Electron-positron pair production in relativistic heavy ion collisions

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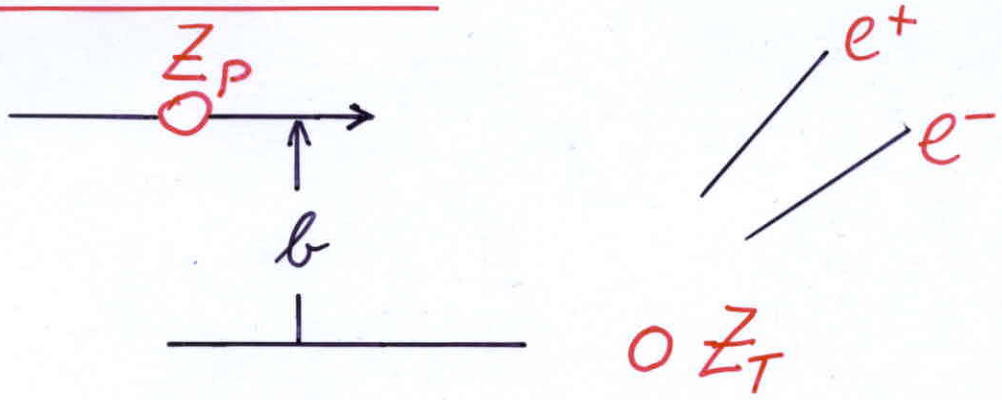
Justus-Liebig-University Giessen:

Oliver Busic

Matthias Gail

Norbert Grün

1. Introduction



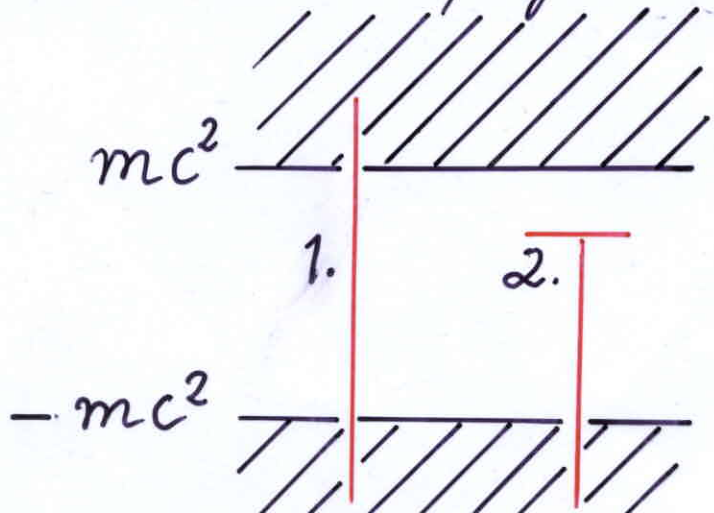
Atomic collisions: $b >$ sum of nuclear radii

high charge numbers Z_P, Z_T: $Z\alpha > 0.5$

relativistic velocities: $T_p \gtrsim 1$ GeV/nucleon

Possible processes

1. Free electron-positron pair production
2. Pair production with capture of electron into a bound state, e.g. into K-shell



Cross section for pair production with capture limits the lifetime of ions in relativistic heavy ion colliders (Gould 1984).

Example: RHIC, Brookhaven

Pair production with capture of electron:

$\sigma = 100$ barn for $U^{92+}(100\,AGeV) + U^{92+}(100\,AGeV)$

Also limitation by Coulomb fission.

Cross section for free electron-positron pair production (Bhabha 1935)

$$\sigma_{e^+e^-} = \frac{28}{27\pi}\alpha^2 Z_T^2 Z_P^2 r_0^2 (\ln(\gamma/4))^3 \qquad \gamma \gg 1$$

$r_0 = e^2/(mc^2) = 2.8$ fm, classical electron radius

proportional to $Z_T^2 Z_P^2$

Pair production yields basis information about the dynamics of the negative continuum under the influence of strong electromagnetics fields.

Theories show that pair production is a nonperturbative process for small impact parameters ($b \lesssim 500\,fm$) and high ion charges ($Z_P, Z_T \gtrsim 70$).

This talk presents nonperturbative calculations in the semiclassical approximation.

Quantum field dynamics of $e^- e^+$-field

$\longrightarrow$ Solution of Dirac equation

$$\left(\vec{\alpha}\vec{p} + \beta - \frac{Z_T e^2}{r} - Z_P e^2 \gamma \left(1 - \alpha_z v_P\right) / r'(t)\right) \psi_\alpha(\vec{r}, t) =$$

$$i\, \partial \psi_\alpha(\vec{r}, t) / \partial t$$

2. Coupled channels calculations

Expansion of wave functions in target system, atomic wave functions around target and projectile nucleus (according to Toshima, Eichler).

$$\psi_\alpha(\vec{r},t) = \sum_{n_T \in T} a_{n_T\alpha}(t)\,\psi_{n_T}(\vec{r},t) + \sum_{n_P \in P} a_{n_P\alpha}(t)\,S^{-1}(v_P)\,\psi_{n_P}(\vec{r}',t')$$

Lorentz boost operator $S^{-1}(v_P) = \left(\frac{\gamma+1}{2}\right)^{1/2} + \left(\frac{\gamma-1}{2}\right)^{1/2} \alpha_z$

$$\longrightarrow \quad i\hat{N}\frac{d\vec{a}_\alpha(t)}{dt} = \hat{V}(t)\,\vec{a}_\alpha(t)$$

with $\vec{a}_\alpha(t) = (\ldots, a_{n_T\alpha}, \ldots, a_{n_P\alpha}, \ldots)$

The states $\psi_n(\vec{r},t)$ contain bound and continuum states. Discretization of continuum with stationary wave packets:

$$\psi_E^{SP} = \frac{\exp(-iEt)}{\sqrt{\Delta E}} \int_{E-\Delta E/2}^{E+\Delta E/2} \psi_{E'}(\vec{r})\,dE'$$

Applications to collisions of
U^{92+} and La^{57+} projectiles on
Au^{79+}, Ag^{47+} and Cu^{29+} targets
at 930 MeV/nucleon $\hat{=}$ $\gamma = 2$

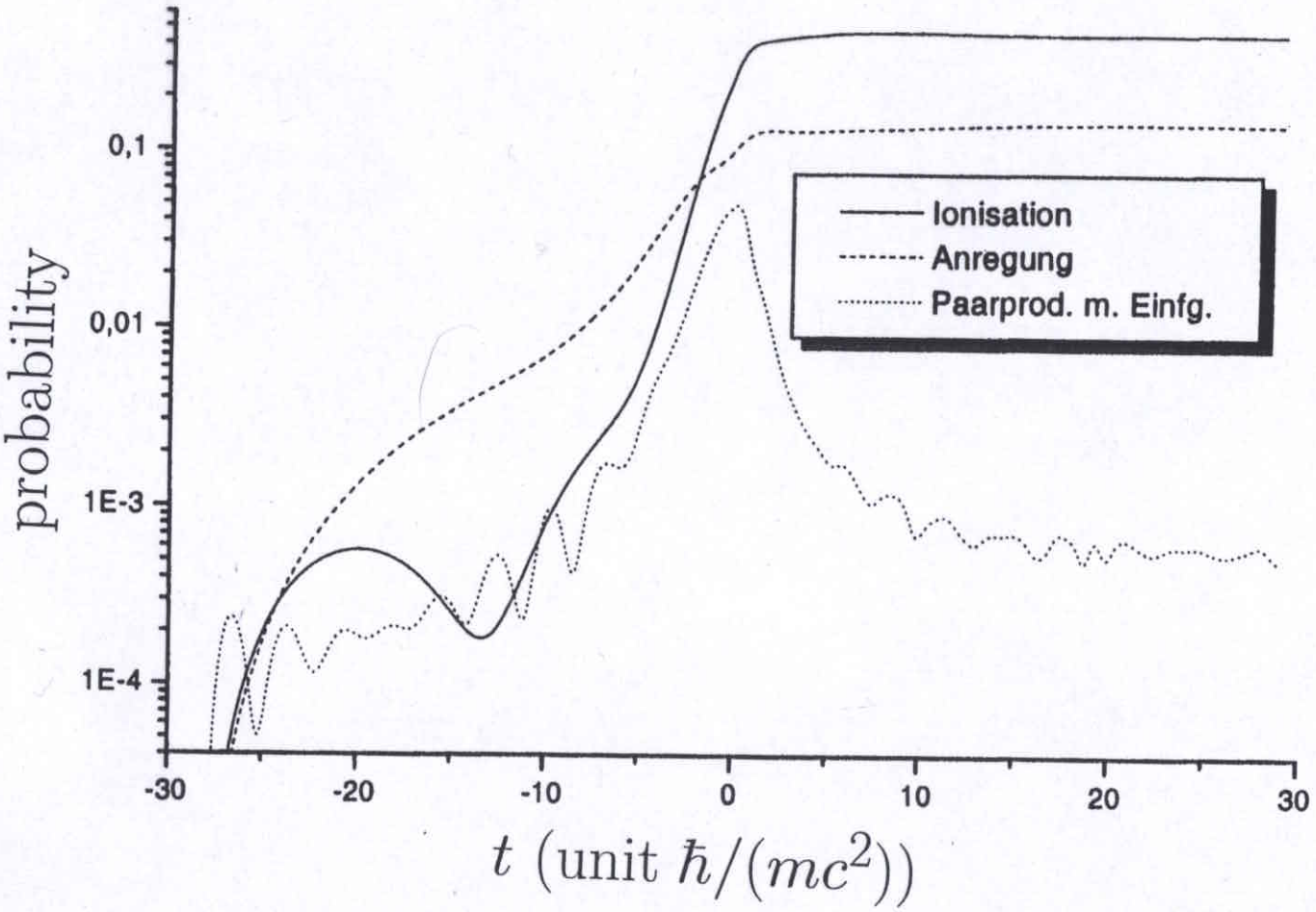
a) Wave functions around the projectile U^{91+}
in the projectile frame

