Towards tests of QED effects in heavy ions

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Outline of the talk

- Lamb shifts
- Hyperfine splitting
- Bound-electron g factor

Highly charged ions

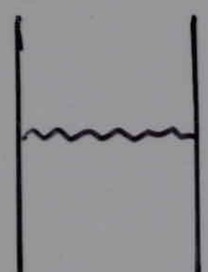
$N << Z$

N is the number of electrons

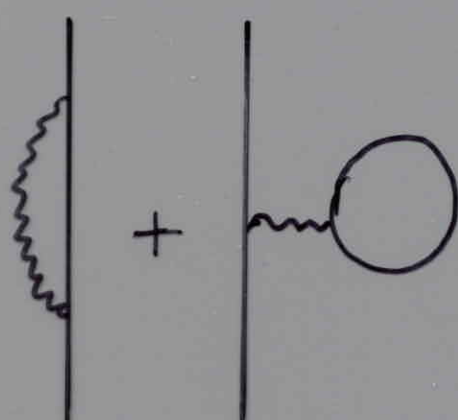
The zeroth-order approximation

$$(c\vec{\alpha}\cdot\vec{p} + \beta mc^2 + V_c)\psi_{njem} = E_{nj}\psi_{njem}$$

Perturbation theory in $\alpha \approx \frac{1}{137}$ and in



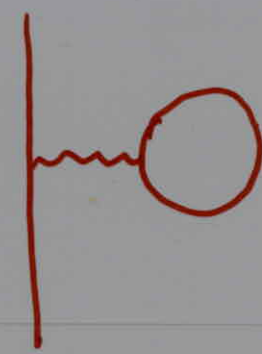
$$\frac{\text{Interelectronic interaction}}{\text{Binding energy}} \sim \frac{1}{Z}$$



$$\frac{\text{Rad. corrections}}{\text{Binding energy}} \sim \alpha(\alpha$$

QED corrections of first order in α

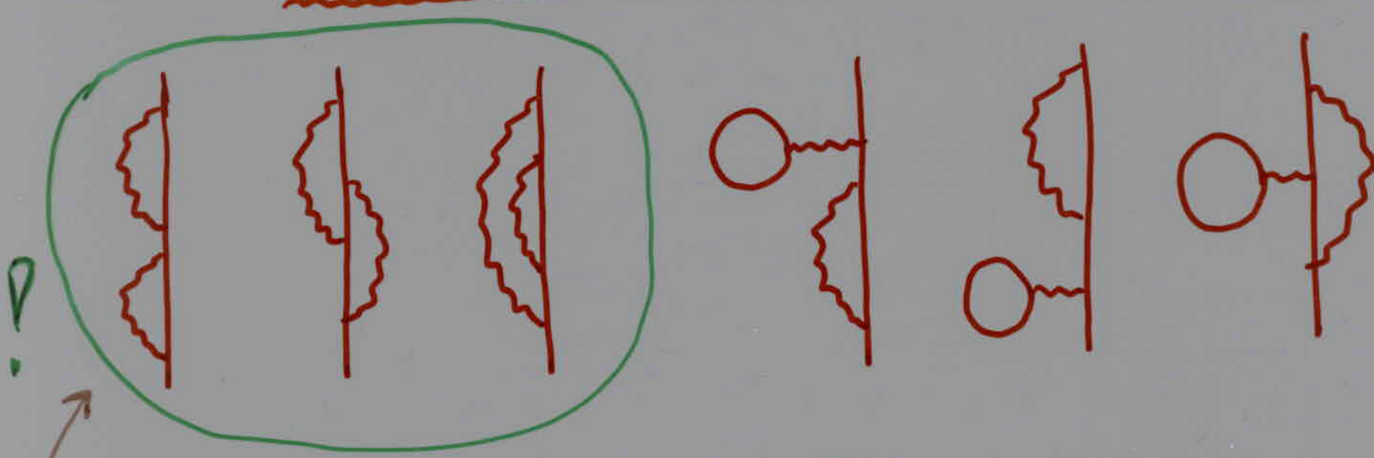
Self energy | Vacuum polarization

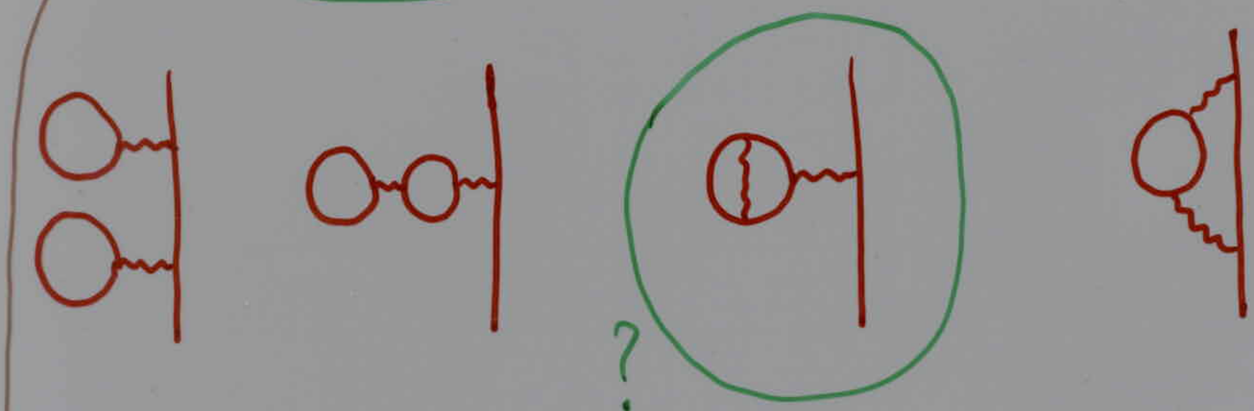


Mohr (1974)

