Free and Bound-free pair production in relativistic heavy ion collisions

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Strong fields in Relativistic HI Collisions

- Study of effects of strong fields at high energy Heavy Ion Collisions:
- RHIC: $\gamma_{coll} = 100$, LHC $\gamma_{coll} = 3400$.
- GSI Future Project: γ_{lab} up to 30, $\gamma_{coll} \approx 4$.
- Colliding mode: γ_{coll} up to 30.
- Results already exist for AGS ($\gamma_{lab} = 10$) and SPS ($\gamma_{lab} = 170$).

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- Colliding mode: γ_{coll} up to 30.
- Results already exist for AGS ($\gamma_{lab} = 10$) and SPS ($\gamma_{lab} = 170$).
- Two aspects: Free pair production, Bound free pair production.
- Some reviews already exist:
  Eichler&Meyerhof: Relativistic Atomic Collisions,
  Bertulani&Baur, Physics Reports 163 ('88) 299,
  Baur, Hencken, Trautmann, Top. Rev., J. Phys. G24 ('98) 1657, Sec. 7
  Baur, Hencken, Trautmann, Sadovsky, Kharlov,
  Physics Reports **364**, 359 ('02), Sec. 7

Free pair production: General theory

- External field approach well justified in heavy ion case.
- Interest in pair production in external field goes back to foundations of QED
  (Schwinger, Feynman, Landau&Lifschitz).
- Peculiarity of HI collisions: Two pure Coulomb fields.
- Pair production occurs only starting from second order.
- At low energies: Molecular effects, Two-center Dirac equation.
  Momberger et al., Rumrich et al., Thiel et al.

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- At low energies: Molecular effects, Two-center Dirac equation.
  Momberger et al., Rumrich et al., Thiel et al.
- Strong fields, but of short duration:
  Perturbative treatment useful!
- Two aspects are of interest in strong external fields:
- Several Fermion lines:
  Multiple pair production, Unitarity corrections.
- Higher order interaction within one Fermion line:
  Coulomb corrections.

Cross section in lowest order calculation

- First case: Cross section for pair production in lowest order. Landau&Lifschitz (1934).
- Total cross section calculated by Racah (1937):

$$\sigma = \frac{Z^4 \alpha^4}{\pi m^2} \frac{28}{27} \left(\ln^3 \gamma^2_{coll} - 2.19 \ln^2 \gamma^2_{coll} + \cdots \right)$$

- Cross sections for e^+e^- pair production are huge:
  200 kb for Pb-Pb at LHC,
  700 b for Ca-Ca at LHC,
  30 kb for Au-Au at RHIC.

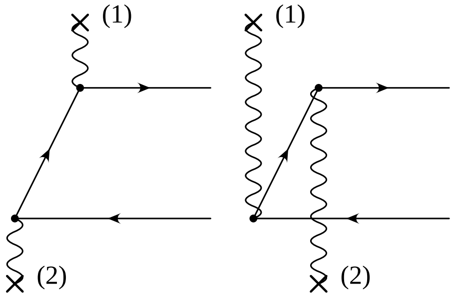
Cross section in lowest order calculation

- First modern approach: Direct MC-integration of 9-dim. integrals

  Bottcher&Strayer PRD39 (1989) 1330.