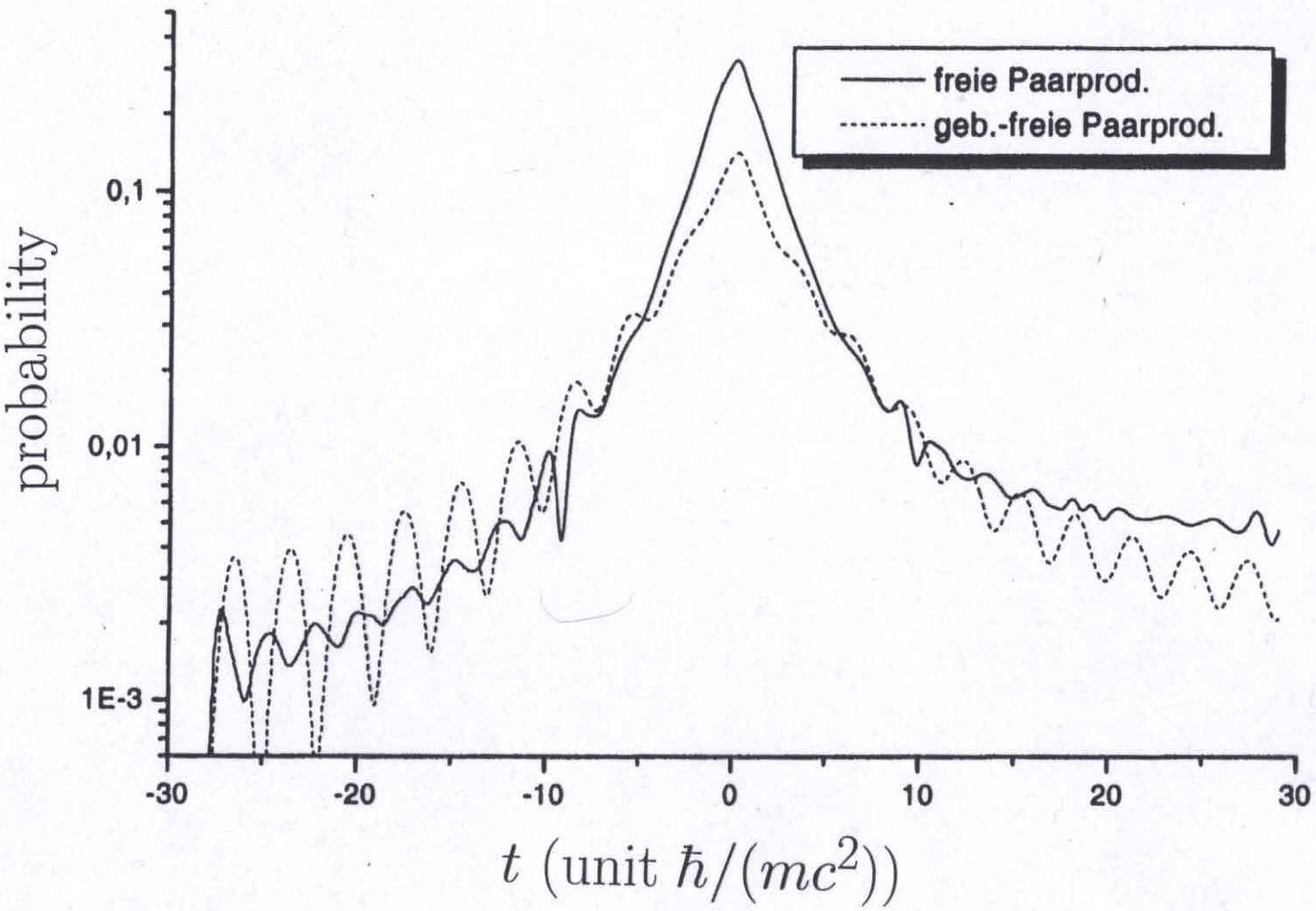
bound states $n \leq 3$, $|\kappa| \leq 2$
continuum states $|\kappa| \leq 2$ $-5mc^2 \leq E$, $E \leq 3mc^2$

α) U^{91+} (930 MeV/nucl.) + Au^{79+}, $b = 0.125 \lambda_C = 48$ fm

probability for pair production with capture
in $1s_{1/2}$ state of U^{91+} with $m = \frac{1}{2}$ by
time reversal method
large peak of pair production at $t = 0$,
adiabatic behaviour of peak

β) Free pair production in $U^{92+} + Au^{79+}$


probability
0,1
0,01
1E-3
1E-4
-30
-20
-10
0
10
20
30
t (unit ħ/(mc²))
Ionisation
Anregung
Paarprod. m. Einfg.


freie Paarprod.
geb.-freie Paarprod.
probability
0,1
0,01
1E-3
-30
-20
-10
0
10
20
30
t (unit $\hbar/(mc^2)$)

b) Wave functions around the projectile in the target system

bound states $n \leq 3$, $|\kappa| \leq 2$

continuum states $|E| \leq 5mc^2$, $|\kappa| \leq 2$

α) Probability for pair production

$U^{92+}(\gamma=2) + Au^{79+}$ $b = 0.125\,\lambda_C$

small width and height of the peak, fast convergence

β) Cross section for free pair production

$\sim Z_T^{2.5}$ in contrast to Z_T^2 of perturbation th.

γ) Collisions of $La^{57+}(\gamma=2)$ with Au^{75+}, Ag^{47+}, Cu^{29+}

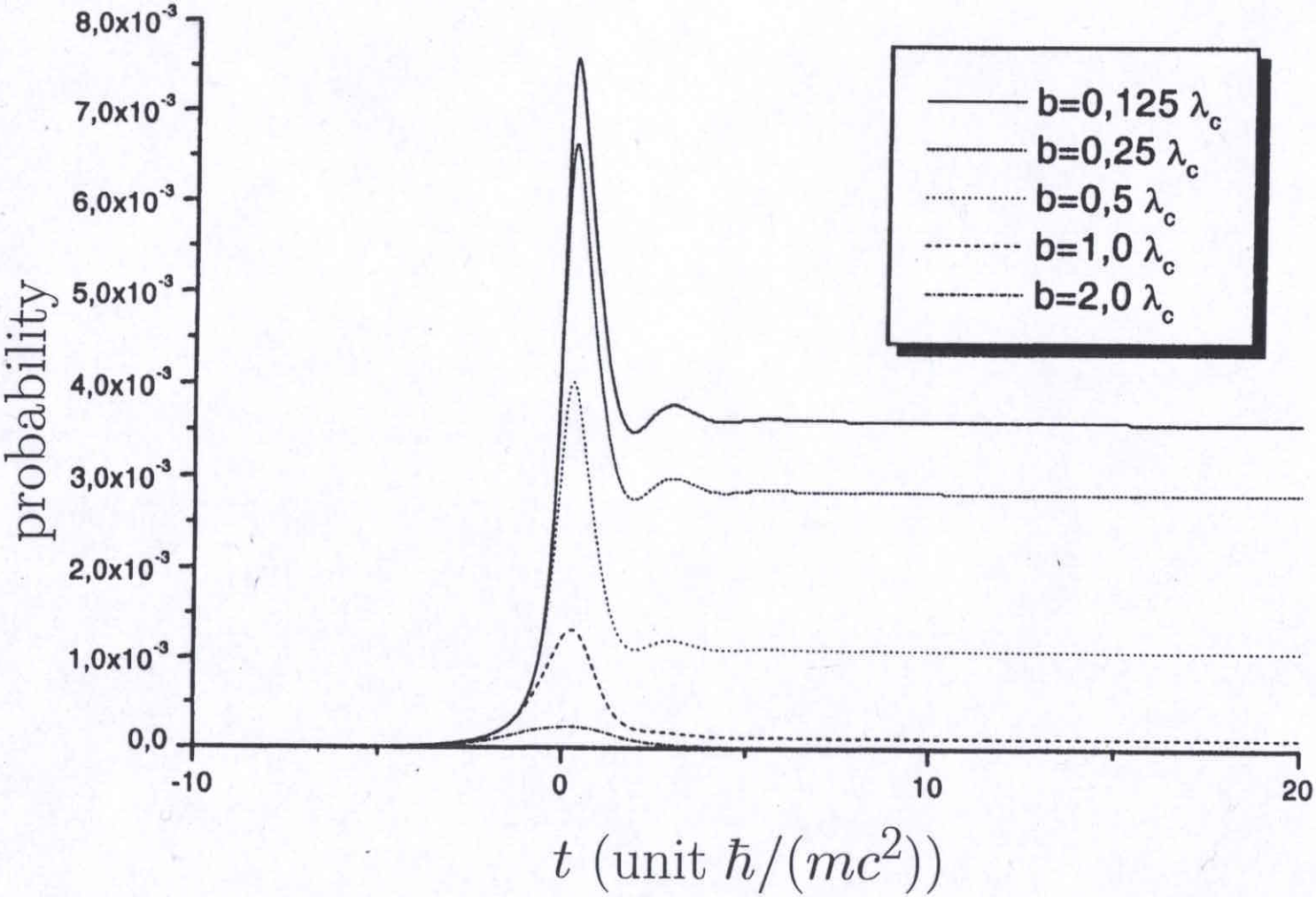
pair production, measurements by Belkacem et al. (1997)

free pair production with Au^{75+} target:

