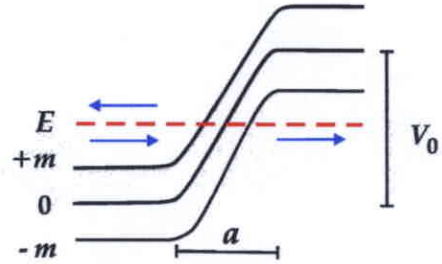
The Physics of Supercritical Fields in *Slow* Heavy-Ion Collisions

- Strong fields in QED – The charged vacuum
- Strong fields in heavy ion collisions
  - The quasimolecular picture
  - Supercritical collisions
  - Experimental observables: K holes
δ electrons
MO Xrays
Positrons
  - Past experiments at GSI-UNILAC
  - [• Nuclear time delay]
- Collisions of bare nuclei
  - Bound-free pair production
  - e^+e^- angular correlations
  - "Spectroscopy" of superheavy quasimolecules

Strong Fields in QED – History

- **Klein's paradox and pair creation**

O. Klein (1929): Anomalous transmission and reflection coefficients at a potential barrier with $V_0 > 2m$.



F. Sauter (1931): Tunneling through smooth barrier $T \sim \exp\left[-\frac{\pi mc^2}{\frac{V_0}{a}\frac{\hbar}{mc}}\right]$

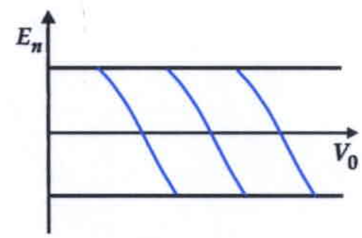
Euler + Heisenberg (1936), **Weisskopf**: Effective QED Lagrangian

Schwinger (1951): Pair production rate $\quad 2\,\mathrm{Im}L_{\mathrm{eff}} = \frac{eE}{(2\pi)^3}\sum_{n=1}^{\infty}\frac{1}{n}\mathrm{e}^{-nE_{\mathrm{cr}}/E}$

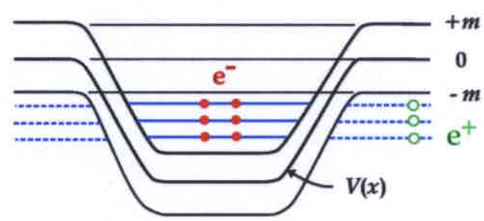
Critical field strength $\quad E_{\mathrm{cr}} = \frac{\pi m^2 c^3}{e\hbar} \sim 10^{16}\frac{\mathrm{V}}{\mathrm{cm}}$

- **Deep potential wells**

Schiff, Snyder, Weinberg (1940): Square well levels go down to $E_n = -mc^2$



Hund (1940s): Filling of the well by pair creation.



- **Supercritical atoms**

Sommerfeld formula: $E_{1s} = mc^2\sqrt{1-(Z\alpha)^2}$.

"Collapse of the wavefunction" at $Z > 1/\alpha = 137$.

Finite nuclei:

Pomeranchuk, Smorodinskij (1945); **Werner, Wheeler** (1958); **Voronkov, Kolesniko** (1961): $\quad E_{1s} \to -m$ at $Z \to Z_{\mathrm{cr}}$

Pieper, Greiner; Müller,... (≥ 1969)
Gershtein, Zeldovich, Popov (≥ 1969)
} Spontaneous pair creation. Charged vacuum. Heavy ion collisions.

Frankfurt group — Dynamics of e^+e^- creation.

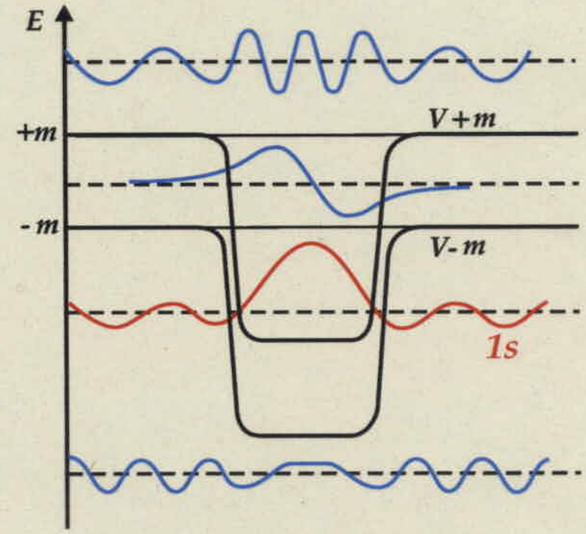
Supercritical Atoms

Bound states reach $E_n = -mc^2$
at $Z_{\rm cr}^{1s} = 172$, $Z_{\rm cr}^{2p_{1/2}}$, ...

$Z > Z_{\rm cr}$: Bound state embedded in continuum.

Empty K-shell: Spontaneous emission of 2 (spin) positrons

Decay width: γ_{1s} $\sim$ 5 keV
Life time: τ_{1s} $\sim$ 10^{-19} s



Relativistic contraction of wave functions: $\rho_{1s}(170)/\rho_{1s}(1) \simeq 10^9$

The Charged Vacuum

The vacuum polarisation charge density:

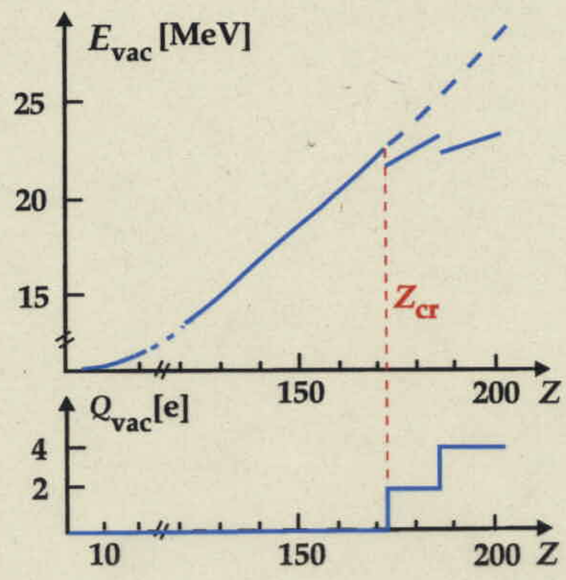
$$\begin{aligned}\rho_{\rm vac}(x) &= \langle 0|\,\hat\rho(x)\,|0\rangle = -{\rm i}e \lim_{x\to x'} {\rm Tr}(\gamma^0 S_{\rm F}(x',x)) \\ &= \frac{e}{2}\sum_{n<F}\varphi_n^\dagger(x)\varphi(x) - \frac{e}{2}\sum_{n>F}\varphi_n^\dagger(x)\varphi(x)\end{aligned}$$