- Analytic expression for differential cross section

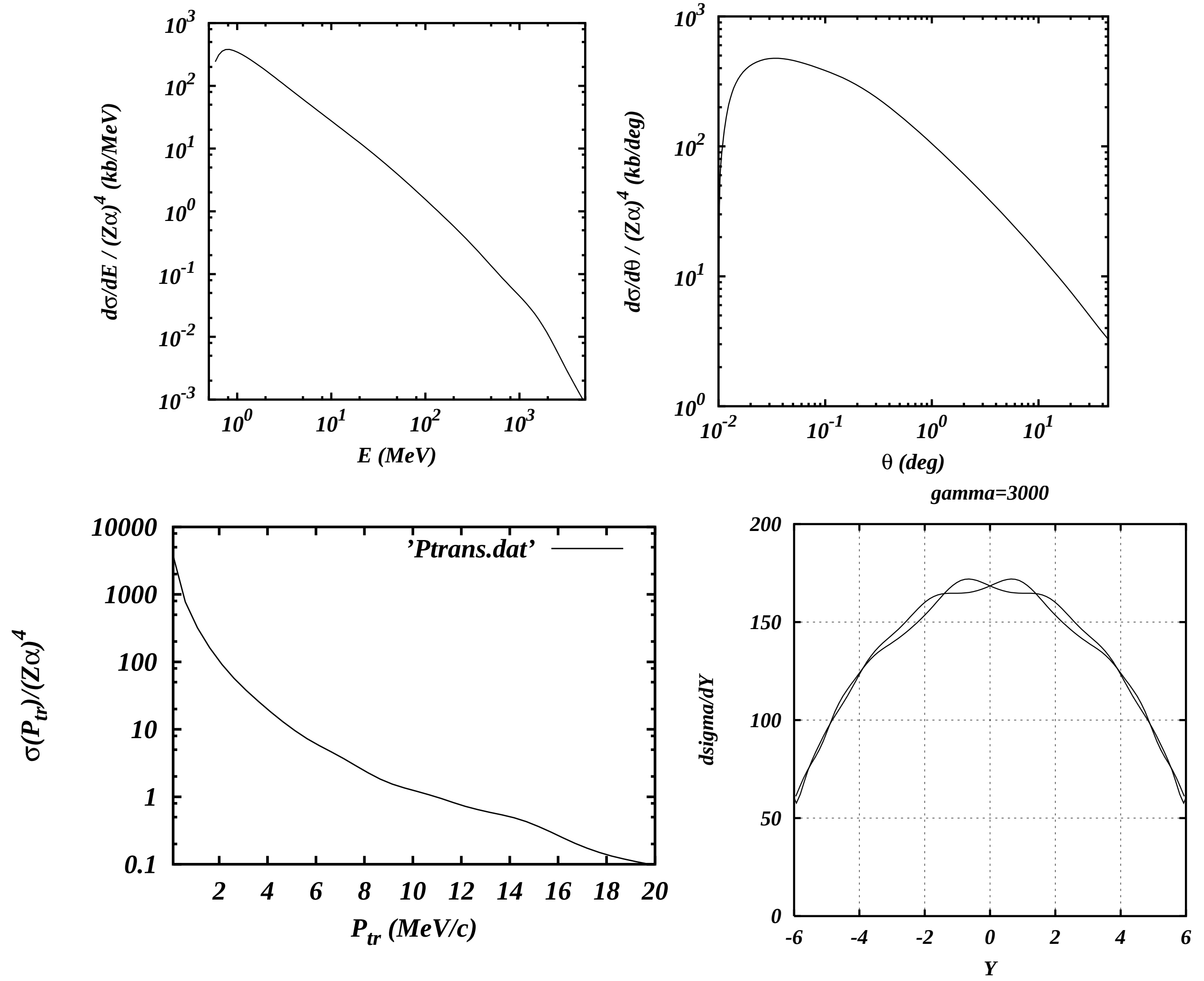
  Hencken et al. PRA51 (1995) 1874, Alscher et al. PRA55 (1997) 396



- Total cross section by MC integration.
  Confirms Racah formula to high accuracy (less than $1^{0}/_{00}$)

Cross section in lowest order calculation

Differential cross section for Pb-Pb@LHC (Collider frame):



Cross section in lowest order

- Study of pair production as background for the L0 trigger at ALICE.

  Hencken, Sadovsky, Kharlov, ALICE-INT-2002-011, 2002
- Integrated into ALIRoot event generator for ALICE.

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- Integrated into ALIRoot event generator for ALICE.
- Relevance for lower γ (GSI):
- “Benchmark” for all higher order effects.
- Differential cross section available. Inclusion of experimental efficiencies easily included.
- Deviations from lowest order results were already detected at AGS.

$P(b)$ lowest order theory

- In semiclassical picture:
  Impact parameter dependent probability well defines.
  Hencken, Trautmann, Baur, PRA49 (1994) 1584.
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- Calculated via Fourier transform of $P(b)$, numerical integration over final states.