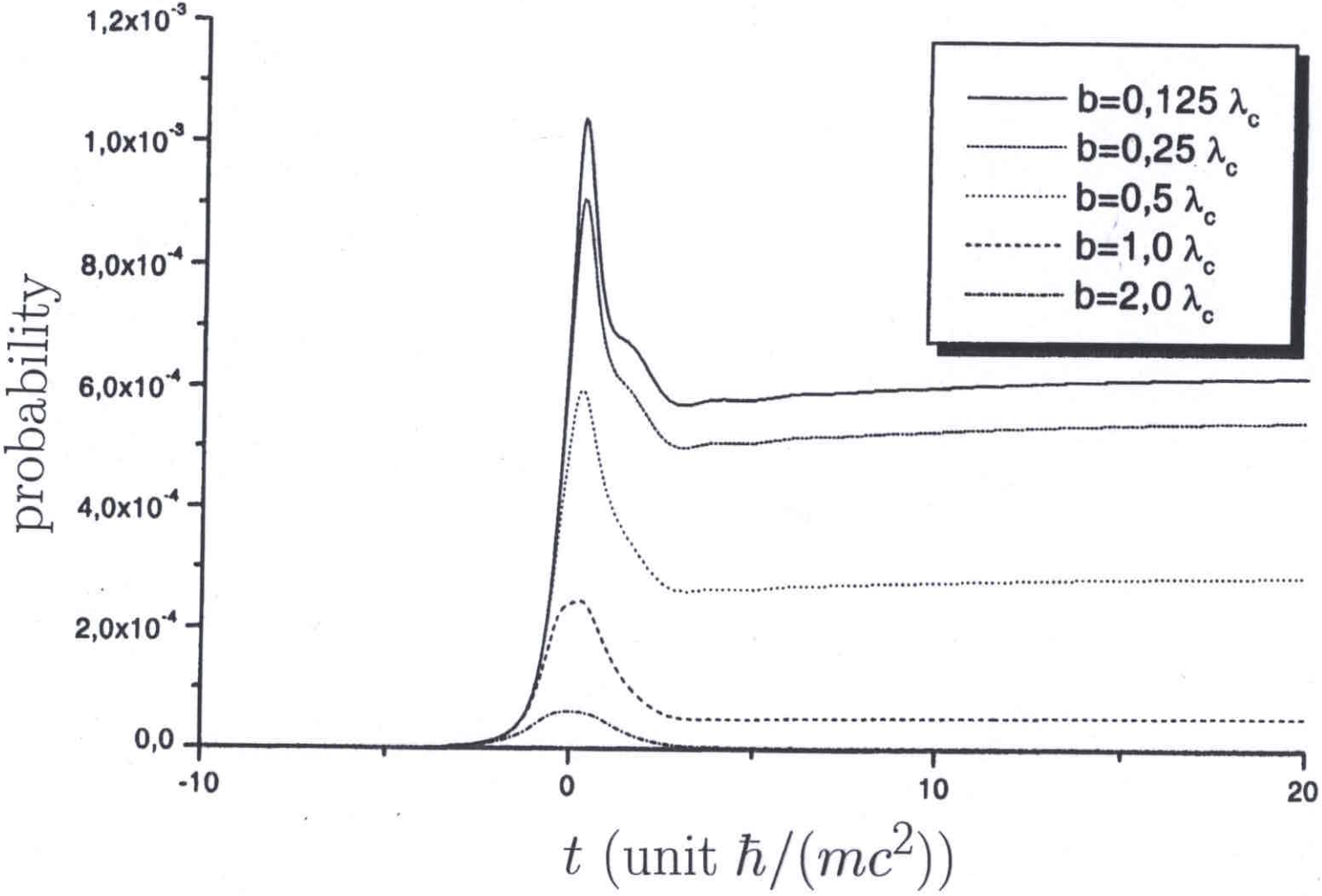
$\sigma_{exp} = 1.6 \pm 0.3\,b$,

with wave functions around La^{57+}: $\sigma_{cal} = 0.84\,b$,

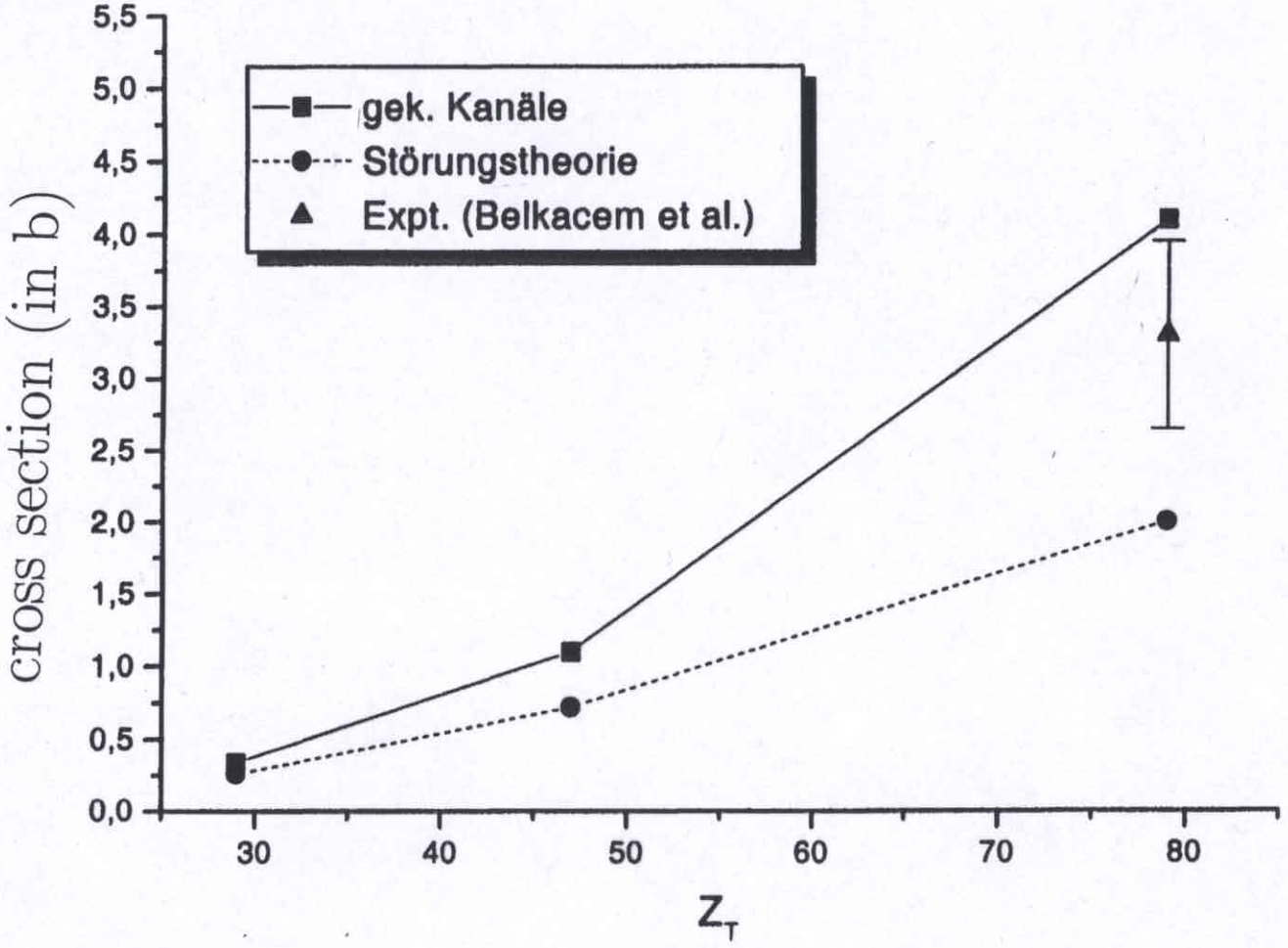
free e^+e^-


8,0x10-3
7,0x10-3
6,0x10-3
5,0x10-3
4,0x10-3
3,0x10-3
2,0x10-3
1,0x10-3
0,0
probability
-10
0
10
20
t (unit ħ/(mc2))
b=0,125 λc
b=0,25 λc
b=0,5 λc
b=1,0 λc
b=2,0 λc