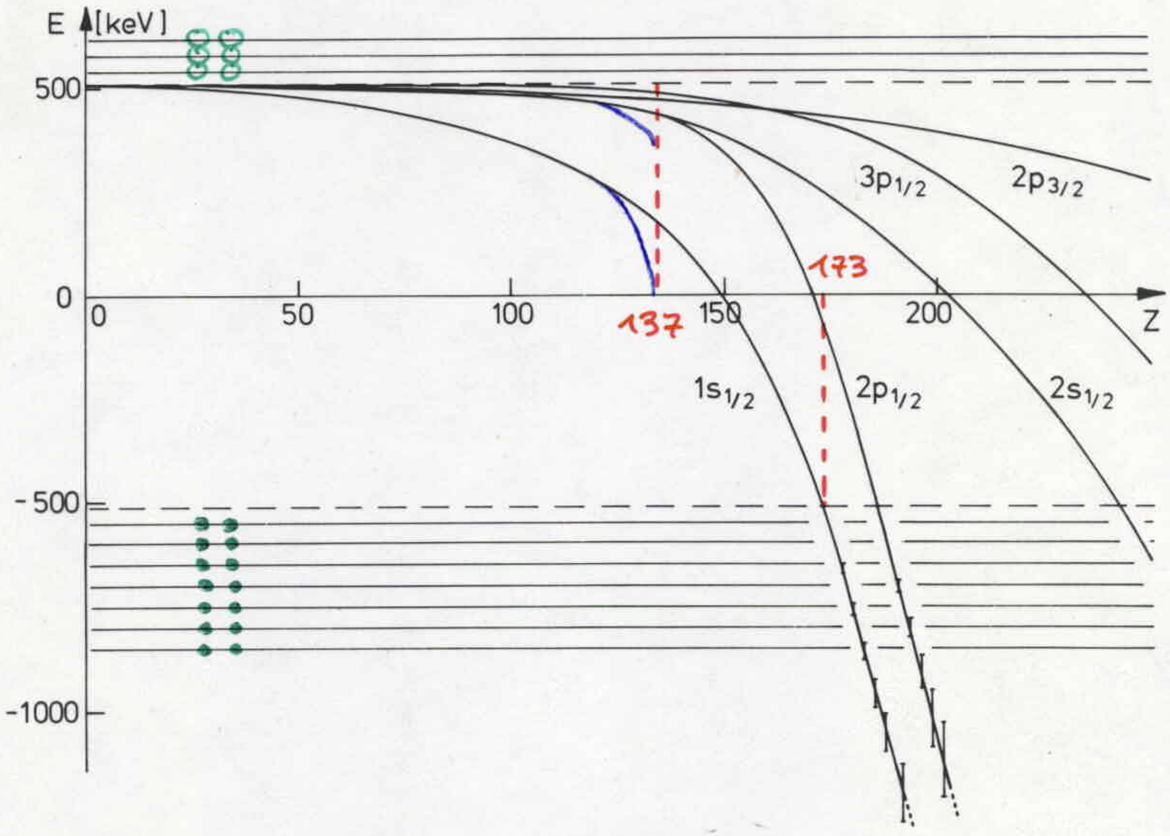
Fermi surface F at $E = -mc^2$.

In weak potentials: $Q_{\rm vac} = 0$

In supercritical systems: $Q_{\rm vac} = 2\,e\,N_{\rm vac}$ Real vacuum polarisation!

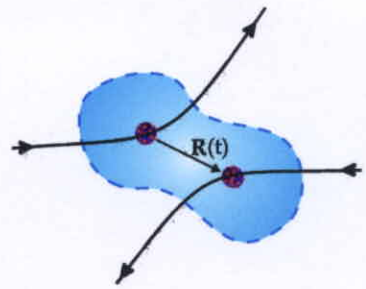
$N_{\rm vac}$: Number of electron levels that have fallen below $E = -mc^2$.





E
[keV]
500
0
-500
-1000
0
50
100
150
200
Z
137
173
$1s_{1/2}$
$2p_{1/2}$
$2s_{1/2}$
$3p_{1/2}$
$2p_{3/2}$

Strong electromagnetic fields in heavy ion collisions



Two Center Dirac Hamiltonian:

$$H_{\mathrm{TCD}}(\boldsymbol{R}) = -\mathrm{i}\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta m + V_1(\boldsymbol{r},\boldsymbol{R})$$

Semiclassical approximation:
$\boldsymbol{R}(t) =$ Rutherford trajectory.

If the velocity $\dot{\boldsymbol{R}}$ is "small" the electrons follow the adiabatic quasimolecular basis

$$H_{\mathrm{TCD}}(\boldsymbol{R})\,\phi_n = E_n(R)\,\phi_n$$

TCD solutions: B. Müller,... (> 1975)

Critical distance for U+U: R_{cr} = 37 fm (point nuclei, unscreened)

The time-dependent problem

$$\mathrm{i}\frac{\partial}{\partial t}\,\psi_n^{(+)}(\boldsymbol{r},t) = H_{\mathrm{TCD}}[\boldsymbol{R}(t)]\,\psi_n^{(+)}(\boldsymbol{r},t)$$

with the asymptotic condition $\psi_n^{(+)}(\boldsymbol{r},t\to-\infty)\to\phi_n$