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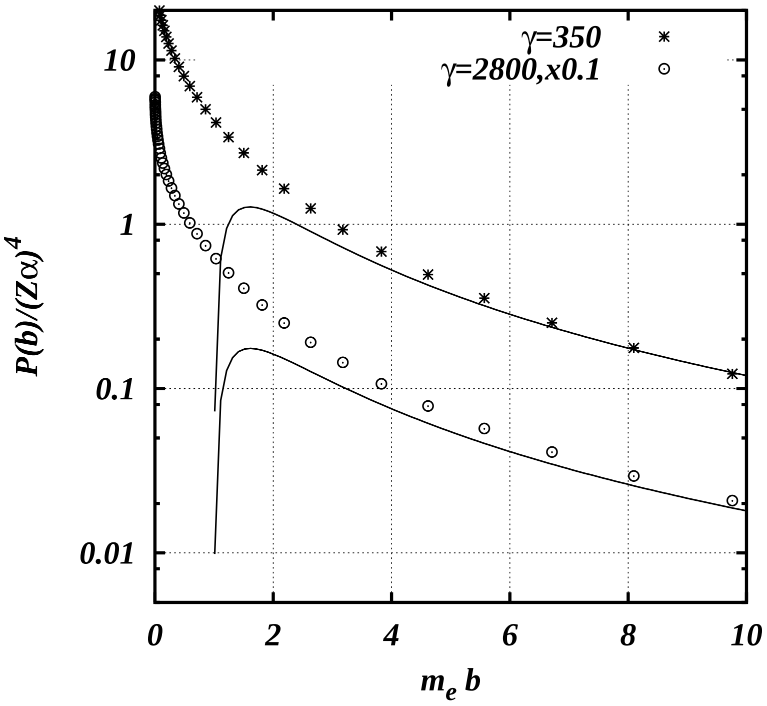
- Calculated via Fourier transform of $P(b)$, numerical integration over final states.
- DEPA calculations are not reliable below the region $b \leq 1/m_e$.
- Recent analytical results by Lee, Milshteyn, Serbo (PRA65): ($R_{cutoff} = 1/m_e$, based on DEPA).
- For $1/m_e < b < \gamma_{coll}/m_e$:

$$P(b) = \frac{28}{9\pi^2} \frac{(Z_1\alpha Z_2\alpha)^2}{(m_e b)^2} \left[2\ln\gamma_{coll}^2 - 3\ln(m_e b)\right] \ln(m_e b)\,,$$

Comparison to DEPA approach



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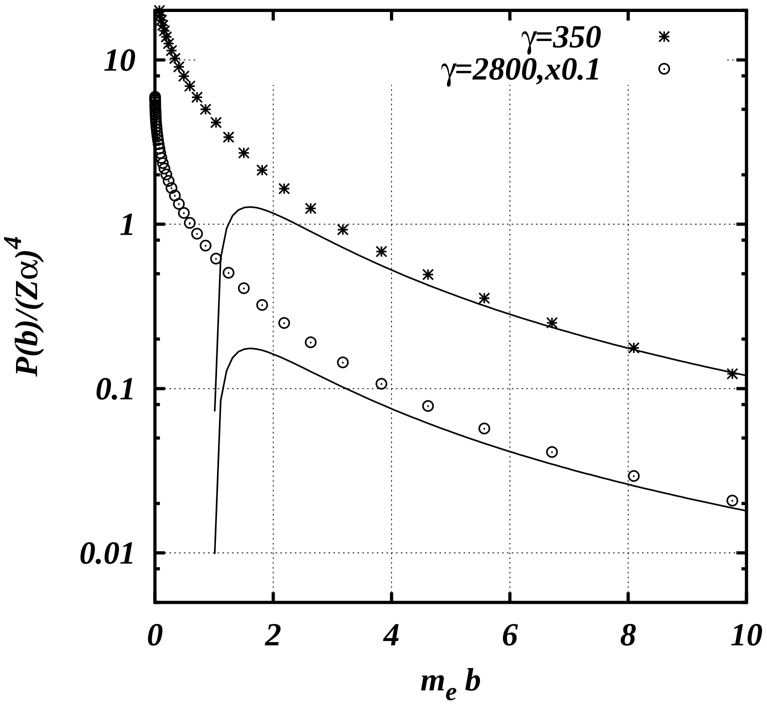
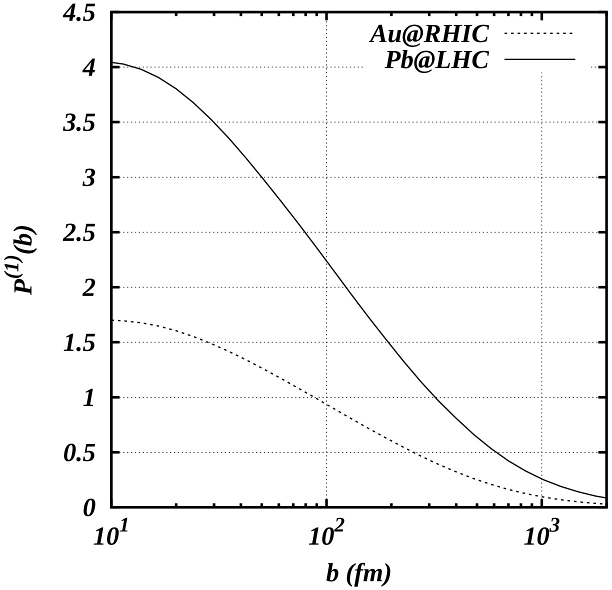