Soff and Mohr (1988)
Manakov et al. (1989)

QED corrections of second order in α

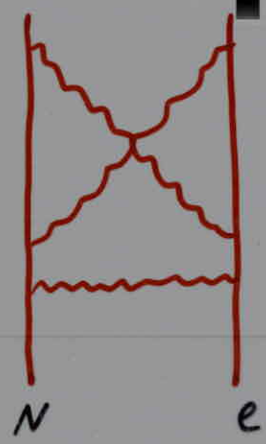




[Mallampalli and Sapirstein (1998)

Nuclear recoil correction

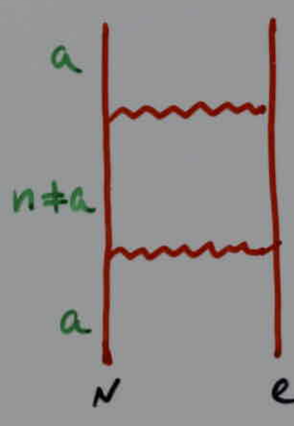
(QED to first order in $\frac{m}{M}$)

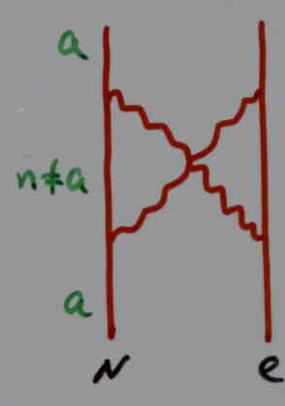


Theory: Shabaev (1985)

Calculation: Artemyev, Shabaev, Yerokhin (1995)