Slow collisions: Expansion w.r.t. adiabatic basis

$$\psi_n^{(+)}(\boldsymbol{r},t) = \sum_k a_{nk}(t)\,\phi_n(\boldsymbol{r},\boldsymbol{R}(t))\,\mathrm{e}^{-\mathrm{i}\chi_k(t)}$$

with the phase $\chi_k(t) = \int^t dt'\,E_k(R(t))$

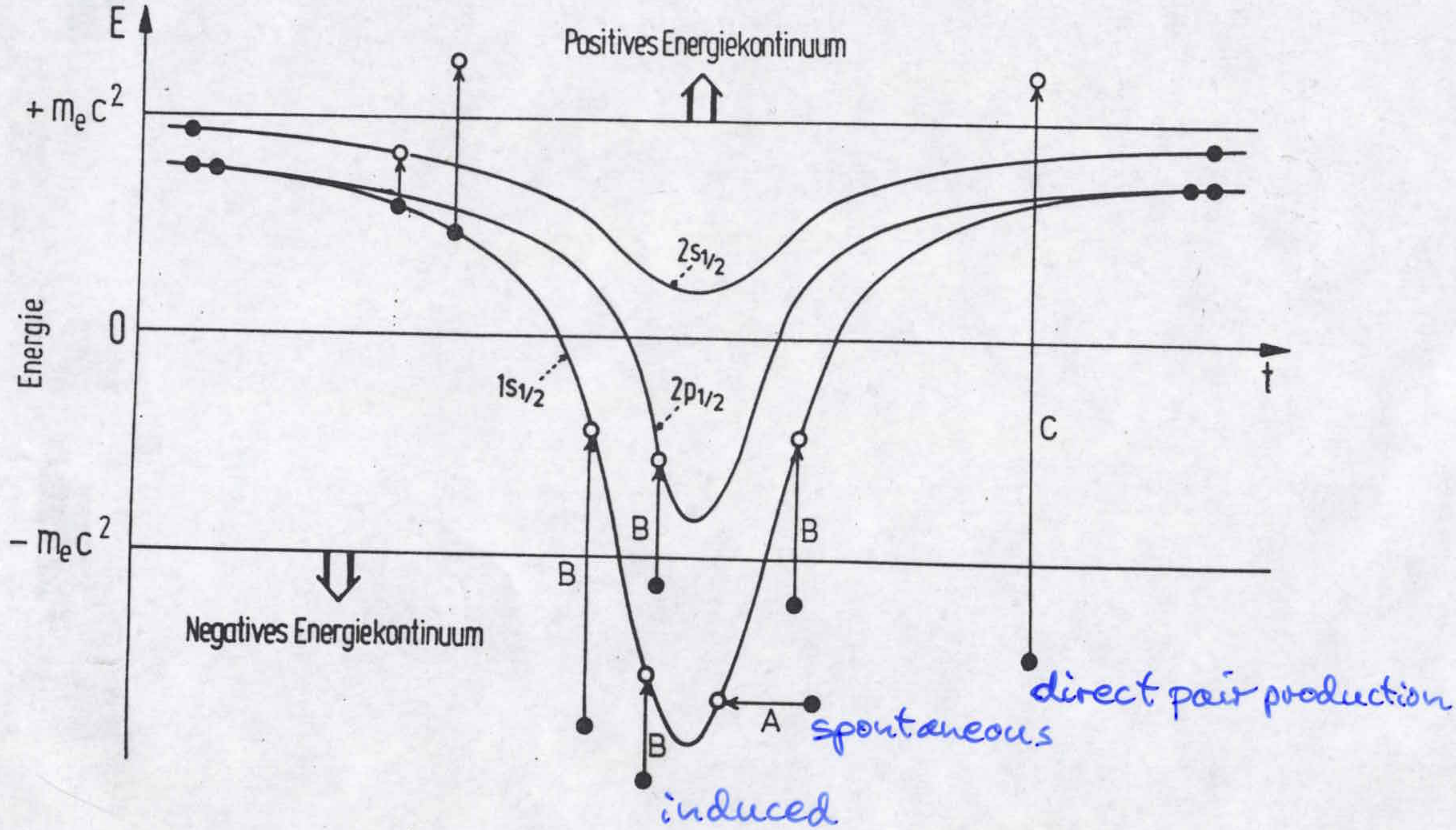
Coupled channel equations:

$$\dot{a}_{nk}(t) = -\sum_{j\neq k} a_{nj}(t)\,\langle\phi_k|\partial/\partial t|\phi_j\rangle\,\mathrm{e}^{\mathrm{i}(\chi_k-\chi_j)}$$

Numerical integration with initial condition $a_{ik}(t\to-\infty)=\delta_{ik}$

Coupling operator: $\dfrac{\partial}{\partial t} = \dot{R}\dfrac{\partial}{\partial R} - \mathrm{i}\boldsymbol{\omega}\cdot\boldsymbol{J}$

Adiabatic energy levels



Field theoretical description

Field operator: $\hat{\psi}(x) = \sum_{n>F} \hat{b}_n^{\rm in}\, \psi_n^{(+)} + \sum_{n<F} \hat{d}_n^{\rm in\dagger}\, \psi_n^{(+)}$

The state vector $|F\rangle$ is prepared as $\begin{cases} \hat{b}^{\rm in}|F\rangle = 0 \;, & n > F \\ \hat{d}^{\rm in}|F\rangle = 0 \;, & n < F \end{cases}$

Expansion in terms of out-operators: $\hat{\psi}(x) = \sum_{n>F} \hat{b}_n^{\rm out}\, \psi_n^{(-)} + \sum_{n<F} \hat{d}_n^{\rm out\dagger}\, \psi_n^{(-)}$

Canonical transformation: $\hat{b}_n^{\rm out} = \sum_{k>F} \hat{b}_k^{\rm in} a_{kn} + \sum_{k<F} \hat{d}_k^{\rm in\dagger} a_{kn}$

Number of created particles in state i:

$$N_i = \langle F|\hat{b}_i^{\rm out\dagger}\hat{b}_i^{\rm out}|F\rangle = \sum_{k<F} |a_{ki}(\infty)|^2$$

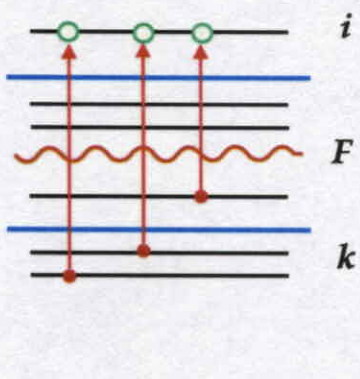
Number of created holes (antiparticles) in state j:

$$\overline{N}_j = \langle F|\hat{d}_j^{\rm out\dagger}\hat{d}_j^{\rm out}|F\rangle = \sum_{k<F} |a_{kj}(\infty)|^2$$