- Again also differential probabilities are available.
- Can one enhance the small b effects by small-b tagging?
- Effects of the photon polarization:
  Correlation of the pairs with direction of $\vec{b}$.

Unitarity violation of the probability

- $P(b)$ violates unitarity for RHIC and LHC energies:



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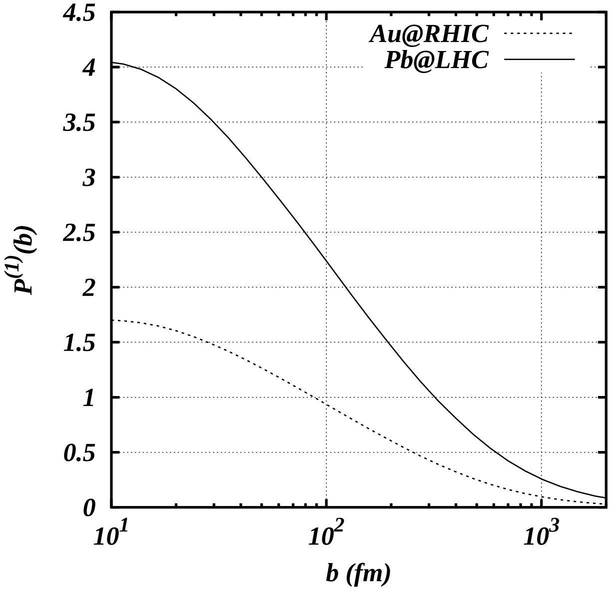
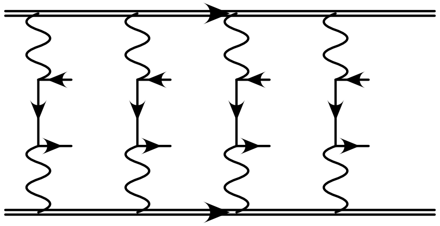


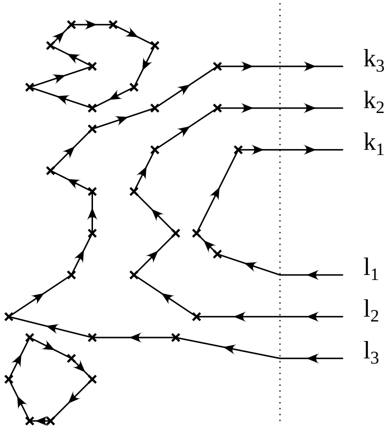


- Considered in a series of papers:
  Baur PRA42, Rhoades-Brown&Weneser PRA44, Best et al. PRA46, Hencken et al. PRA51, Baltz et al. NPA695, Aste et al. EPJC23, Bartos et al. hep-ph/0109281
- Higher order effects are important:
  Multiple pair production in one collision.

Unitarity violation of the probability

- External field approach leads to matrix element

$$S_N = \langle 0|\, S \,|0\rangle \sum_\sigma \text{sgn}(\sigma) s^{+-}_{k_1 l_{\sigma(1)}} \cdots s^{+-}_{k_N l_{\sigma(N)}},$$