e^+e^- with capture in U^{91+}

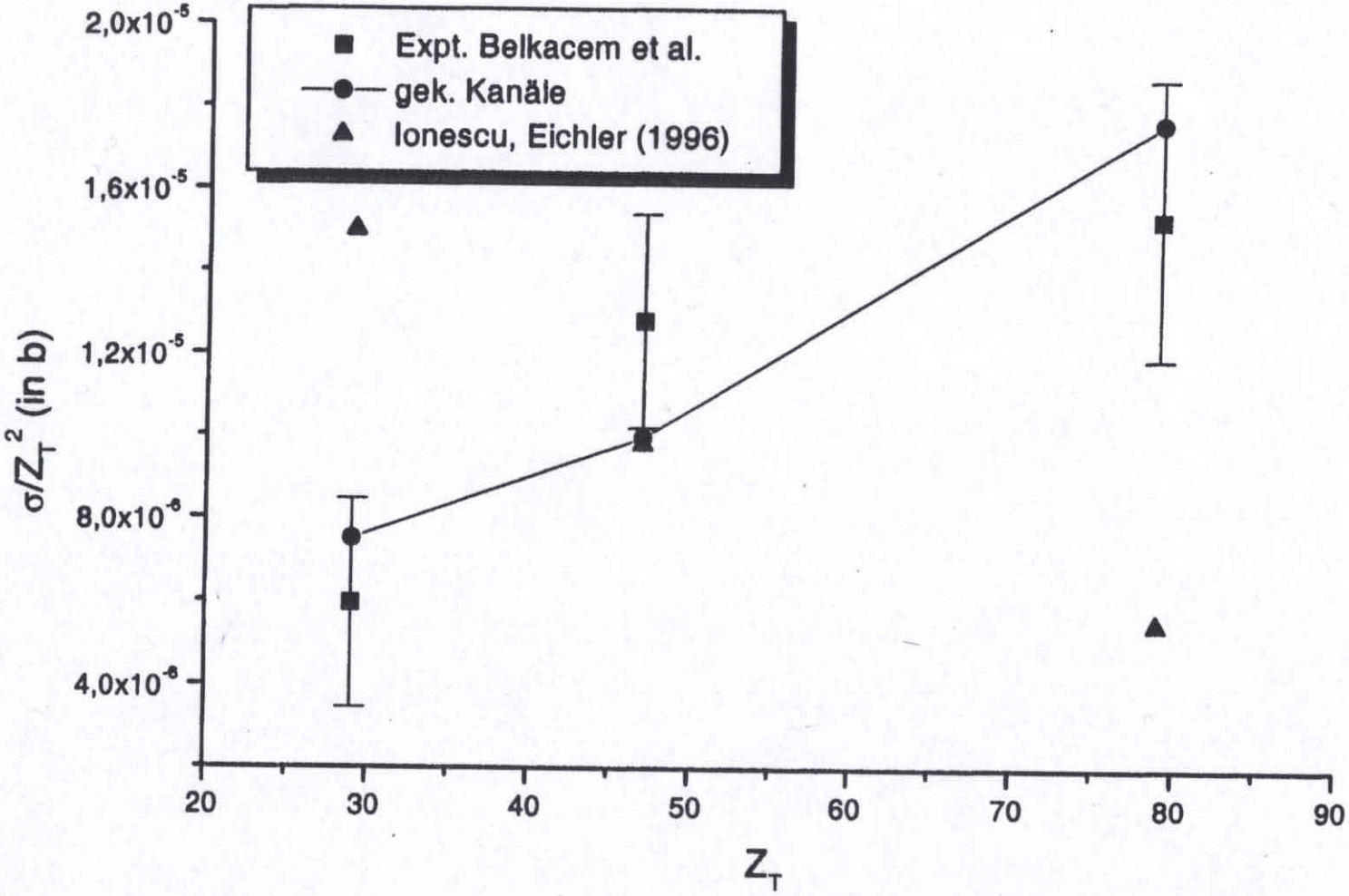


free e^+e^-, $U^{92+}(\gamma=2) + Au^{79+}, Ag^{47+}, Cu^{29+}$

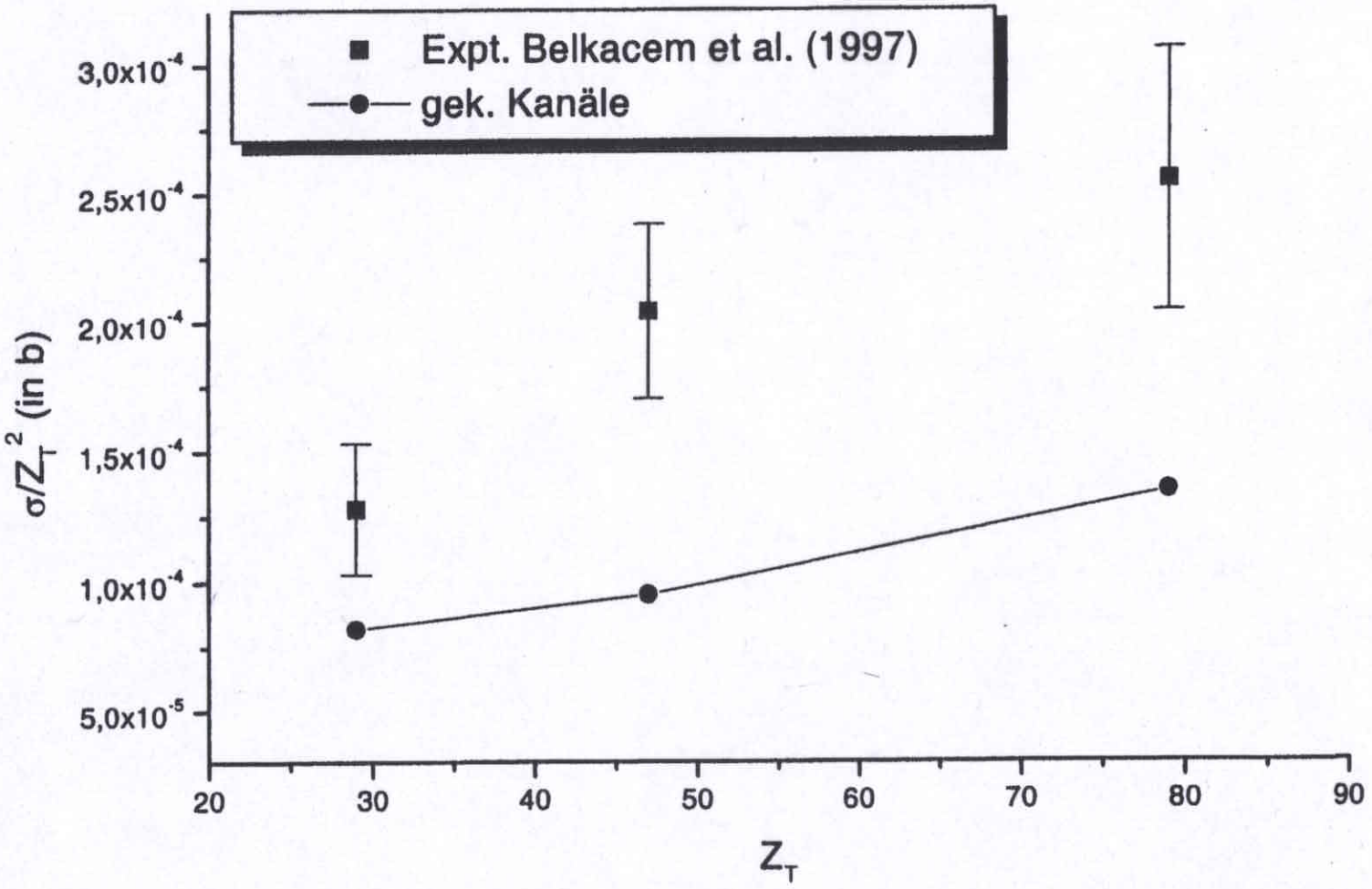


projectile La^{57+} ($\gamma=2$)

e^+e^- with capture



projectile La^{57+} (y=2)
free e^+e^-



c) Two-center wave functions

α) Bound states around target ($n \leq 3, |\kappa| = 2$) and bound ($n \leq 3, |\kappa| = 2$) and continuum ($|E| \leq 5mc^2$) states around projectile

new channel: charge transfer

$U^{91+}(\gamma = 2) + Au^{79+}$ at $b = 0$:

probability for	Momberger	here
transfer	0.015	0.020
pair prod. w. capt.	3.9×10^{-4}	3.0×10^{-4}