Number of particle-hole pairs :

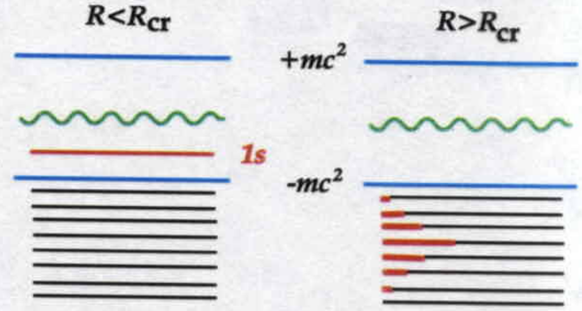
$$\begin{aligned} \overline{N}_{ij} &= \langle F|\hat{b}_i^{\rm out\dagger}\hat{b}_i^{\rm out}\hat{d}_j^{\rm out\dagger}\hat{d}_j^{\rm out}|F\rangle \\ &= N_i \cdot \overline{N}_j - \sum_{k<F}\sum_{l>F} a_{ki}^*\, a_{kj}\, a_{lj}^*\, a_{li} \end{aligned}$$

Incoherent sum of single particle probabilities.
(Neglecting correlation effects)



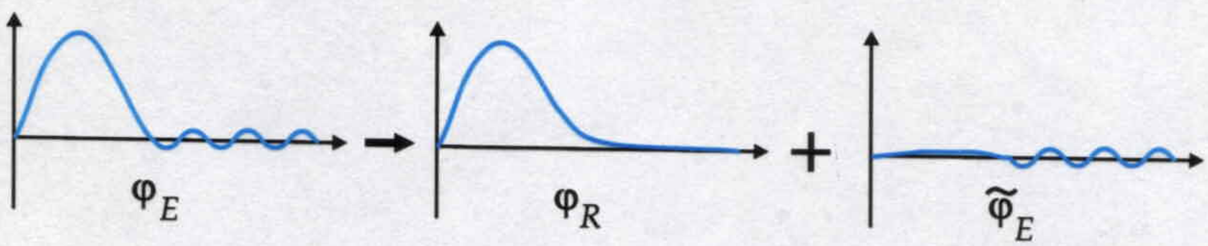
Dynamics of supercritical collisions

A bound state is lost from the discrete spectrum and moves through the antiparticle continuum as a narrow resonance.



Projection operator formalism

Extract quasistationary state φ_R out of the continuum φ_E. Generate new nonresonant continuum $\tilde{\varphi}_E$



so that $\int dE\,|\varphi_E\rangle\langle\varphi_e| \;=\; \int dE\,|\tilde{\varphi}_E\rangle\langle\tilde{\varphi}_E| \;+\; \int dE\,|\varphi_R\rangle\langle\varphi_R|$

$$I \;=\; P \;+\; Q$$

The modified continuum is defined through

$$(E - PHP)\,\tilde{\varphi}_E \;=\; 0$$

or

$$(E - H)\tilde{\varphi}_E \;=\; -\langle\varphi_R|H|\phi_E\rangle\,\phi_R$$

Consequence: H is **nondiagonal** in the projected basis.

Additional coupling:

$$\langle\varphi_E|\frac{\partial}{\partial t}|\varphi_R\rangle \quad\longrightarrow\quad \langle\tilde{\varphi}_E|\frac{\partial}{\partial t}|\varphi_R\rangle \quad+\quad \mathrm{i}\,\langle\tilde{\varphi}_E|H|\varphi_R\rangle$$

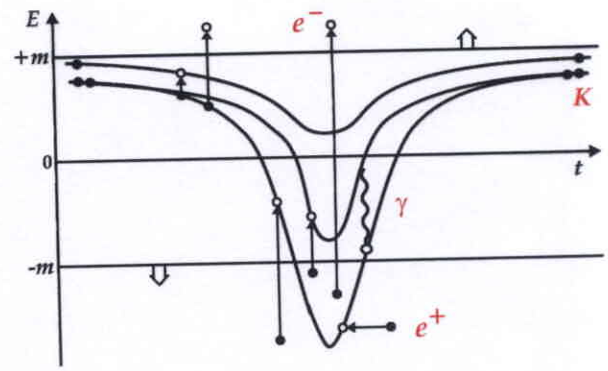
Coherent sum of "induced" and "induced" couplings

In the static limit : φ_R decays exponentially with a width

$$\Gamma = 2\pi|\langle\varphi_R|H|\tilde{\varphi}_E\rangle|^2$$

Experimental observables

In systems with $(Z_1 + Z_2) > 137$: "Collapse" of wave functions leads to strong $\partial/\partial R$ couplings. Multi-step processes become important.



- **K-hole production**

High ionisation rates: $P_{1s\sigma} \simeq 10\%$.

Approximate scaling behaviour:

$$\boxed{P_{1s\sigma}(b) \simeq D(Z)\,\mathrm{e}^{-2R_{\min}q_{\min}}} \quad \text{where} \quad q_{\min} = \frac{E^B_{1s\sigma}}{\hbar v_{\text{ion}}}$$

$\rightarrow$ "Spectroscopy" of superheavy quasimolecules.

- **δ-electron production**

The high-energy tail (up to $E_e > 2\ \mathrm{MeV}$) probes the high-momentum components of the the quasimolecular wave functions.

- **Quasimolecular X rays (MOX)**

Broad photon spectra. No "end point", quasistatic picture not applicable.