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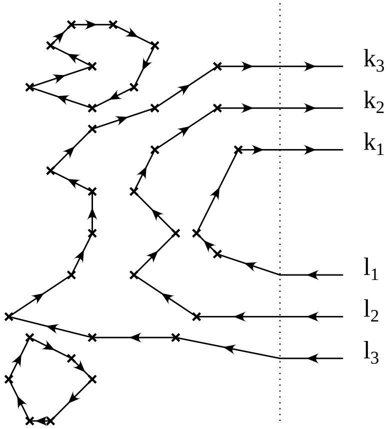
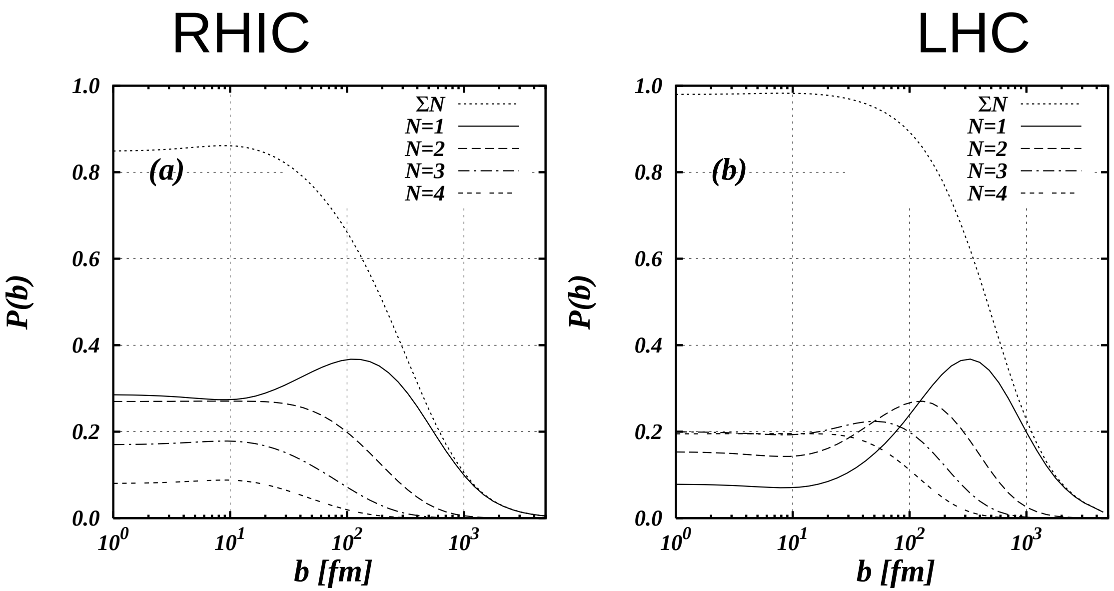


- Well described by Poisson distribution:

$$P(N,b) = \frac{\left[P^{(1)}(b)\right]^N}{N!} \exp\left[-P^{(1)}(b)\right]$$

$$\langle N(b) \rangle = P^{(1)}(b)$$

Multiple Pair Production Probability



- At small b: $\approx 1-2$ (Au@RHIC), $\approx 3-4$ (Pb@LHC).

Cross section for one and multiple pairs

Cross section for single and multiple pairs:

N	$\gamma_{coll} = 8.9$, PbPb	γ_{coll}=100, AuAu	γ_{coll} =3400, PbPb
1	2.28k	34k	200k
2	101.9	893	3.9k
3	6.66	113	780
4	0.508	18.9	219

- Multiple pair production was searched for at AGS.
- Only upper limit (above theoretical prediction) given.
  Vane et al. PRA56, 3682 ('97).
- Multiple pair production of interest as possible trigger for UPCs at LHC
  Hencken, Sadovsky, Kharlov, ALICE-INT-2002-011, 2002

Coulomb corrections: BMD

- Calculation in lowest order. But $Z\alpha \approx 0.6$!
- Bethe Maximon Davis theory: $\gamma + A \to e^+e^- + A$
  Coulomb corrections well known in this case:
  $\sigma = \frac{28}{9}\frac{Z^2\alpha^2}{m_e^2}\left[\ln\frac{2\omega}{m_e} - \frac{109}{42} - f(Z\alpha)\right]$

