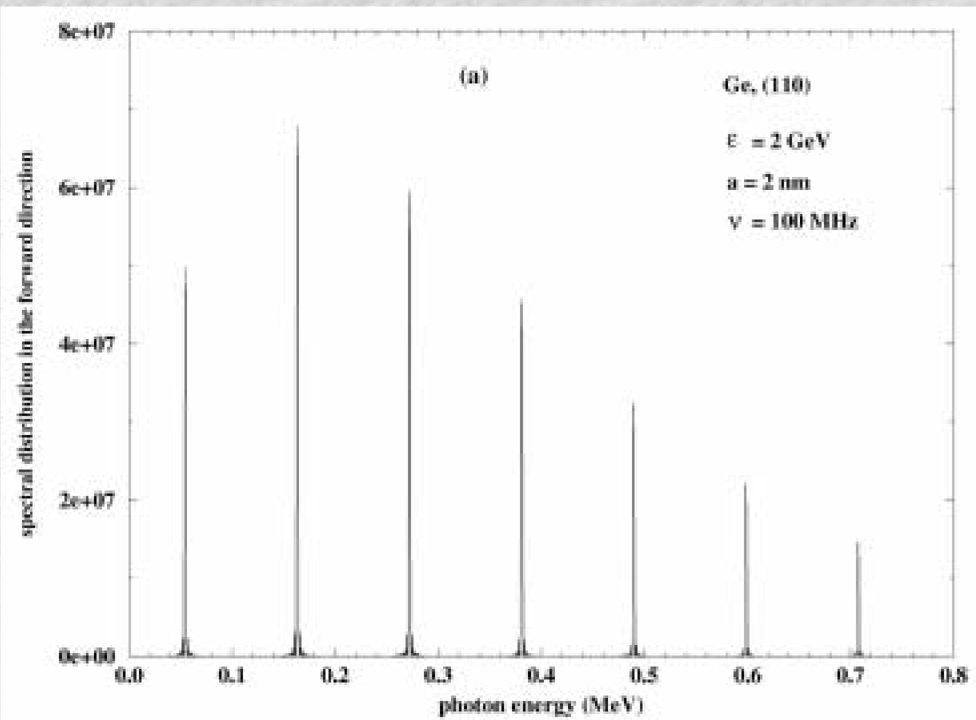
p is undulator parameter
Dipole regime: p << 1
Non-dipole regime: p >> 1

Spectral and angular distribution of spontaneous radiation

$$\frac{\mathrm{d}E_N}{\mathrm{d}\omega\,\mathrm{d}\Omega_{\mathbf{n}}} = D_N(\eta)\,F(\omega;\gamma,p;\theta,\varphi)$$

$$D_N(\eta) = \left(\frac{\sin N\pi\eta}{\sin \pi\eta}\right)^2$$

$$\eta = \frac{\omega}{\Omega_u}\left(\frac{1}{2\gamma^2} + \frac{\vartheta^2}{2} + \frac{p^2}{4\gamma^2}\right)$$