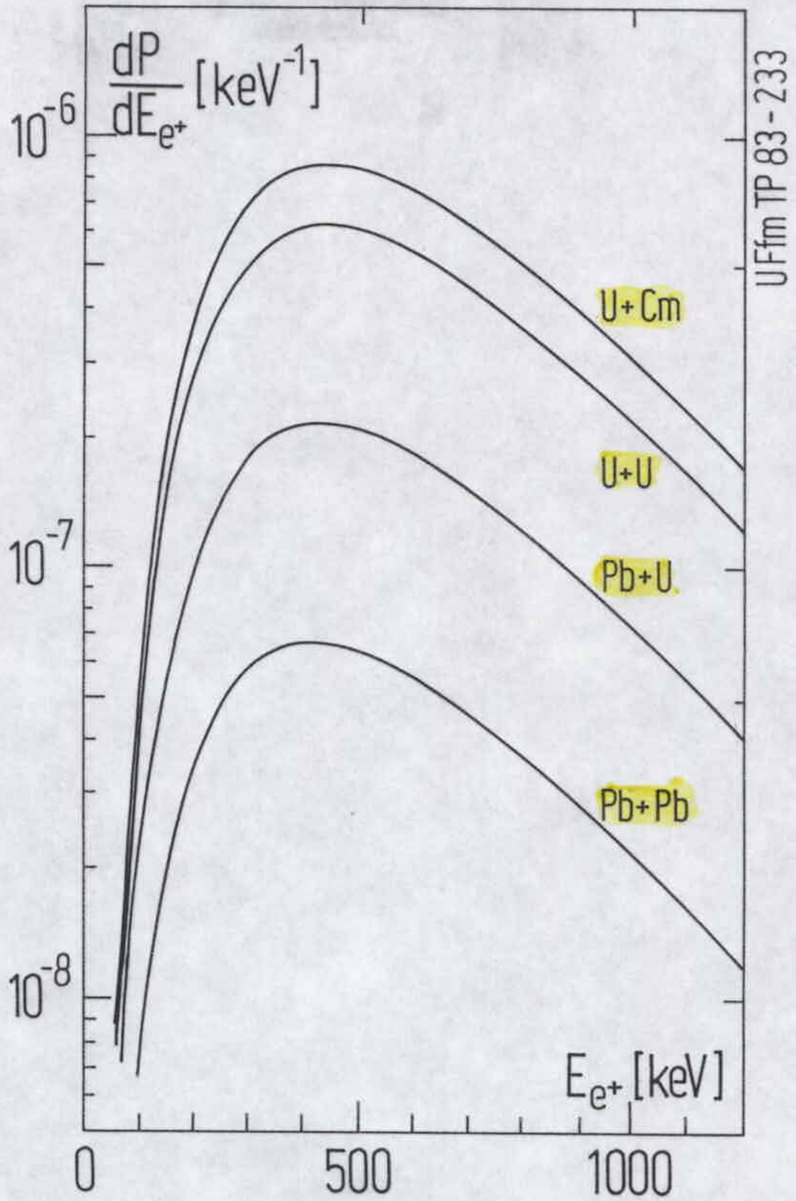
- **Positron creation**

Drastic increase of positron yield with nuclear charge Z:

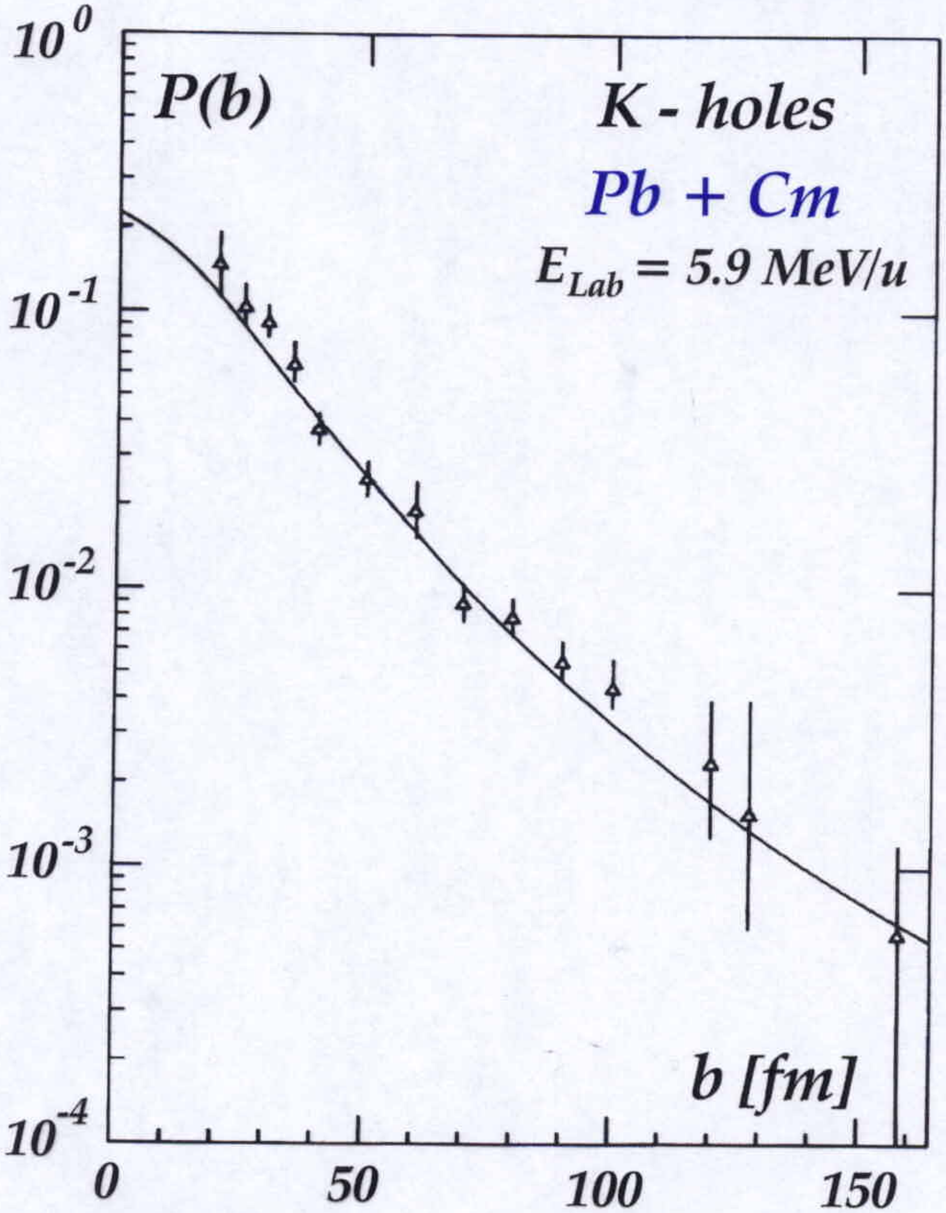
$$\boxed{P_{e^+} \propto Z^{20}}$$

- *No qualitative signal* for level diving expected.
- Good quantitative agreement with experiments: $P(b), P(Z), dP/dE_{e^+}$.