β) Bound and continuum states at both centers

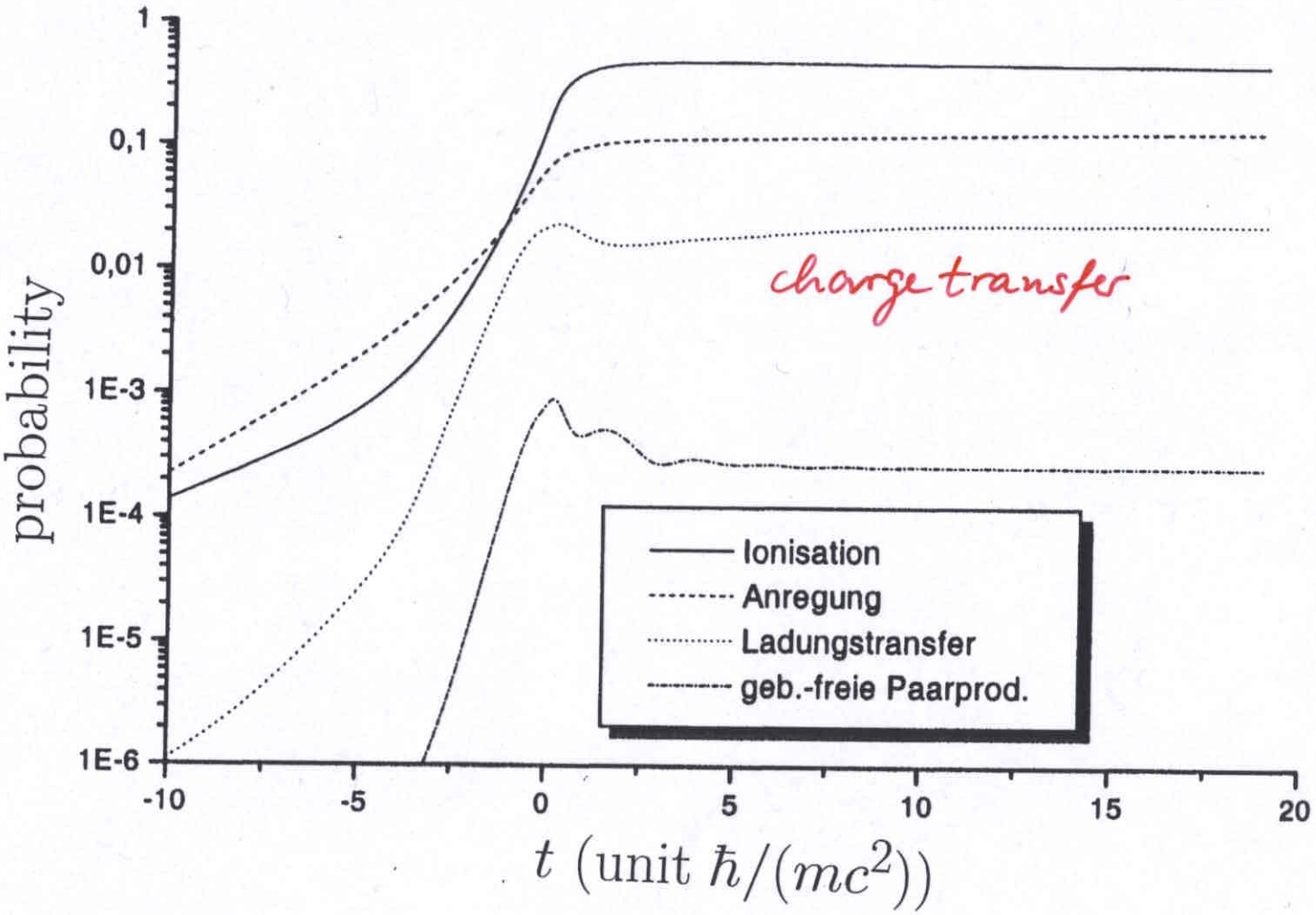
bound states $n \leq 3, |\kappa| \leq 2$

continuum states $|\kappa| \leq 1$, $|E| \leq 5mc^2$

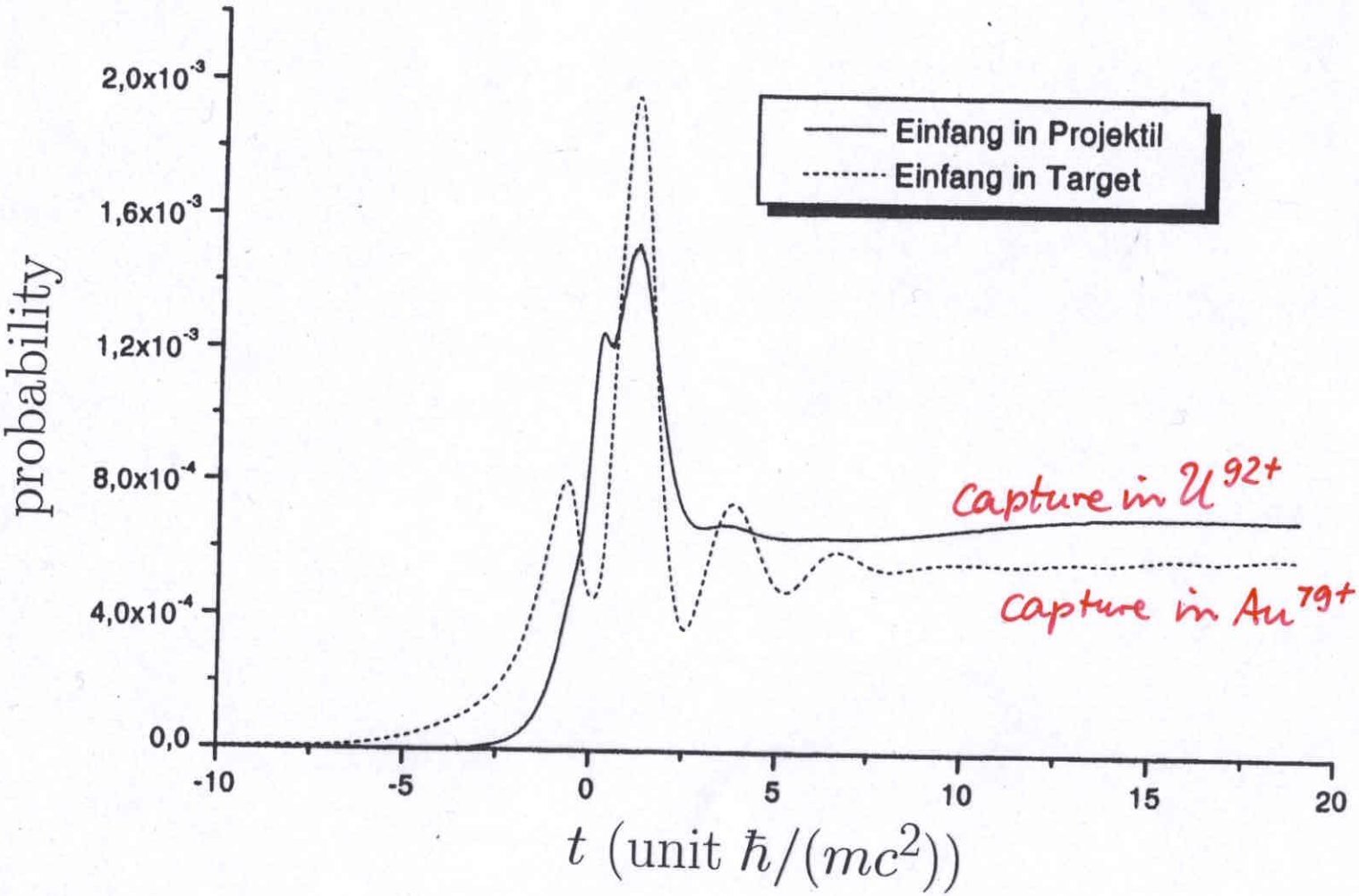
capture into target not stationary in the considered times, too small basis

Conclusions: Coupled channels calculations give a basis for studies at slightly higher