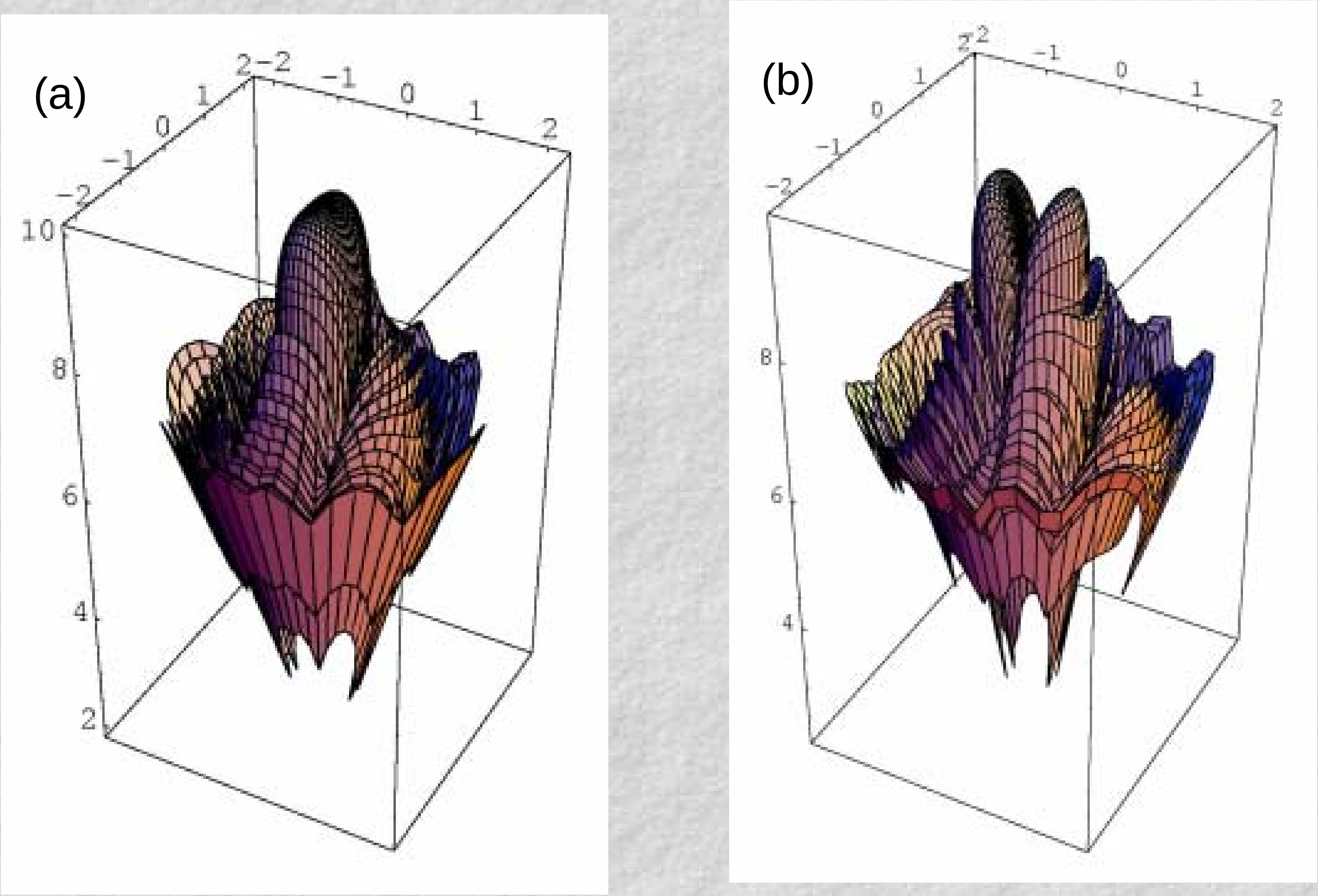
Here, $F(\omega;\gamma,p;\theta,\varphi)$ is a smooth function of the arguments

$$\omega^{(K)} = \frac{4\gamma^2\Omega_u\,K}{2 + 2\theta^2\gamma^2 + p^2}, \quad K = 1, 2, \ldots$$