Positron Spectrum



Experiment: D. Liesen et al.
F. Bosch et al.



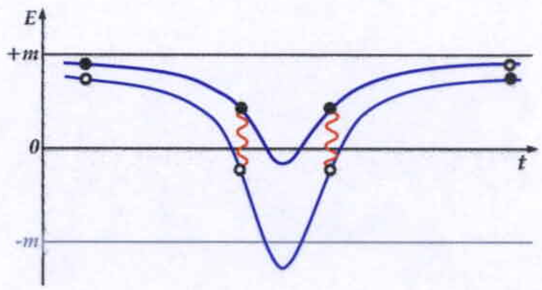
Collisions involving fully stripped ions

1. K-K Vacancy transfer

Interference involving the phase integral $\phi = \int_{-\infty}^{+\infty} dt\, \Delta E_{1s,2p_{1/2}}(R(t))$

2. Quasimolecular X rays

Interference between transitions on the incoming/outgoing half of the trajectory: Oscillating MOX spectrum.



3. Positron creation

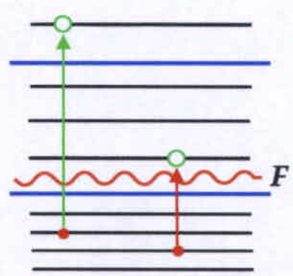
Advantages of bare ions

- No electron-electron interaction $\rightarrow$ Reduced theoretical uncertainty.
- Stronger Coulomb field, deeper electron binding.

E.g. for $Z = 188$, $R = 15\text{fm}$: $E_{1s}(\text{bare}) = -1498\text{keV}$, $E_{1s}(50^+) = -1377\text{keV}$

- Removal of Pauli Blocking

Normally $E_p \rightarrow 1s$ transitions require ≥ 2 steps:
Suppression because $\overline{N}_{1s\sigma} \lesssim 0.05$.
In bare collisions bound-free electron positron pair production becomes possible as a one-step process.



"Electron capture from the vacuum"

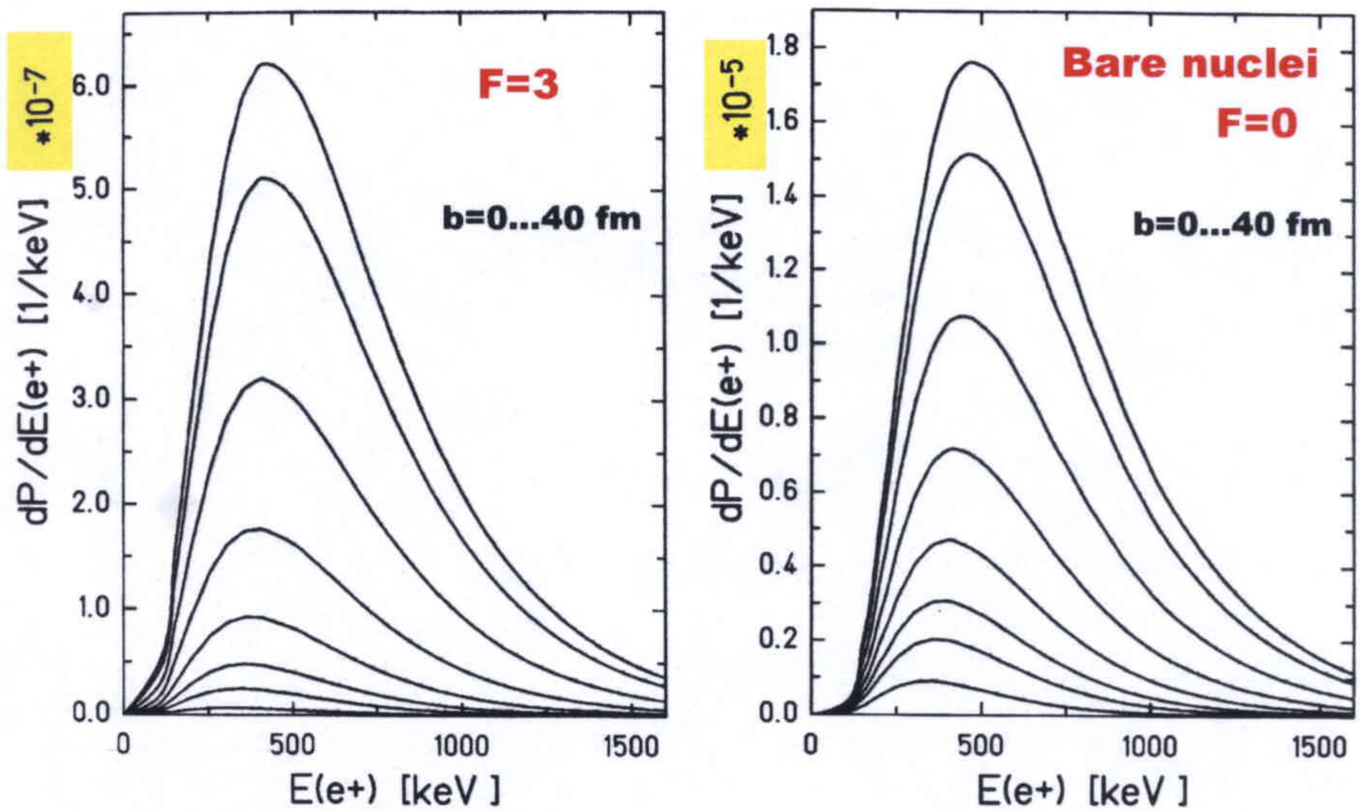
- Positron rates get enhanced by factors of 30 . . . 100.
- Largest contribution: Capture into the $1s\sigma$ state.
- Sensitivity to wave function and binding energy of $1s\sigma$.

Theory: U. Müller et al. Phys. Rev. A37, 1449 (1988)

Positron spectra

U + U

E = 6.2 MeV/u



Approx. two orders of magnitude increase of intensity in bare collisions!