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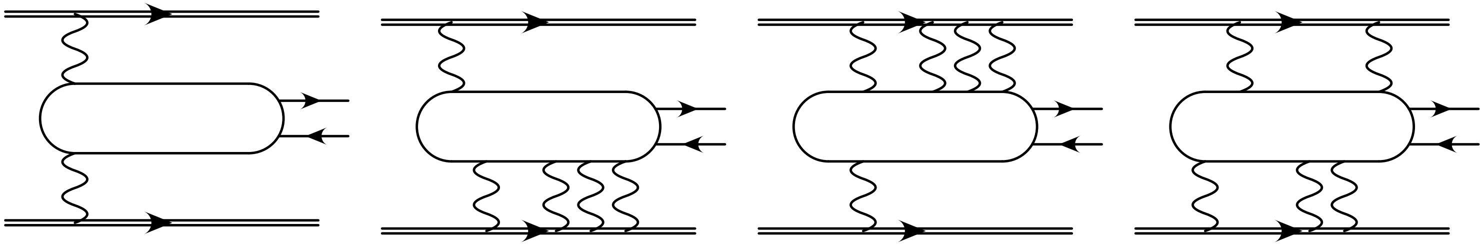
- Applied to $\sigma(N=1)$ AA collisions (Serbo et al.).
- Distinguish $n_i = 1$,$n_i > 1$.



- $n_i = 1$: $P(b) \sim 1/b^2$: $\ln\gamma_{coll}^2$ enhancement.

Coulomb corrections: BMD

- $n_A = n_B = 1$: $\sigma \sim \ln^3 \gamma_{coll}^2$.
- $n_A = 1, n_B > 1$: $\sigma \sim \ln^2 \gamma_{coll}^2$.
- ($n_A > 1, n_B > 1$: $\sigma \sim \ln \gamma_{coll}^2$. Neglected.)
- Reduction by -25% (RHIC), -16% (LHC).

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- $n_A = 1$, $n_B > 1$: $\sigma \sim \ln^2 \gamma_{coll}^2$.
- ($n_A > 1$, $n_B > 1$: $\sigma \sim \ln \gamma_{coll}^2$. Neglected.)
- Reduction by -25% (RHIC), -16% (LHC).
- Analysis in terms of n_i, that is, $\ln \gamma_{coll}^2$ not helpful for multiple pairs:

$$n_A, n_B > 1$$

- Not helpful for small $b < \lambda_e$: No $\ln \gamma_{coll}^2$ enhancement.
- Effects grow with smaller γ_{coll}, $\ln \gamma_{coll}^2$-ordering no longer useful.

Coulomb corrections from Glauber theory

- High energy limit of pair production:
  (Segev&Wells PRA57 ,Baltz&McLerran PRC58,Eichmann et al. PRA59)
- Consider pair production with retarded boundary condition.
- Certain class of diagrams are dominant.
- Eikonal approximation leads to “photon-like” propagator:

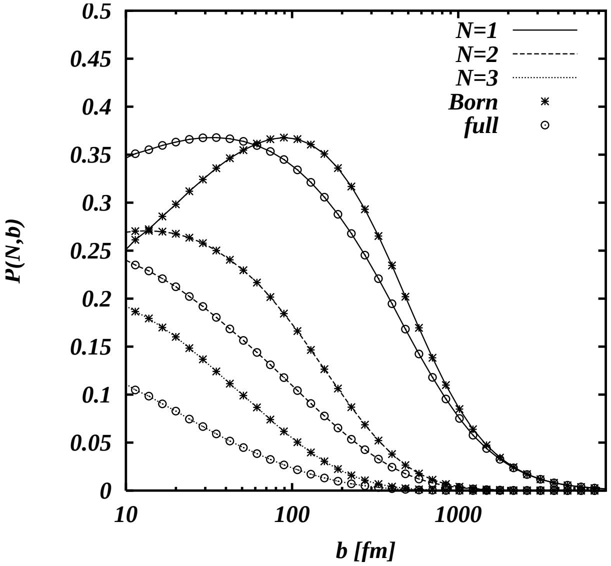
$$1/q^2 \to 1/(q^2)^{(1-iZ\alpha)}$$

- Can be calculated numerically within our approach.
  Hencken, Trautmann, Baur, Phys. Rev. C**59**, 841