The characteristic frequencies (harmonics) of the undulator radiation

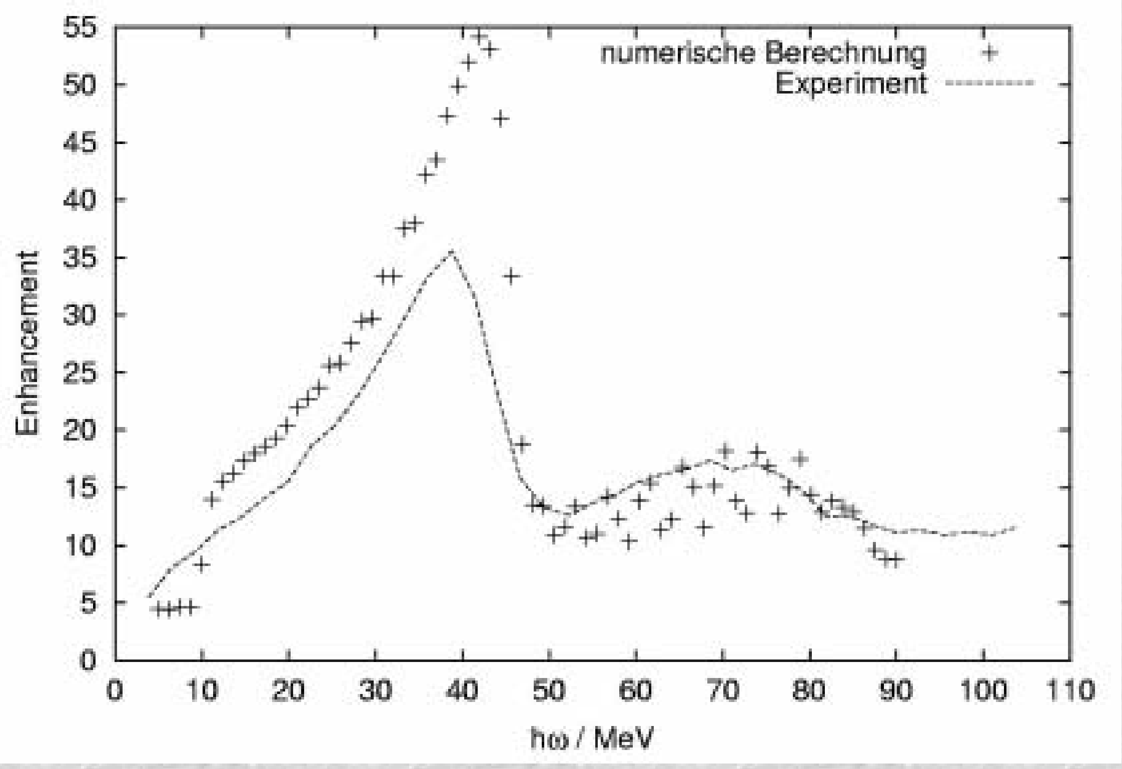
Typical pattern of the spectral-angular distribution of the radiation formed in the crystalline undulator by positrons, $\theta = 0^{o}$

Angular distribution of photons



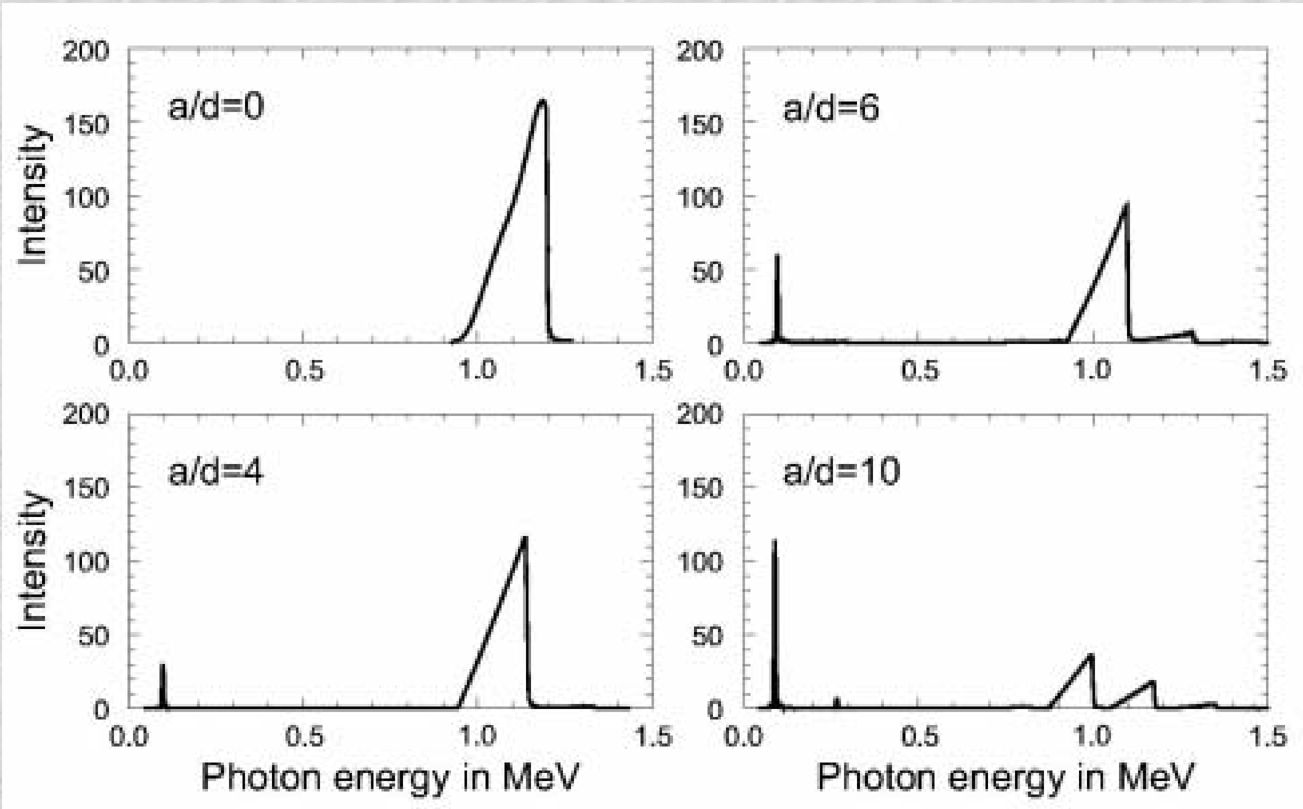
Spatial distribution of the radiation emitted in: (a) the odd harmonic with K=7, (b) the even harmonic with K=6 by 50 GeV positron channelling in a diamond C,(110), A=2.35nm, ν=20MHz