Pair production in bare collisions near the Coulomb barrier

$U^{92+} + U^{92+}$ at $E_{\mathrm{cm}} = 680$ keV i.e. $E_{\mathrm{proj}} = 5.7$ MeV/u

b [fm]	$\Sigma_{\mathrm{bound}} N_i$	$\int dE\, \overline{N}_{\mathrm{pos}}(F=0)$	$\int dE\, \overline{N}_{\mathrm{pos}}(F=3)$	Ratio bare/normal
0	1.04 (-2)	1.06 (-2)	2.93 (-4)	36
10	6.05 (-3)	6.12 (-3)	1.45 (-4)	42
20	2.33 (-3)	2.34 (-3)	3.65 (-5)	64
30	8.80 (-4)	8.80 (-4)	8.24 (-6)	107
40	3.42 (-4)	3.42 (-4)	1.88 (-6)	182
σ	105 mb	105 mb	2.3 mb	46

U. Müller, T.H.J. deReus, J. Reinhardt, B. Müller, W. Greiner, G. Soff: Phys. Rev. A37, 1449 (1988)

In slow collisions electron capture into bound states is the dominant channel for pair production.

Note: Bound-free pair production was observed in *relativistic collisions* at BEVALAC:
A. Belkacem, H. Gould, B. Feinberg, R. Bossingham, W.E. Meyerhof: Phys. Rev. Lett. 71, 1514 (1993)

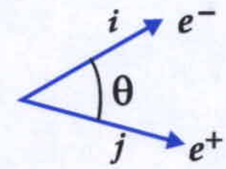
0.956 GeV U^{92+} + Au

$$\sigma_{\mathrm{bound-free}} = 2.2\ \mathrm{b}$$
$$\sigma_{\mathrm{free-free}} = 3.3\ \mathrm{b}$$

Electron-positron pair correlations

Interference $s_{1/2}$ and $p_{1/2}$ waves leads to angular correlations .

$$
\begin{aligned}
N_{ij} &= N_i \cdot \overline{N}_j \\
&\quad +2 \sum_{\kappa=\pm 1} \Big| \sum_{n<F} a_{ki}^{(\kappa)} a_{kj}^{(\kappa)*} \Big|^2 \\
&\quad -4 \cos\theta \, \mathrm{Re} \Big[\mathrm{e}^{\mathrm{i}\Delta} \big(\sum_{k<F} a_{ki}^{(-)*} a_{kj}^{(-)} \big) \big(\sum_{k<F} a_{ki}^{(+)} a_{kj}^{(+)*} \big) \Big] \\
&\equiv S + C + A \cos\theta
\end{aligned}
$$



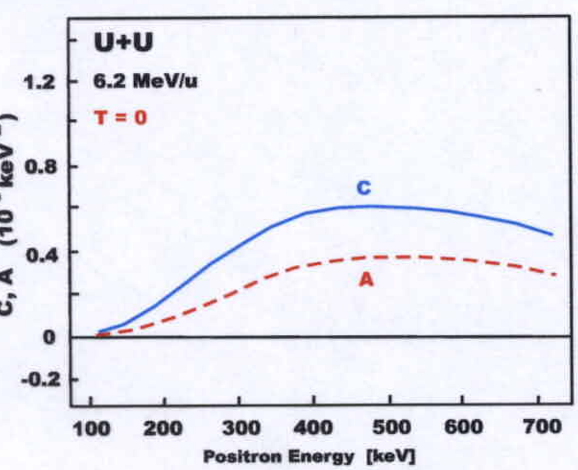
where $\left.\begin{array}{l} (-): \; s_{1/2} \text{ states} \\ (+): \; p_{1/2} \text{ states} \end{array}\right\}$ interference

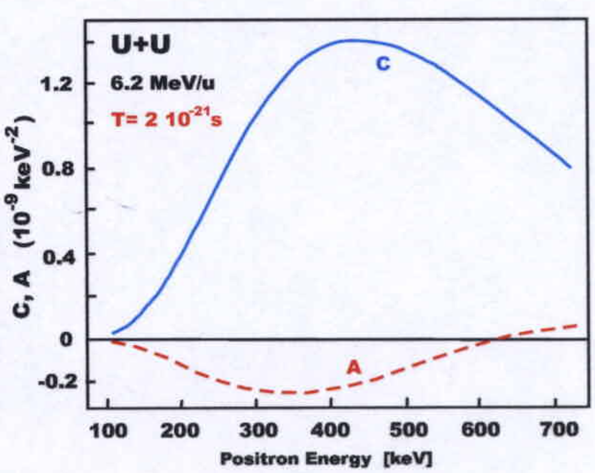
$\Delta = (\delta_i^{(-)} - \delta_i^{(+)}) - (\delta_j^{(-)} - \delta_j^{(+)})$ Scattering phase shifts

Theory: O. Graf, J. Reinhardt, B. Müller, W. Greiner, G. Soff: Phys. Rev. Lett. 61, 2831 (1988); Z. Physik 28, 81 (1987)

In ordinary collisions : Most electrons originate from ionization.
→ The "random" term S is dominant. Isotropic angular distribution.

In collision of bare nuclei : Each electron is accompanied by a positron. S is negligible. Angular correlation carries information on mixture of $s_{1/2}$ and $p_{1/2}$ contributions.





Qualitative signal for spontaneous pair creation:
Negative anisotropies in time delayed collisions of bare nuclei.

“Spectroscopy” of superheavy quasimolecules

Exploit sensitive dependence of excitation rates on transition energies.