Nuclear polarization



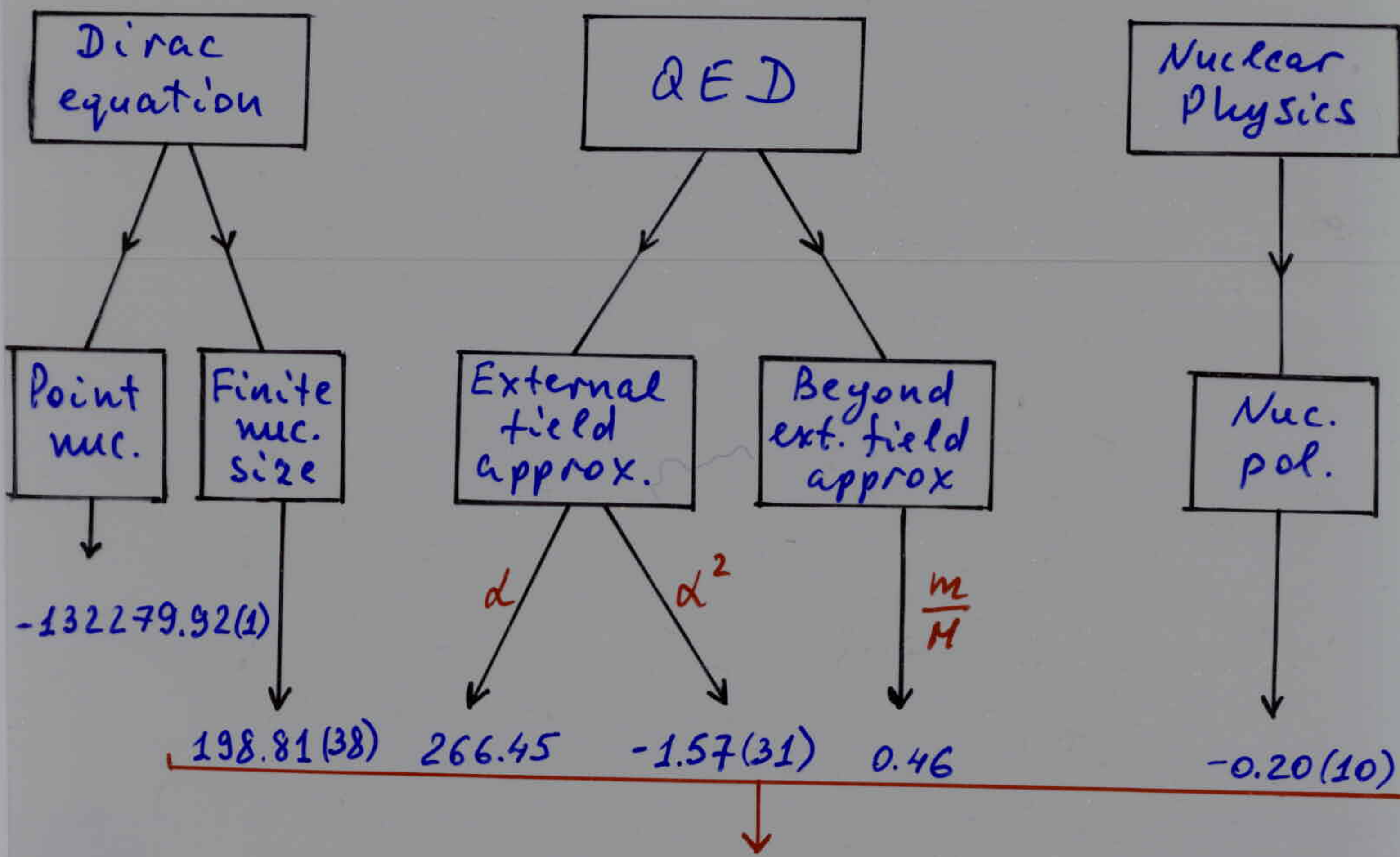


Plunien and Soff (1996)

Nefiodov, Labzowsky, Plunien, and Soff (1996)

The 1S Lamb shift in $^{238}U^{91+}$, in eV

Theory



463.95(50) eV [1]

Experiment: 468(13) eV [2]

470(16) eV [3]

[1] Yerokhin and Shabaev, 2001.
[2] Stöhlker et al., 2000.
[3] Beyer et al., 1995.

Lamb shift in Li-like ions

Experiments:

Schweppe et al. (1991)

$2p_{\frac{1}{2}} - 2s$ in U^{89+}

Precision ~ 0.2% of QED

Beiersdorfer et al. (1998)

$2p_{\frac{3}{2}} - 2s$ in Bi^{80+}

Precision ~ 0.15% of QED

Bosselmann et al. (1999)

$2p_{\frac{1}{2}} - 2s$ in Ag^{44+}, Xe^{51+}

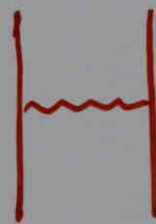
Precision ~ 0.2% of QED

Brandau et al. (2001)

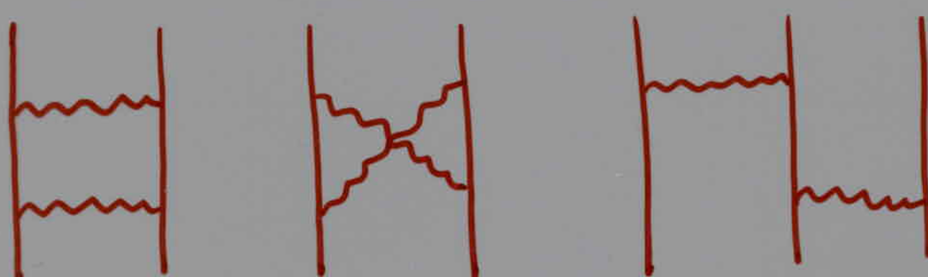
$2p_{\frac{1}{2}} - 2s$ in Au^{76+}, Pb^{79+}, Bi^{80+}, U^{89+}

Accurate QED calculations

First order in α



Second order in α

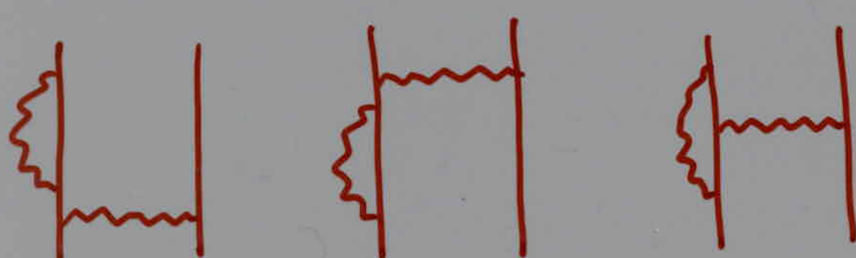


Yerokhin etal

(2000)

Andreev et al.

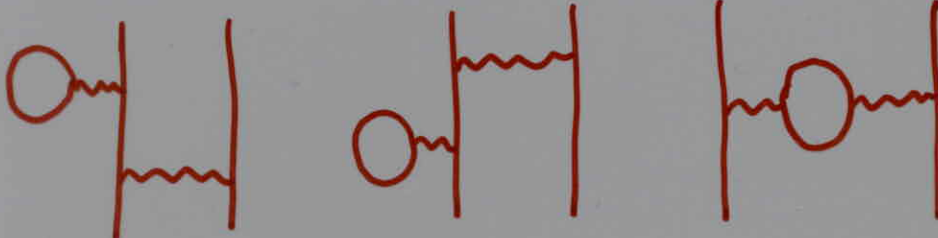
(2001)



Yerokhin et al.