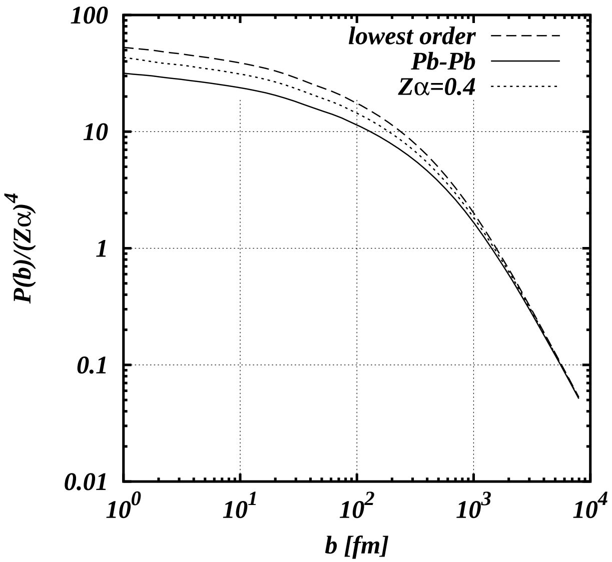
Coulomb corrections from Glauber theory

- Leads to modification of probability at small b:

AuAu@RHIC



PbPb@LHC,$P(b)/(Z\alpha)^4$



N	Born (barn)	full (barn)	Born (barn)	full (barn)	Born (barn)	full (barn)
	$\gamma_{coll} = 10$, Pb ($\eta = 0.59$)		$\gamma_{coll} = 100$, Au ($\eta = 0.57$)		$\gamma_{coll} = 3400$, Pb ($\eta = 0.59$)	
1	4.21k	4.21k	34k	34k	200k	200k
2	123	84.4	893	624	3.9k	2.9k
3	8.61	3.88	113	53.9	780	420
4	0.713	0.212	18.9	6.04	219	86

Problem with regularization of this approach

- High energy limit “predicts”:
  Wells et al., Baltz et al.

$$\sigma(\text{all order}) = \sigma(\text{Born})$$

- In contradiction with BMD corrections. (limit $Z_A\alpha \to 0$).
  (Lee&Milstein PRA61)
- Problem due to regularization of propagator from sudden limit:

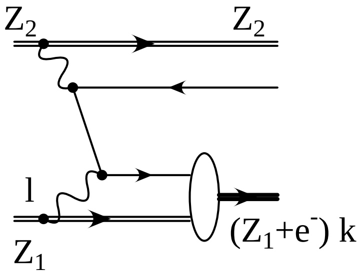
$$\frac{1}{q_\perp^2} \to \frac{1}{q_\perp^{2(1-iZ\alpha)}}, \qquad \text{regularized as:} \frac{1}{q_\perp^2 + q_l^2} \to \frac{1}{(q_\perp^2 + q_l^2)^{(1-iZ\alpha)}}$$

- More careful regularization of integrals solves problem.
  Lee et al., Eichmann et al., Baltz et al.
- Multiple pair production sensitive to small b.

Bound free pair production

- Pair production with electron produced into a bound state:

$$A + A \rightarrow A + (Ae^-)_{K,L,\ldots} + e^+$$



- Analogous process has been used to produce and detect antihydrogen (CERN, FERMILAB):

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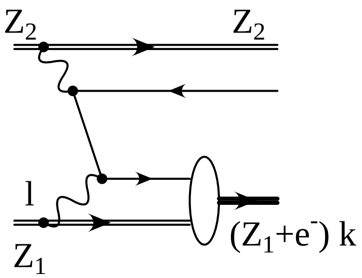