Channelling radiation: comparison with the experiment



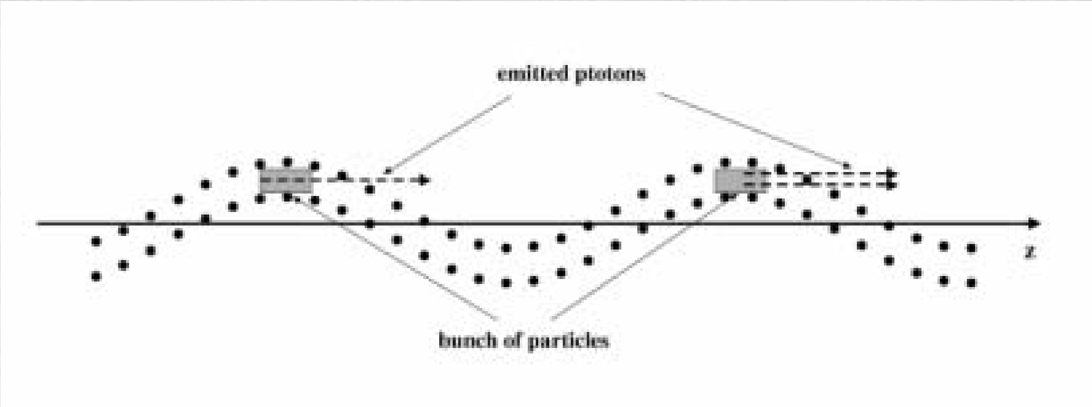
Comparison of the experimentally measured spectrum (J. Bak, J.A. Ellison, E.Uggerhoj et al (1985)) and the results of calculation (A.Korol, W.Krause, A.V.Solov'yov, W.Greiner (2001)) for 6.7 GeV positrons in Si(110).

Total spectral distribution



Spectral distribution of the total radiation emitted in the forward direction for ε=0.5GeV ($\gamma = 10^3$) positron channelling in Si along (110) crystallographic planes calculated at different a/d ratios as indicated. The AW frequency is fixed at ν=200 MHz, the crystal length is L=3.5×10^{-2}cm.