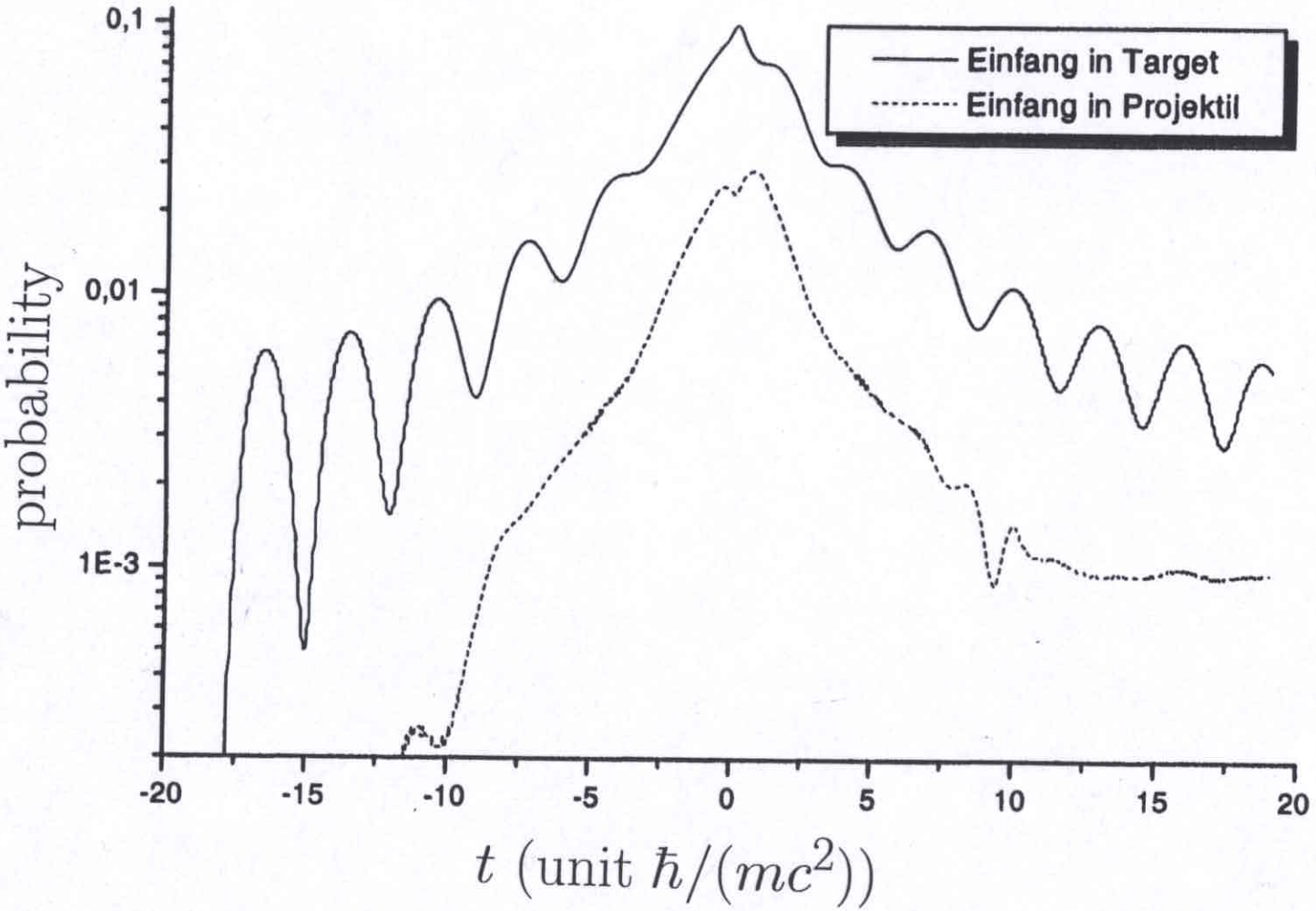
$U^{91+}(\gamma=2) + Au^{79+}$



e^+e^- with capture



$U^{92+}(\gamma=2) + Au^{79+}$



3. Finite element calculations

Solution of time-dependent Dirac equation in three-dimensional coordinate space on a three-dimensional grid.

Time-reversed method is used to get pair production with capture of electron in a bound state, e.g. state of K-shell.

here: time development of $1s_{1/2}$-state.

Projectile moves along a line $\vec{R} = (0, b, v_p t)$,
scattering plane: yz-plane.

Spin quantized in x-direction →
state with $m = 1/2$ calculated with positive b
state with $m = -1/2$ calculated with negative b

Hamiltonian and wave function transformed with phase transformation (Eichler 1987), potentials fall off faster at large distances.

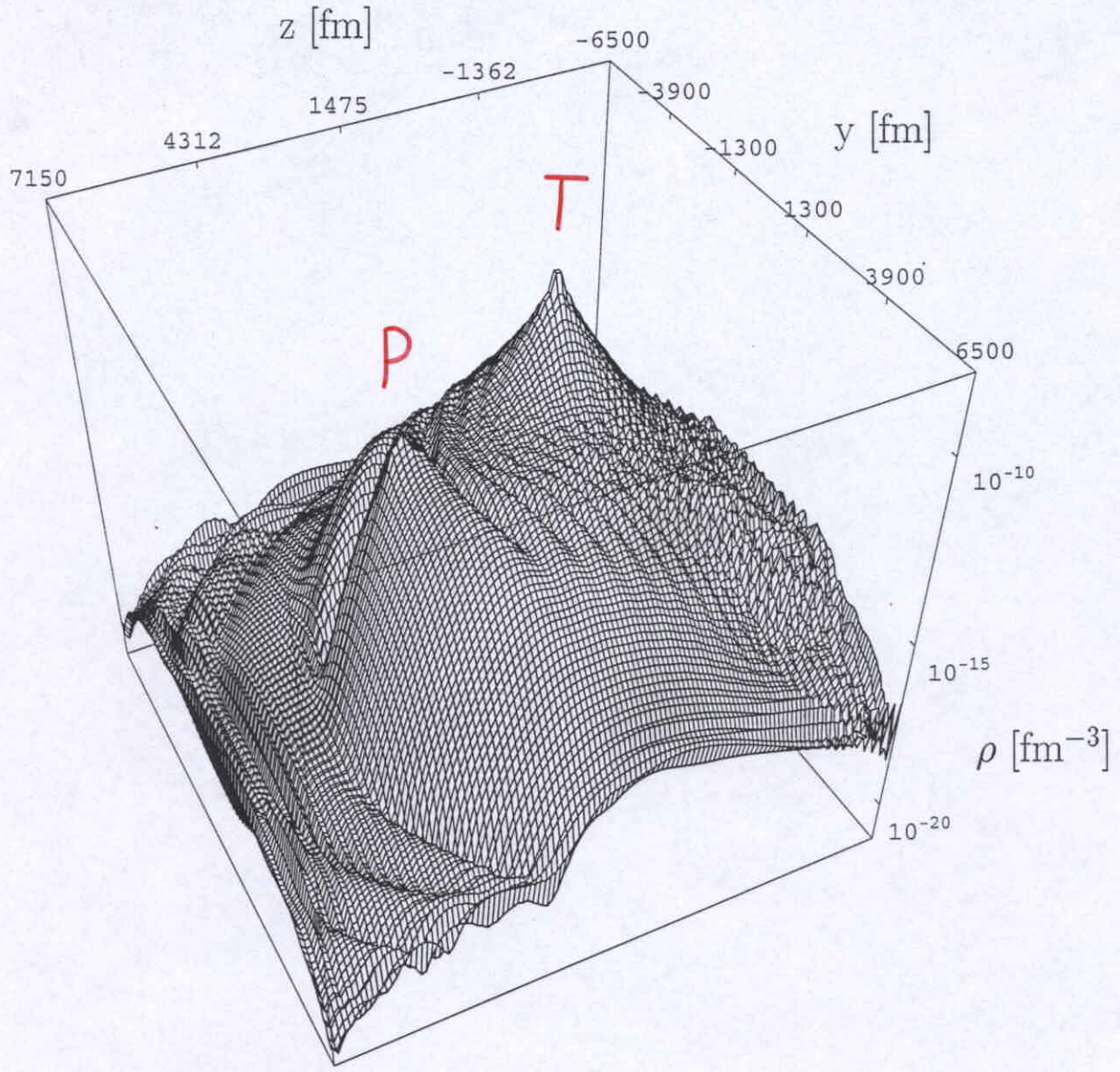
a) U^{92+} (466 MeV/nucleon) + U^{91+}, $\gamma = 1.5$

grid:

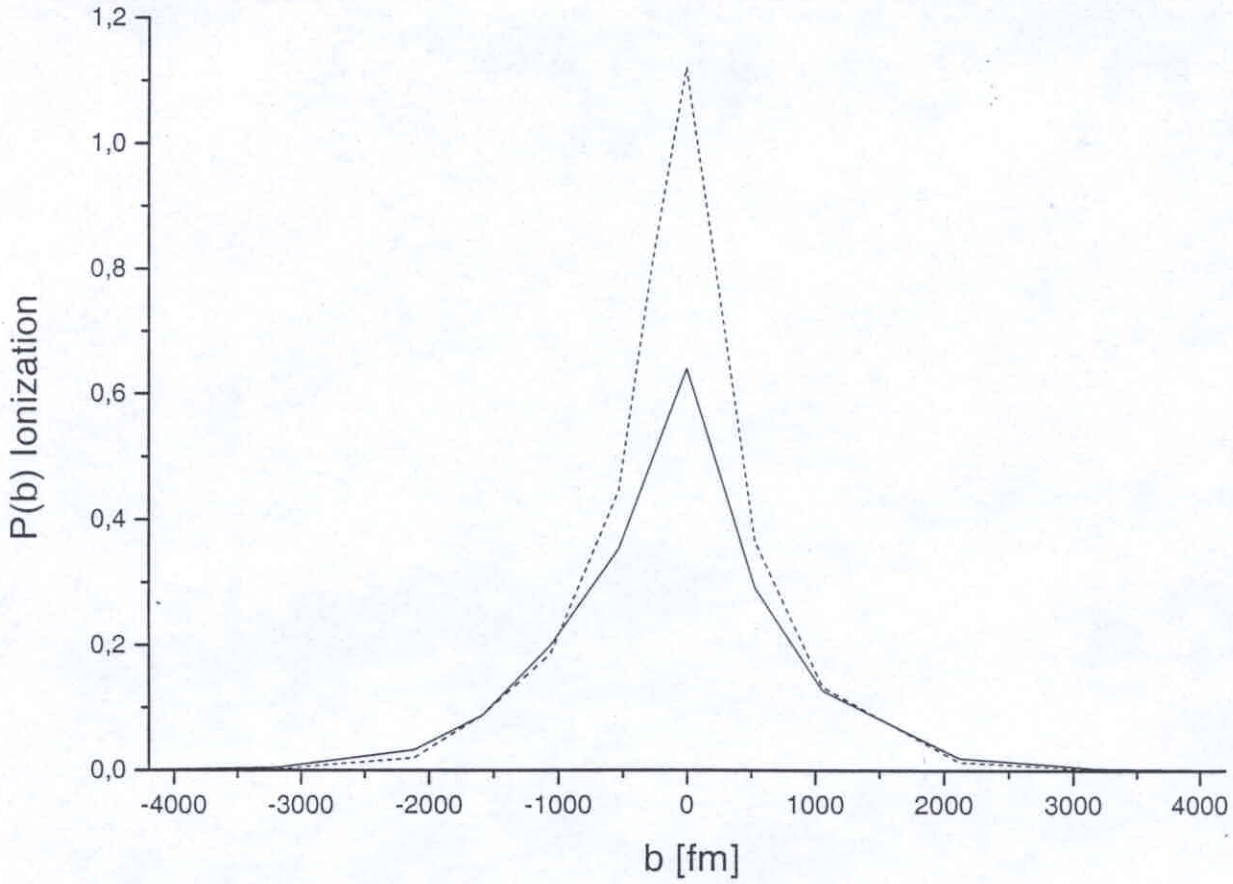
z-direction: 118 points, −4200 to 7150 fm
y-direction: 136 points, −6500 to 6500 fm
x-direction: 68 points, −6500 to 0 fm

time evolution up to 7th order of Hamiltonian

excitation	$\sigma_{exc} = 1.36 \cdot 10^4$ b
ionization	$\sigma_{ion} = 1.9 \cdot 10^4$ b
transfer	$\sigma_{transf} = 1.2 \cdot 10^3$ b