(1999)

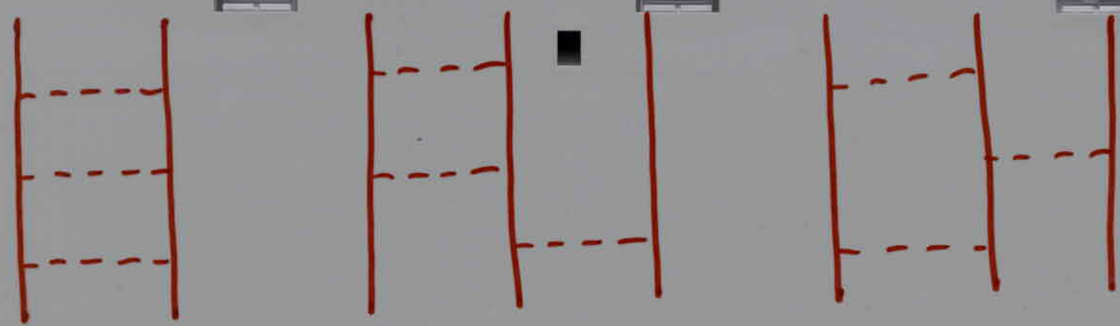
Sapirstein and Cheng (2001)



Artemyev etal

(1999)

Third order in $\frac{1}{Z}$



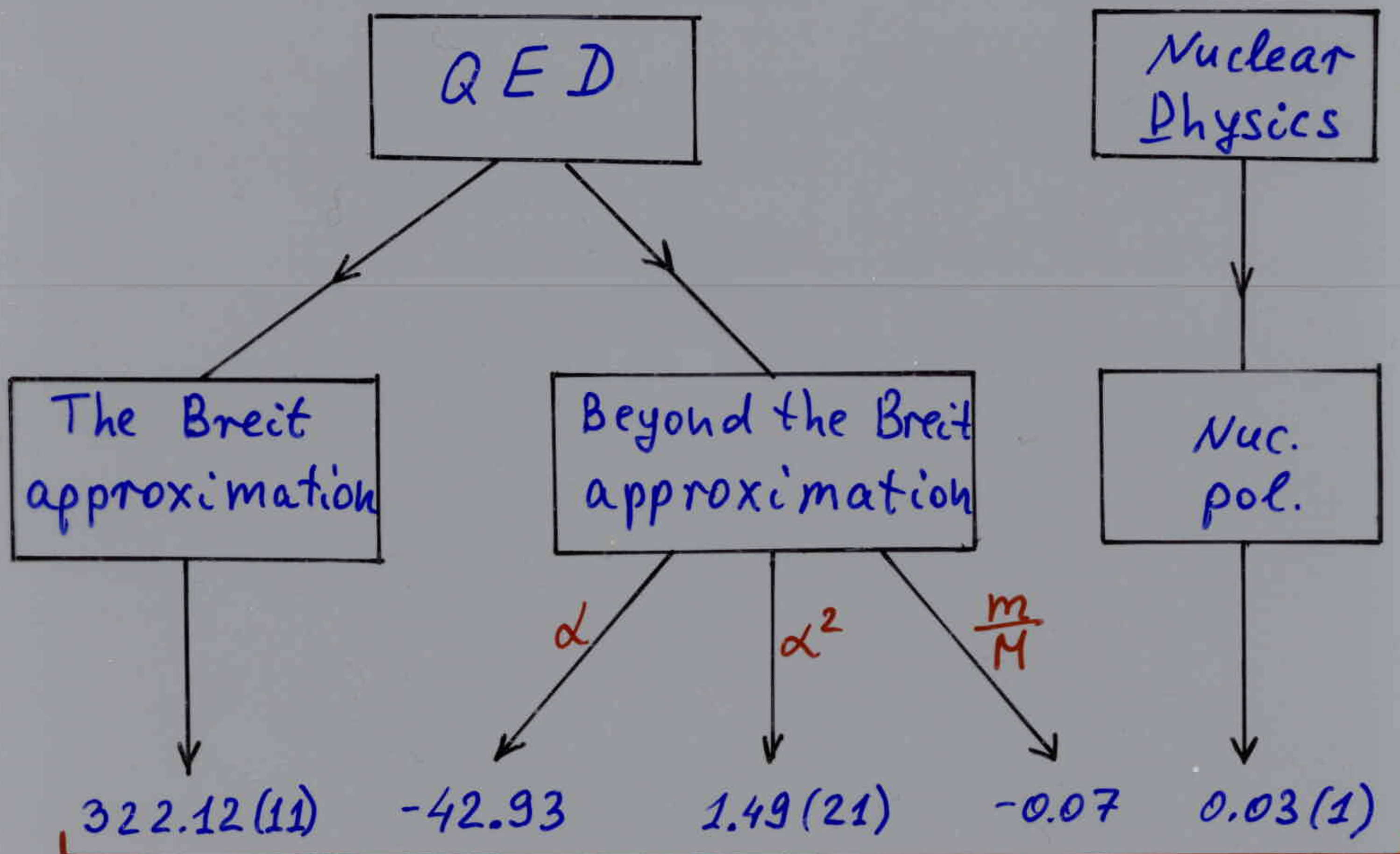
$+ \cdots$

Zherebtsov, Shabaev, Yerokhin (2000)

Andreev, Labzowsky, Plunien, Soft (2001)

The $2p_{\frac{1}{2}}-2s$ transition energy in $^{238}U^{89+}$, in

Theory



280.64(24) eV [1]

Experiment: 280.59(10) eV [2]