Stimulated photon emission



$$\sigma_{\rm em} = 2\pi \lambda_{ph}^2 \left[\frac{{\rm d}E(\mathbf{n})}{{\rm d}\omega\, {\rm d}\Omega_{\mathbf{n}}}\right]_{\varepsilon} \Delta\omega\, \Delta\Omega_{\mathbf{n}}$$

$$\sigma_{\rm ab} = 2\pi \lambda_{ph}^2 \left[\frac{{\rm d}E(\mathbf{n})}{{\rm d}\omega\, {\rm d}\Omega_{\mathbf{n}}}\right]_{\varepsilon+\hbar\omega} \Delta\omega\, \Delta\Omega_{\mathbf{n}}$$

Mechanism of the stimulated radiation in the crystalline undulator

$$ {\rm d}N_{ph} = g(\omega)\, N_{ph}\, {\rm d}z $$

The gain factor defines the increase or the decrease per cm in the total number of the emitted photons

The general expression for the gain reads as

$$g(\omega) = n\, \left[\sigma_{\rm em}(\varepsilon, \varepsilon - \hbar\omega) - \sigma_{\rm ab}(\varepsilon, \varepsilon + \hbar\omega)\right]$$

The gain factor

Assuming $\hbar\omega \ll \varepsilon$, one gets the following expression for the gain:

$$g(\omega) = -(2\pi)^3 \frac{c^2}{\omega} n \frac{\mathrm{d}}{\mathrm{d}\varepsilon} \left[\frac{\mathrm{d}E(\mathbf{n})}{\mathrm{d}\omega\,\mathrm{d}\Omega_{\mathbf{n}}} \Delta\omega\,\Delta\Omega_{\mathbf{n}} \right]_{\theta=0}$$

Substituting explicit expressions for all quantities, after transformations, one derives

$$g(\omega) \approx (2\pi)^3 \frac{Z^2}{M} r_{\mathrm{cl}} \frac{N^2\lambda}{\gamma^3} n \begin{cases} p^2 & \text{for } p^2 < 1 \\ 1 & \text{for } p^2 > 1 \end{cases}$$

Here, $r_{cl} = e^2/m_e c^2 = 2.8\times10^{-13}$cm is the classical radius of the electron, Z an M are the charge and the mass of the projectile measured in the units of m_e

Estimate of the gain for a proton beam

$\nu \sim 10^3$ MHz, $\lambda \sim 3\times10^{-4}$ cm, $a \sim 3\times10^{-7}$cm

$L \sim 3$cm, $N_u \sim 10^4$, $a/\lambda \sim 10^{-3}$, $p \sim 6\times10^{-3}\,\gamma$

$g(\mathrm{cm}^{-1}) \sim 4\times 10^{-14}\, n/\gamma$

These estimates demonstrate that for $\gamma \sim 4$ ($\varepsilon \sim 4$Gev) and $n > 10^{14}$ cm^{-3} it is meaningful to discuss the stimulated photon emission