$U^{92+}(y=1.5) + U^{91+}(1s_{1/2})$



$U^{92+}(\gamma = 1.5) + U^{91+}(1s_{1/2})$

b) Au^{79+} (930 MeV/nucleon) + U^{91+}

grid:

y- and z-directions: 224 points, -6500 to 6500 fm

X-direction: 112 points, -6500 to 0 fm

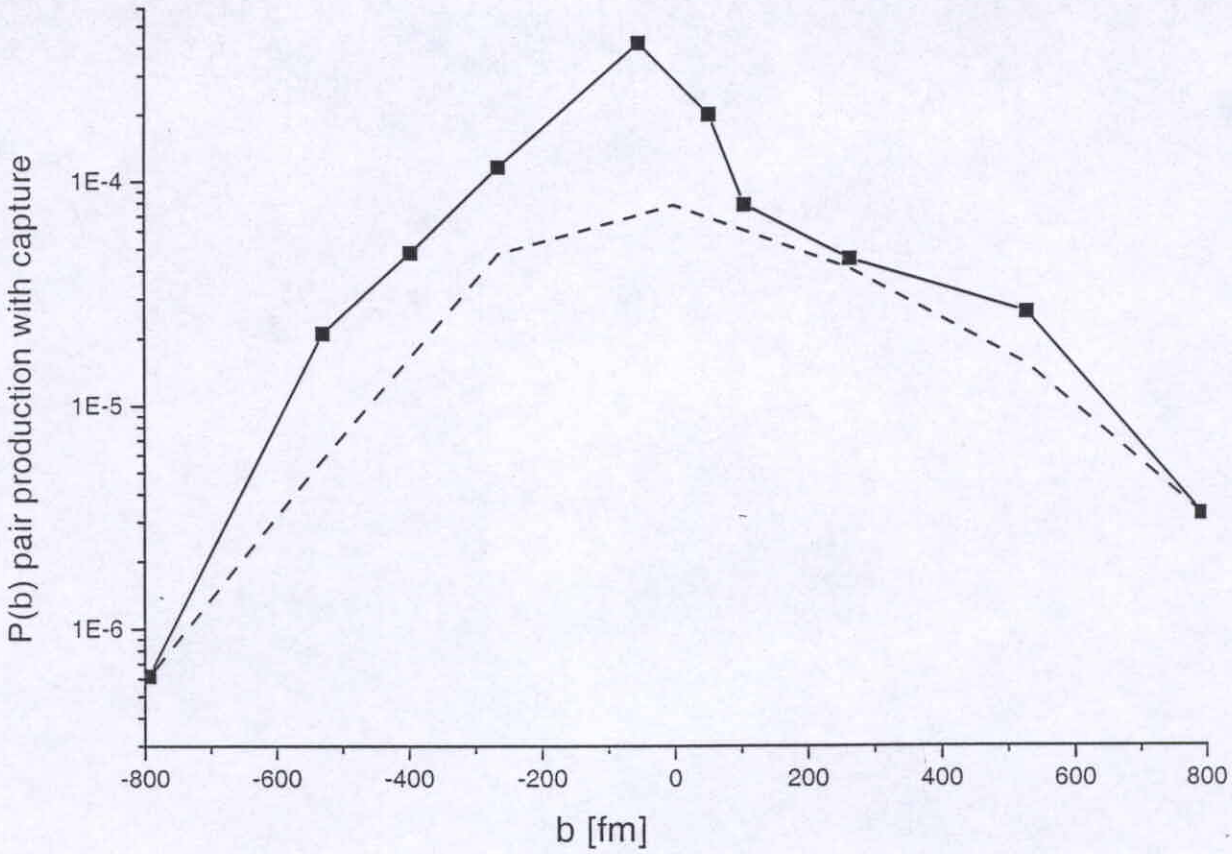
cross section of pair production with capture: 1.3 b, projection on states of negative continuum around target up to $|\kappa| = 8$.

Bad convergency for large impact parameters if projecting is not restricted in κ.

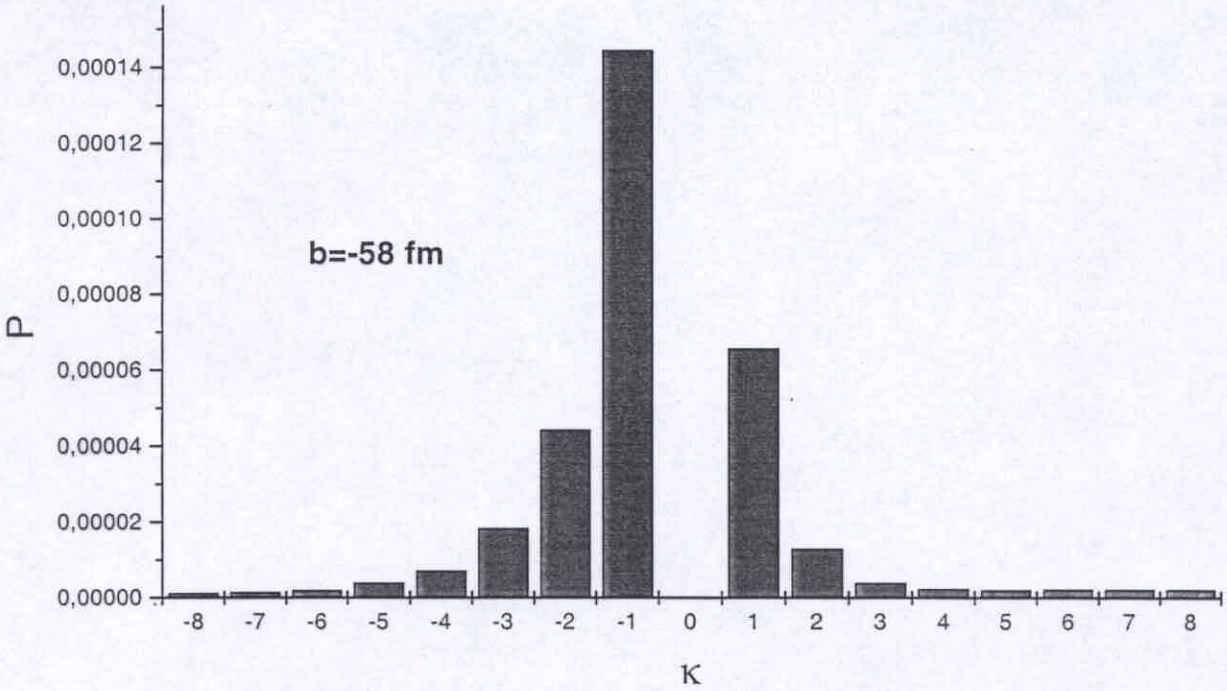
Subtracting of bound $1s_{1/2}$ projectile wave function $\longrightarrow \sigma = 1.1\,b$ compared with $\sigma_{exp} = 2.19\,b$.

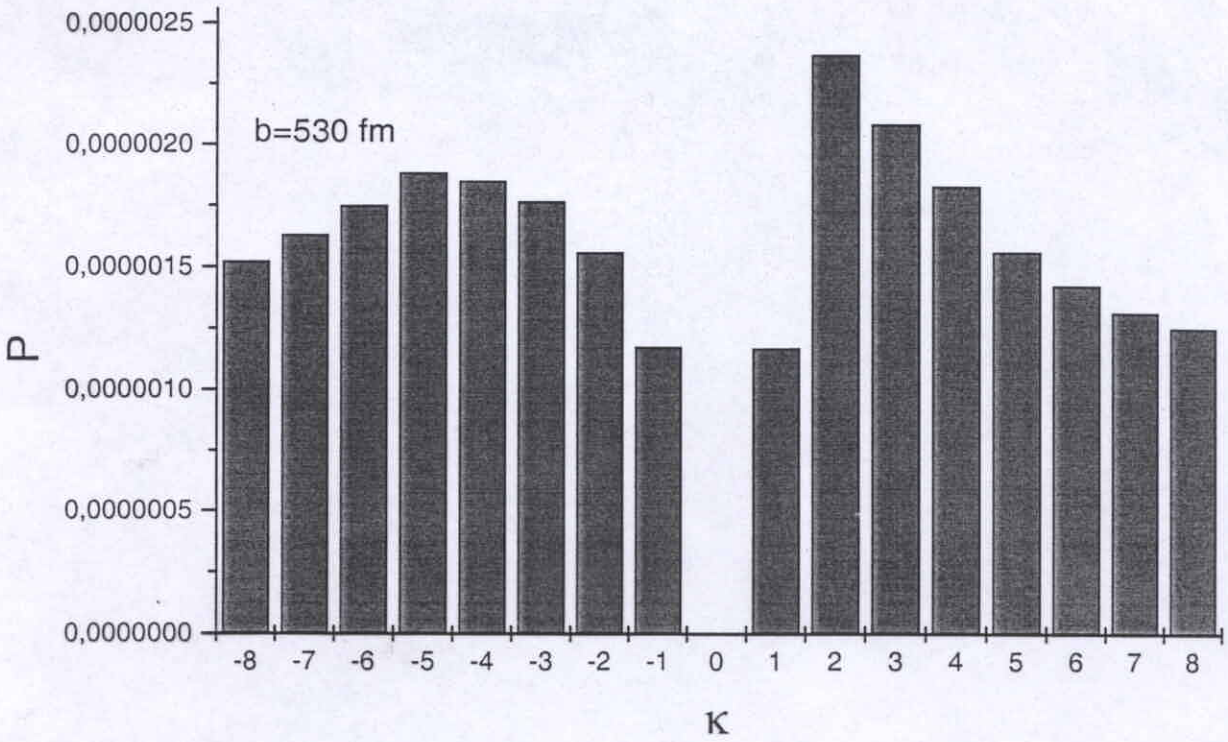
Summary: Precise calculations are possible. Problems with projection on physical states, because no analytical dynamical molecular states are known.