[1] V.A. Yerokhin and V.M. Shabaev, 2001

[2] J. Schweppe et al, 1991

Testing QED: To first order in α ~0.5%

To second order in α ~15%

Future prospects for the Lamb shift measurements

	1s, H-like U	$2p_{\frac{1}{2}}-2s$, Li-like U
QED	265.34(31) eV	-41.51(21) eV
Nuclear physics limit	~0.10 eV (~0.03% of QED)	~0.013 eV (~0.03% of QED)
Current experimental accuracy	13 eV	0.10 eV

⇓

To achieve the nuclear physics limit (~0.03% of QED) we should improve the experimental precision

by factor <u>130</u> in <u>H-like U</u>

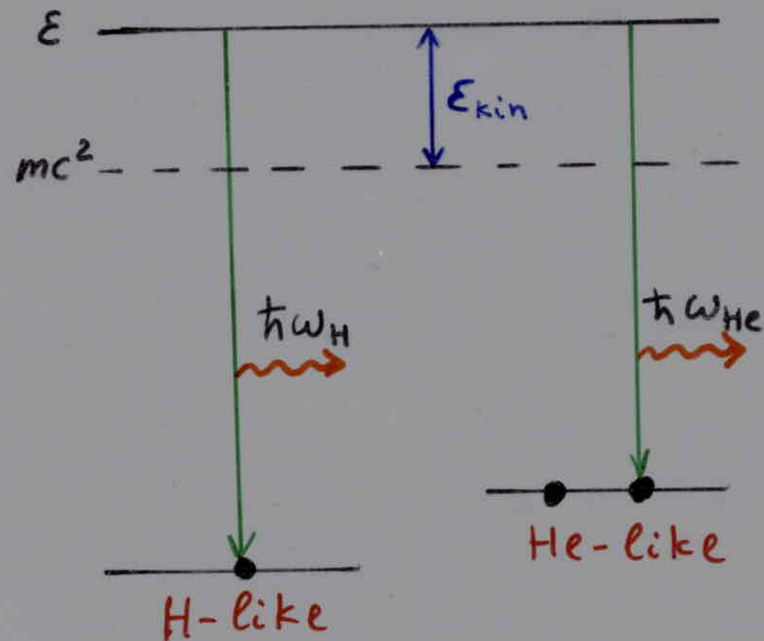
or

by factor <u>8</u> in <u>Li-like U</u>

He-like ions

Two-electron contribution to the ground state energy

Experiment: Marrs, Elliott, Stöhlker (1995)
Super-EBIT (LLNL)