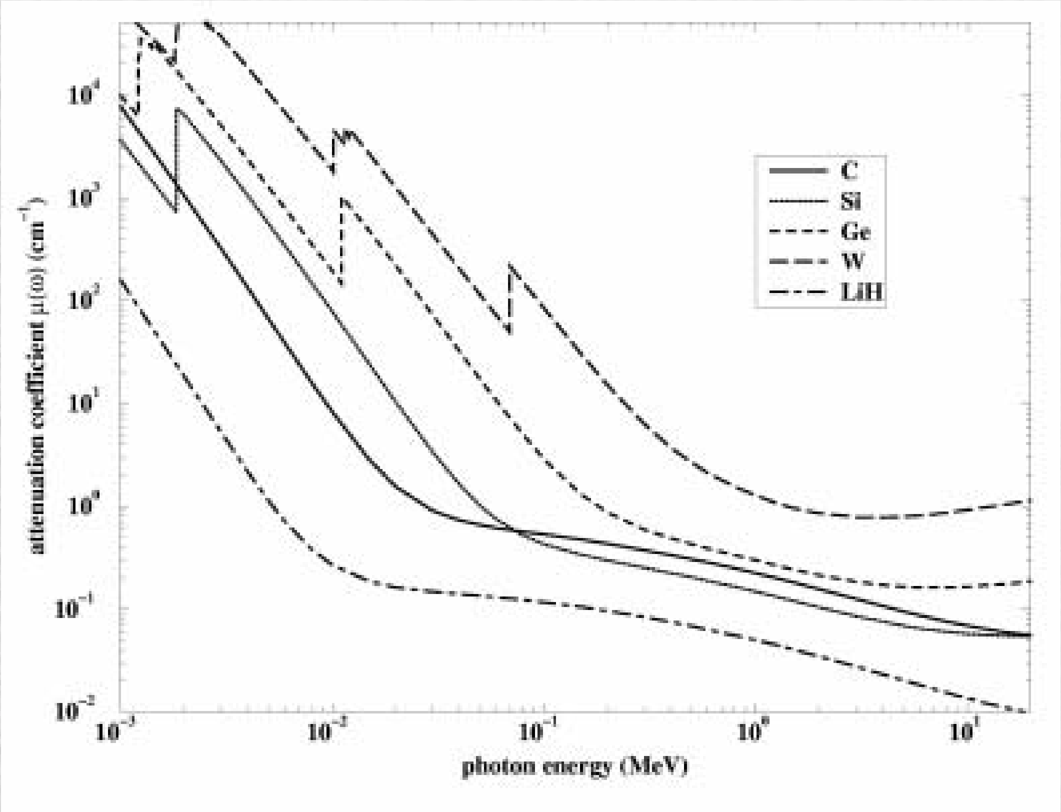
One needs to increase the density of a beam in order to have the stimulated photon emission at higher energies

Conclusions

- A crystal, which is periodically bent either by a transverse AW or by any other method can be used for the construction of an undulator for a beam of ultra-relativistic particles channeled in the lattice
- This undulator can be used for the generation of the radiation of high energy photons
- The parameters of this undulator can be tuned by varying the AW amplitude and frequency, the energy of the projectile and by using different types of crystals and its channels
- It is meaningful to discuss the possibility to create a powerful source of a free electron laser type stimulated radiation in the energy range of eV to the MeV region
- Both light and heavy particles can be used for the effect considered
- Experimental efforts for the verification of the theory predictions are needed

THE END

Photon attenuation

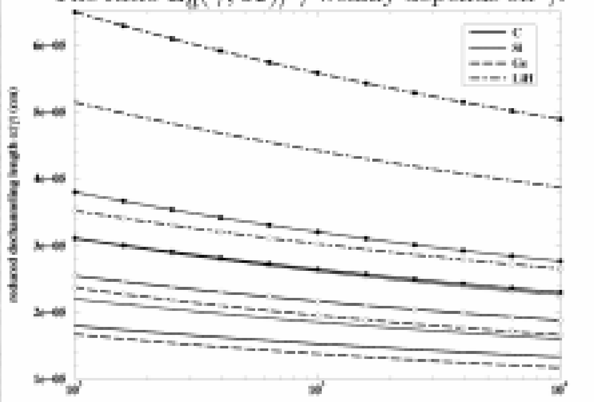


The attenuation coefficient $\mu(\omega)$ for various crystals.

Dechanneling lengths

Values of $L_d(\gamma,\infty)$ (**in cm**) for a **positron** with $\varepsilon = 5$ GeV					
Channel	C	Si	Ge	W	LiH
(100)	0.13	0.16	0.13	0.12	0.39
(110)	0.19	0.23	0.19	0.17	0.27
(111)	0.23	0.28	0.23		0.49

The ratio $L_d(\gamma,\infty)/\gamma$ weakly depends on γ.



Dependence of $\alpha = L_d(\gamma,\infty)/\gamma$ on γ of a **positron** channelled in various planar channels of C, Si, W and LiH.

$$g \approx (2\pi)^3\, r_{\mathrm{cl}}\, \frac{L^2}{\lambda\gamma^3}\, n \begin{cases} p^2 & \text{for } p^2 < 1 \\ 1 & \text{for } p^2 > 1 \end{cases}$$

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