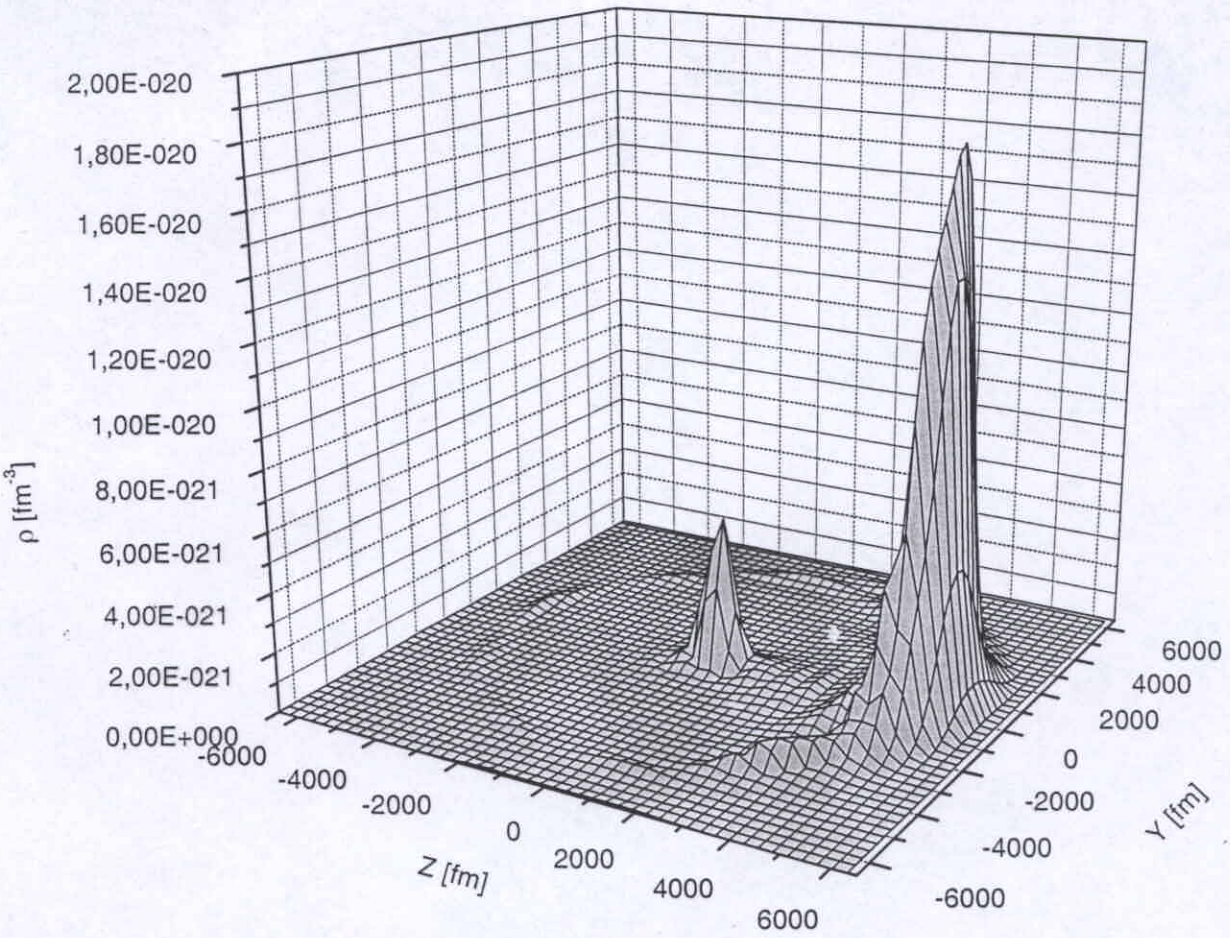
Extension to higher incident energies is carried out.


P(b) pair production with capture
1E-4
1E-5
1E-6
-800
-600
-400
-200
0
200
400
600
800
b [fm]

$Au^{79+}(y=2) + U^{92+}$


b=-58 fm
P
0,00014
0,00012
0,00010
0,00008
0,00006
0,00004
0,00002
0,00000
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
K


b=530 fm
P
0,0000025
0,0000020
0,0000015
0,0000010
0,0000005
0,0000000
-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8
K


2,00E-020
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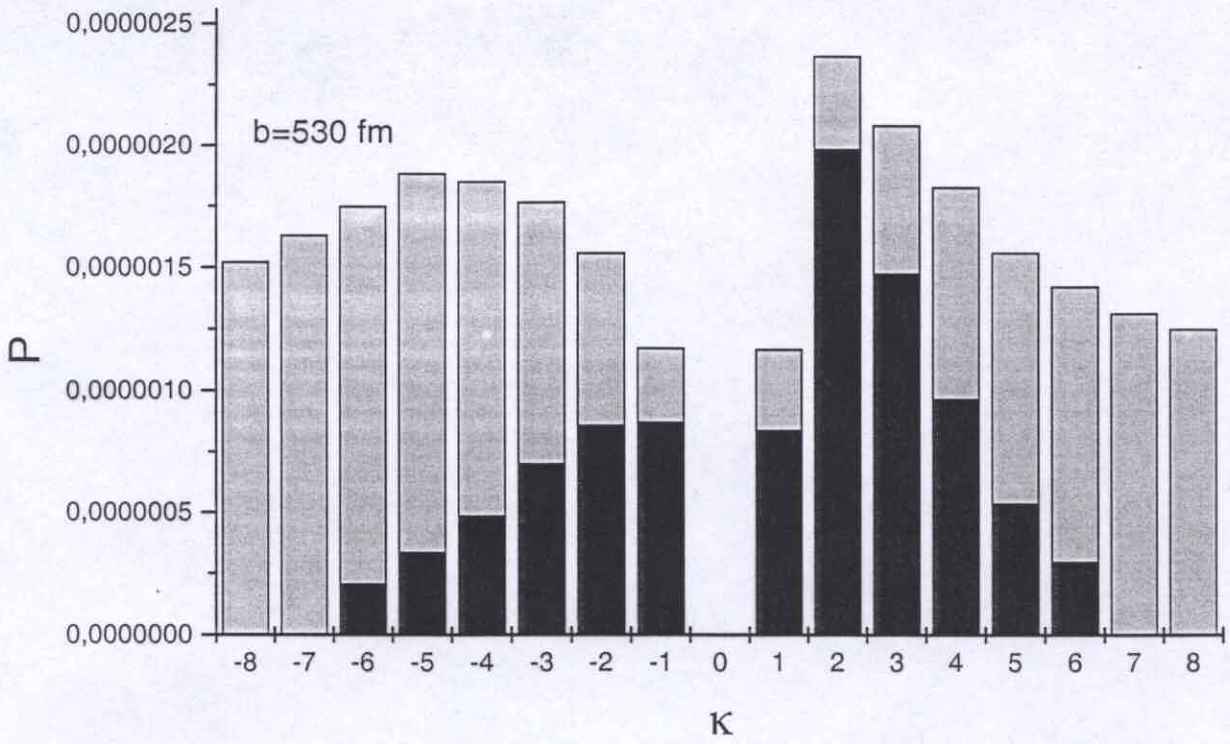
1,60E-020
1,40E-020
1,20E-020
1,00E-020
8,00E-021
6,00E-021
4,00E-021
2,00E-021
0,00E+000
ρ [fm-3]
-6000
-4000
-2000
0
2000
4000
6000
Z [fm]
6000
4000
2000
0
-2000
-4000
-6000
Y [fm]


0,0000025
0,0000020
0,0000015
0,0000010
0,0000005
0,0000000
b=530 fm
P
-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8
κ

4. Summary and conclusions

Electron-positron pair production with K-shell capture is the most important atomic process for the limitation of the lifetime of ultrarelativistic heavy ion colliders.

For high ion charges and small impact parameters the probability for pair production can only be obtained with a nonperturbative theory.

Nonperturbative theories are the coupled channels and finite element (difference] methods presented here.

The pair production yields basic information about the dynamics of the negative continuum under the influence of time-dependent strong electromagnetic fields. Information about the dynamics would be gained from the measurement of the impact parameter dependence or the angular distribution of the pair production.

At small impact parameters occurs a spontaneous electron-positron pair creation during the collision. This point has to be investigated in more detail.

This field needs more experimental data.