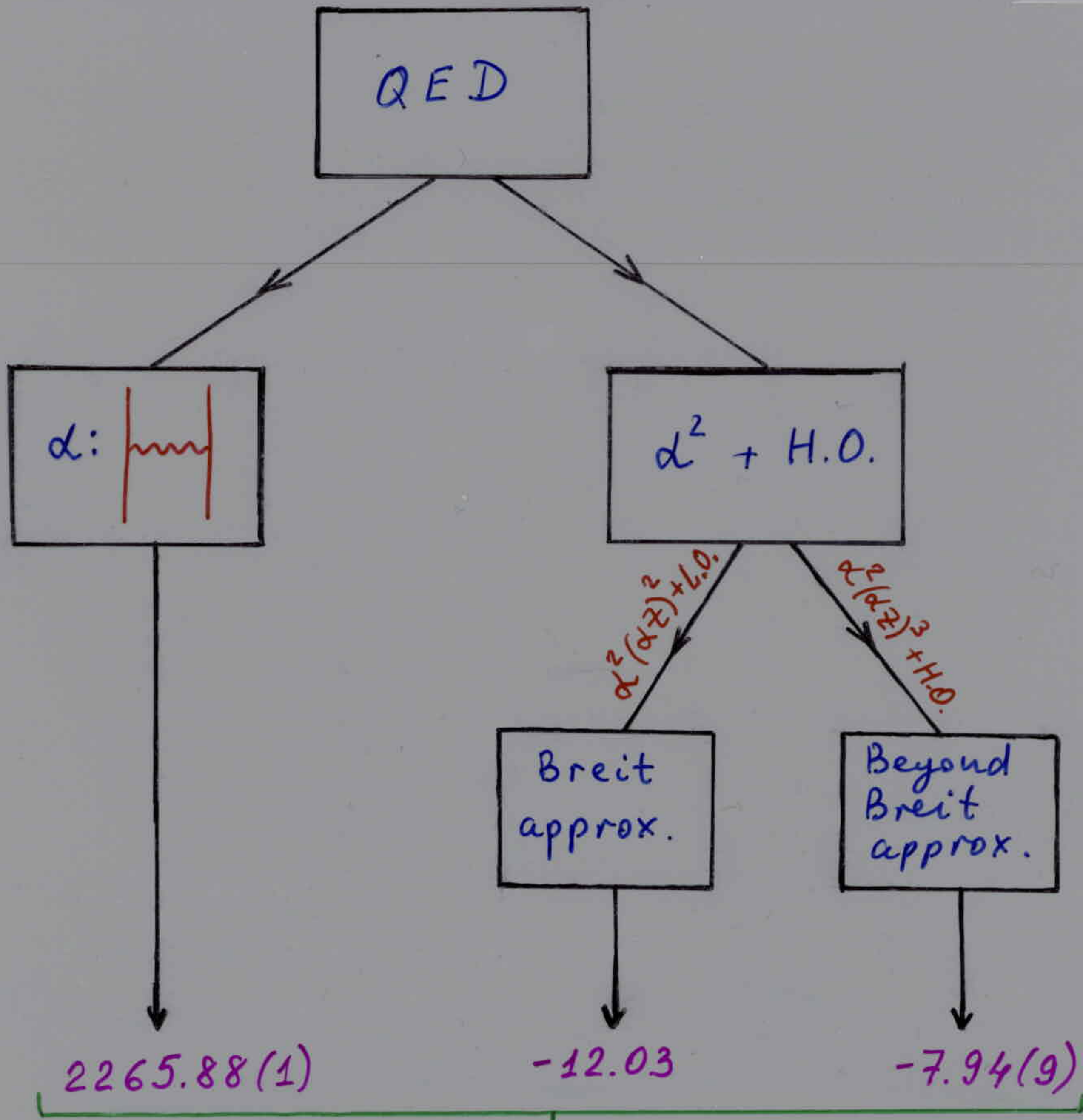


$$\hbar\omega_H = \varepsilon - \varepsilon_H$$

$$-\quad \hbar\omega_{He} = \varepsilon + \varepsilon_H - \varepsilon_{He}$$

$$\hbar(\omega_H - \omega_{He}) = \underbrace{\varepsilon_{He} - 2\varepsilon_H}_{\text{two-electron contribution}}$$

Two-electron Lamb shift in U^{90+} (in eV)



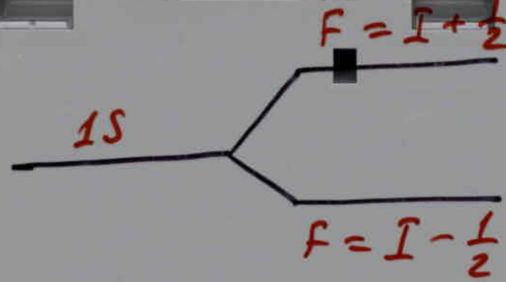
Theory: 2245.92(9) eV [1]

Exper.: 2248(9) eV [2]

[1] Yerokhin, Artemyev, Shabaev (1997)
Persson, Salomonson, Sunnergren, Lindgren (1996)

Hyperfine splitting in H-like ions

$\vec{F} = \vec{J} + \vec{I}$



$$\Delta E = \Delta E_{Dirac}(1-\varepsilon) + \Delta E_{QED}$$

where ε is the Bohr-Weisskopf correction:

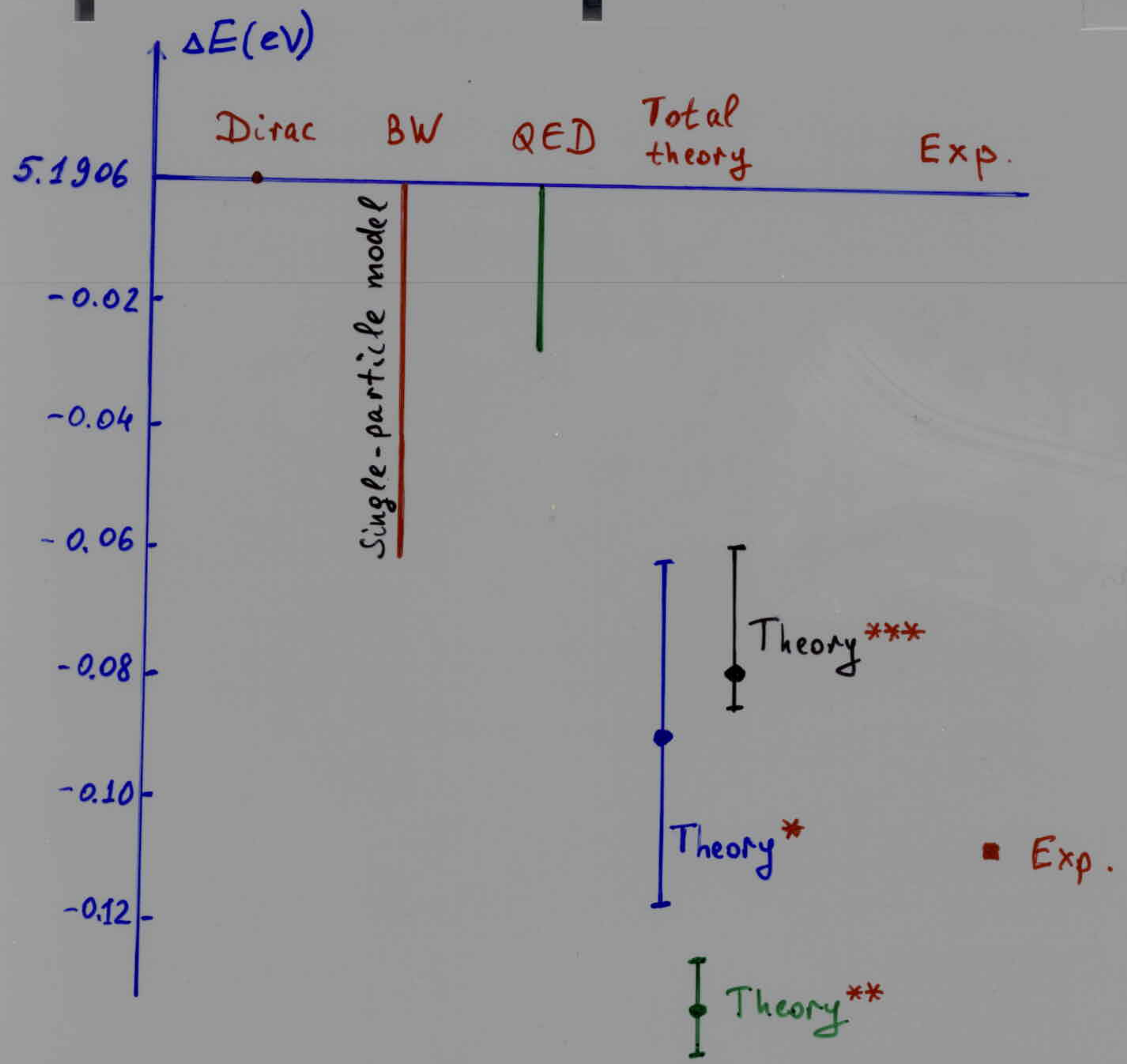
Point-dipole nuclear magnetism → Extended nuclear magnetism