$1s\sigma$ ionisation

Ionisation amplitude in 1st order perturbation theory :

$$a_{1s,E} = -\int_{-\infty}^{\infty} dt\, \dot{R}\, \langle E|\partial/\partial R|1s\rangle\, \mathrm{e}^{-\mathrm{i}\int^t dt'\, (E - E_{1s})}$$

Radial matrix element (monopole approximation)

$$\langle E|\partial/\partial R|1s\rangle = \sqrt{\frac{D(Z)}{4\pi^2 m}} \left(\frac{m}{E}\right)^{\gamma/2} \frac{1}{R}$$

Approximate solution

$$|a_{1s,E}|^2 \simeq \frac{D}{4m} \left(\frac{m}{E}\right)^{\gamma} \mathrm{e}^{-\frac{2\tau_0}{\hbar}(E - E_{1s}(R_0))}$$

where
$$\tau_0 = \frac{1}{v}\left[b + a(\pi - \text{arctan}(b/a))\right] \simeq R_0/v$$

R_0: distance of closest approach
τ_0: characteristic collision time

K-hole rate:

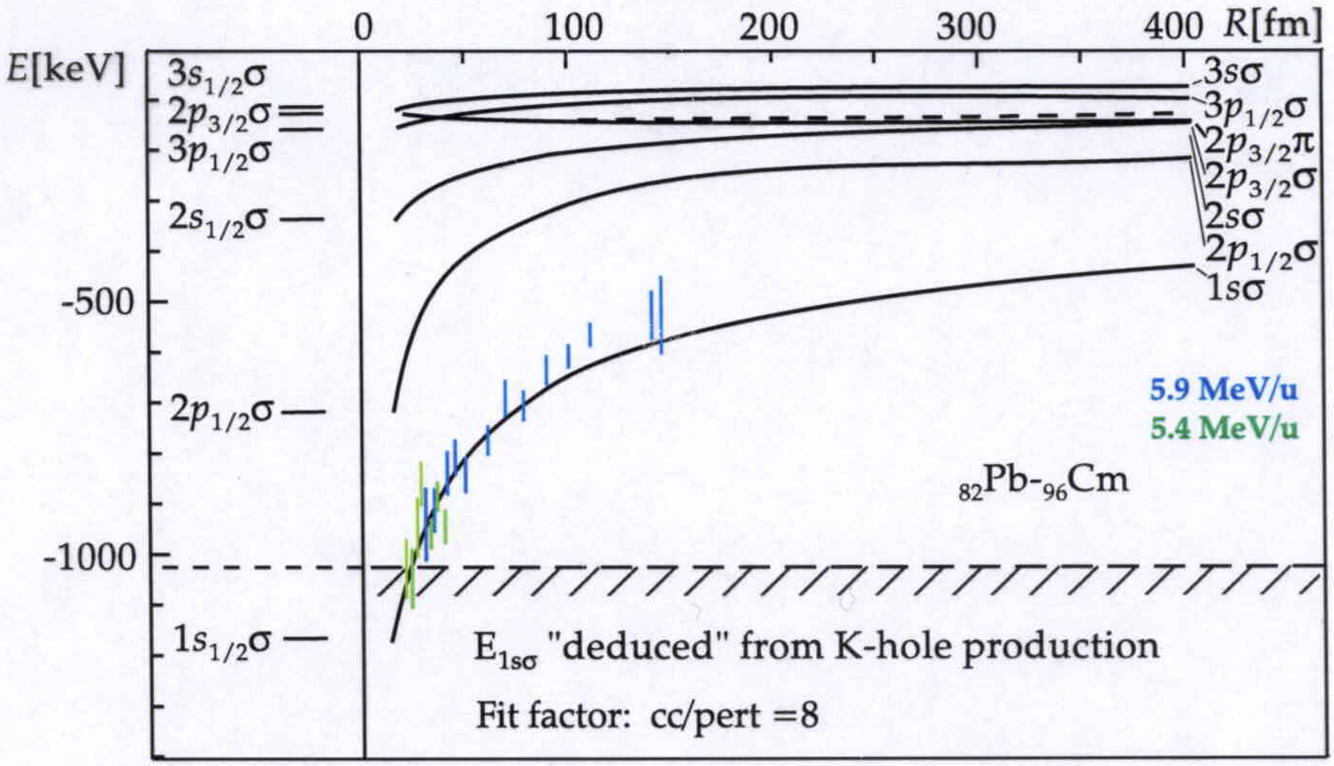
$$P_{1s\sigma}(b) = 2\int_m^{\infty} dE\, |a_{1s,E}|^2 = D(Z)\, N(Z,b,v)\, \mathrm{e}^{-\frac{2\tau_0}{\hbar}(m - E_{1s}(R_0))}$$

B. Müller, J. Reinhardt, W. Greiner, G. Soff, Z. Physik A311, 151 (1983)

$\longrightarrow$ Deduce $E_{1s}(R)$ from $P_{1s}(b)$.

Problem in (nearly) symmetric: *Vacancy sharing* of $1s\sigma$ and $2p_{1/2}\sigma$ levels.

Deficiency of the method: Perturbation theory not well justified.
Receipe: Replace $D \to D_{\text{eff}}$ (Fit to coupled channel results).



Large systematic uncertainties.

Systematic way to check QED predictions:

Perform coupled channel calculations with modified energies of the basis state(s):

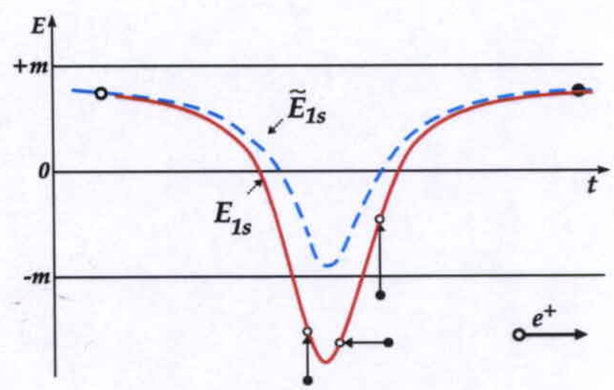
$$\widetilde{E}_n(R) = E_n(R) + \delta E_n(R)$$

QED modification

better:

$$\widetilde{H}(R) = H(R) + \delta H(R)$$

(Also wave functions get modified.)



Comparison Experiment/Theory $\longrightarrow$ Deviation $\Delta[E_n]$

Determine $E_n(R)$ from minimization of Δ.
The functional derivative $\frac{\delta \Delta}{\delta E_n}$ provides information on the sensitivity to local energy variations.

Optimal sensitivity of $E_{1s}(R)$ can be expected for bound-free pair production

Conclusions

- Slow heavy ion collisions offer a unique opportunity to study Strong Fields in QED .
- The collision dynamics seems to be well understood using the quasimolecular picture.
- Supercritical binding and spontaneous pair creation not yet proven experimentally.
- Collisions of bare + bare or bare + neutral ions offer new opportunities:

  Direct access to the quasimolecular $1s\sigma$ state.

  Bound-free electron-positron pair creation.
  Interferences in MOX rays.
  Structures in K-K vacancy transfer.