Single-particle model: Shabaev et al. (1997)

Many-particle model: Tomaselli et al. (1995, 1998)

Sen'kov and Dmitriev (2002)

Hyperfine splitting in $^{209}Bi^{82+}$



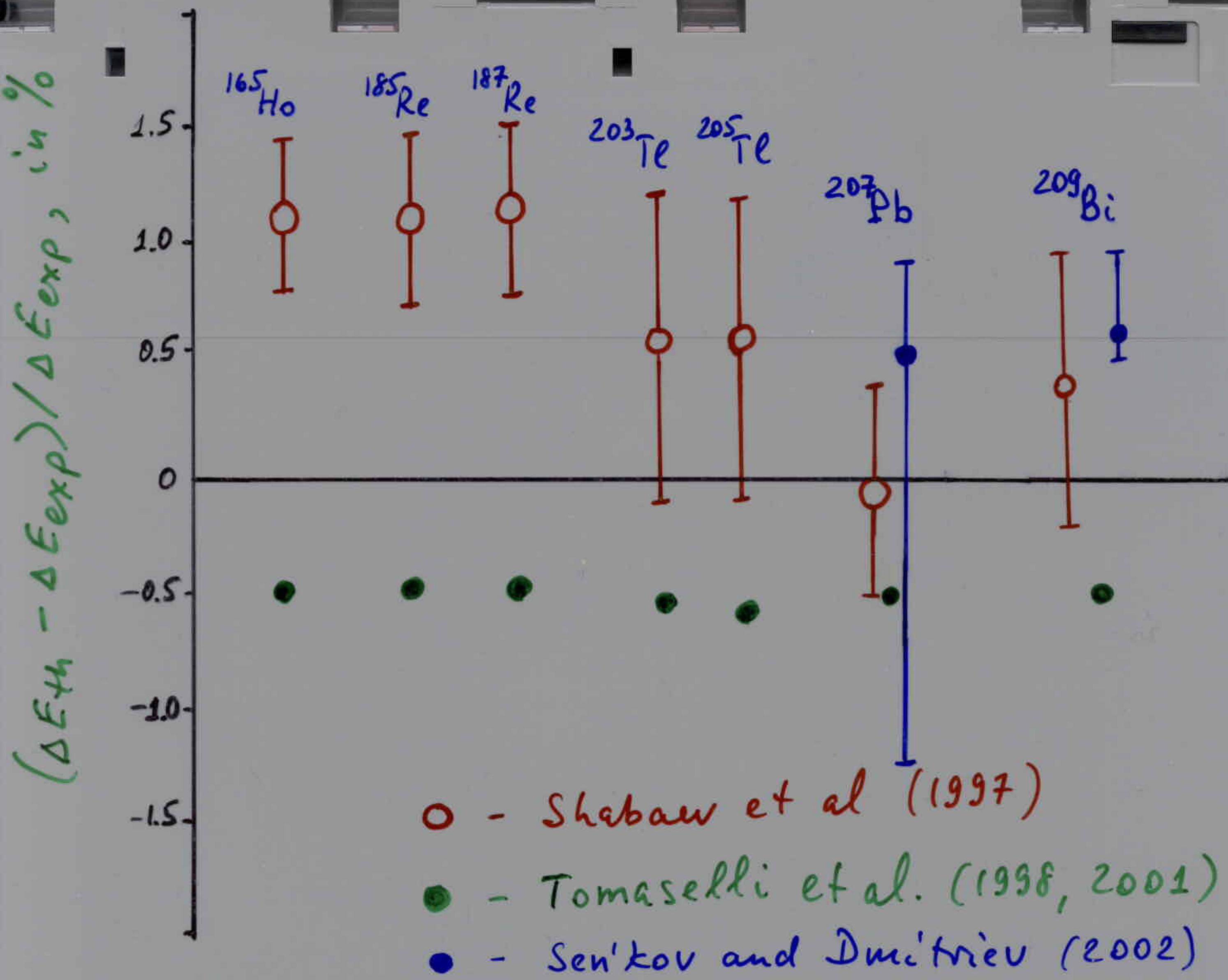
* Shabaev et al. (1997)

** Tomaselli et al. (1995)

*** Sen'kov and Dmitriev (2002)

Exp.: Klaft et al. (1994)

Comparison of theory with experiment on HFS in H-like ions



Experiments:

[Bi] Klaft et al. (1994)

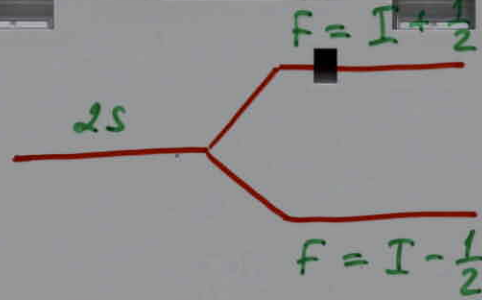
[Ho] Crespo López-Urrutia et al (1996)

[Re] Crespo López-Urrutia et al (1998)

[Pb] Seelig et al. (1998)

[Tl] Beiersdorfer et al (2001)

Hyperfine splitting in Li-like ions



$$\Delta E^{(2s)} = \Delta E^{(2s)}_{Dirac} \left(1 - \varepsilon^{(2s)}\right) + \Delta E_{int} \left(1 - \varepsilon^{(int)}\right)$$
$$+ \Delta E^{(2s)}_{QED} + \Delta E_{int\text{-}QED}$$

$^{209}Bi^{80+}$

Theory

$\Delta E_{th} = 797.1(2)$ meV — Shabaev et al. (2000)

$\Delta E_{th} = 797.15(13)$ meV — Sapirstein and Cheng (2001)

Experiment

$\Delta E_{exp} = 820(26)$ meV — Beiersdorfer et al. (1998)

$\Delta E_{exp} = ?$ — Winter et al. (1999)

Towards a test of QED effects on HFS

(Shabaev et al., PRL, 2001)

We consider

$$\Delta' E = \Delta E^{(2s)} - \xi \Delta E^{(1s)}$$

where the parameter ξ is chosen to cancel the B-W effect.

In the case of Bi, $\xi = 0.16885$.

Theoretical contributions to $\Delta' E$, in meV

non-QED	-61.52(4)
QED	0.24(1)
Total	-61.27(4)
Experiment	?

This method allows for a test of QED on level of a few percent, provided the HFS is measured to accuracy $\sim 10^{-6}$.

The g factor of H-like ions

$$g = \frac{\langle \mu_z \rangle}{\mu_B \langle J_z \rangle}$$

$$g = g_D + \Delta g_{QED} + \Delta g_{rec} + \Delta g_{NS}$$

1s state

$$g_D = \frac{2}{3}\left[1 + 2\sqrt{1-(\alpha Z)^2}\right] = 2\left[1 - \frac{1}{3}(\alpha Z)^2 + \cdots\right]$$

The g factor of $^{12}C^{5+}$

Dirac value	1.998 721 354 4
Nuclear size	0.000 000 000 4
QED, order α/π [1]	0.002 323 663 9(1)
QED, order $(\alpha/\pi)^2$ [2]	−0.000 003 516 2(3)
Recoil [3]	0.000 000 087 6
Total theory	2.001 041 590 1(3)
Experiment [4,5]	2.001 041 596 3(10)(44) ← me

[1] Yerokhin, Indelicato, Shabaev (2002)
[2] Czarnecki, Melnikov, Yelkhovsky (2001)
[3] Shabaev and Yerokhin (2002)
[4] Häffner et al. (2000)
[5] Beier et al. (2002)

Determination of the electron mass from the g-factor experiment on C

$$m_e = 0.000\,548\,579\,909\,29\,(29)(8)\ u$$

$$\Downarrow$$

$$m_e = 0.000\,548\,579\,909\,3\,(3)\ u$$

Yerokhin, Indelicato, Shabaev (2002)

Old value:

$$m_e = 0.000\,548\,579\,911\,1\,(12)\ u$$

Farnham, Van Dick, Schwinberg (1995)

Future prospects for the g-factor measurements

1) Determination of the fine structure constant

$$\frac{\delta\alpha}{\alpha} \sim \frac{1}{(\alpha Z)^2}\frac{\delta g}{g}$$

2) Determination of the nuclear magnetic moments

$$g_{atom} = g^{(e)} \frac{F(F+1)+J(J+1)-I(I+1)}{2F(F+1)}$$

$$- \frac{m_e}{m_p} g^{(N)} \frac{F(F+1)+I(I+1)-J(J+1)}{2F(F+1)}$$

$$\frac{m_e}{m_p} \sim \frac{1}{2000}$$

3) Determination of RMS radii

$$\delta g^{(e)} \equiv g_1^{(e)} - g_2^{(e)} \simeq A(\alpha Z)\,\delta \langle r^2 \rangle^{1/2}$$

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