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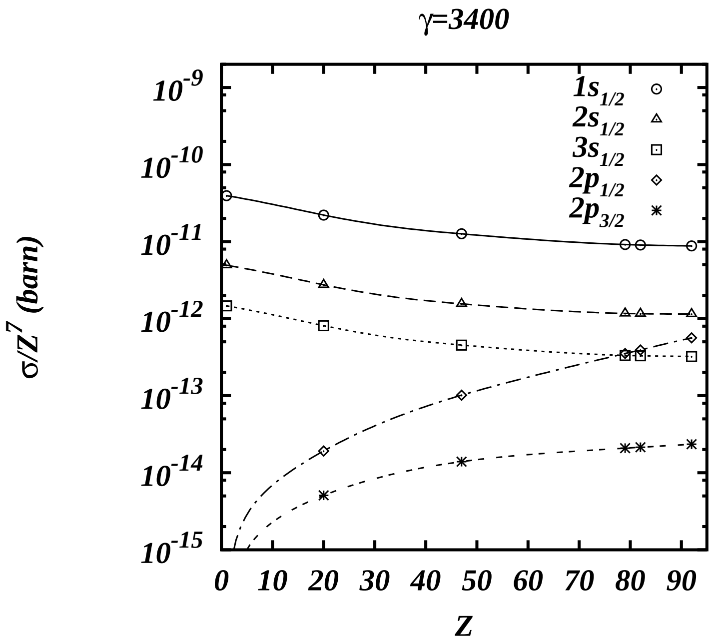
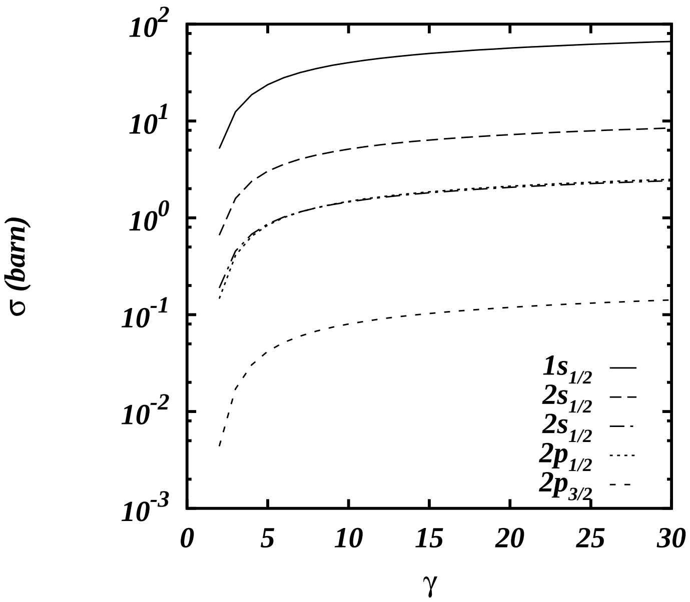
- One of the (two) dominant loss processes at high energies: Z/A ratio is changed to $(Z-1)/A$.
- Large energy deposit in a very small spot.(S. Klein NIM A459)
- Maximum Luminosity for Pb@LHC is limited by magnet cooling. Brandt, LHC Report 450
- A good knowledge of this process is important for the heavy ion beam run at LHC.

Lowest order in Z_P

- Exact 1st order calculations at LHC energies:
  (H. Meier et al., Phys. Rev. A 63)
- Full Dirac wave functions.
- No extrapolation to large γ.
- Full Born approximation.
- Also capture into p-states, higher s-states calculated.





Comparison with existing data

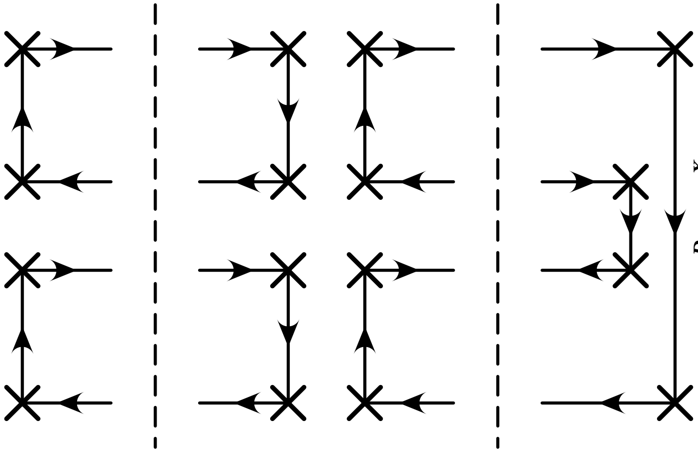
- Up to now process was measured at Bevalac, AGS and SPS.
- Bevalac Au-U at 1 GeV/A.
  $\sigma_{exp} = 2.19(0.25)b$
- At low γ deviation from first order calculation expected.
- AGS Au-Au at 10.8 GeV/A ($\gamma_{coll} = 2.6$).
  $\sigma_{exp} = 8.8b$, $\sigma_{calc} = 11.86b$ (1s only)
- SPS Pb-Au at $\gamma_{lab} = 168$, $\gamma_{coll} = 9.2$.
  $\sigma_{exp} = 44.3b$, $\sigma_{calc} = 45b$ (1s only).
- Difference of about 20% (capture to higher states?).
- For Au-Au@RHIC: $94.9b$ for $1s$, $114.3b$ up to $3s$
- For Pb-Pb@LHC: $225b$ for $1s$, $272b$ up to $3s$
- At higher energies higher order effects are found to be small.
  Baltz, PRL78
- Agreement with other calculations (extrapolated, EPA) in general good. Still there are some exceptions.

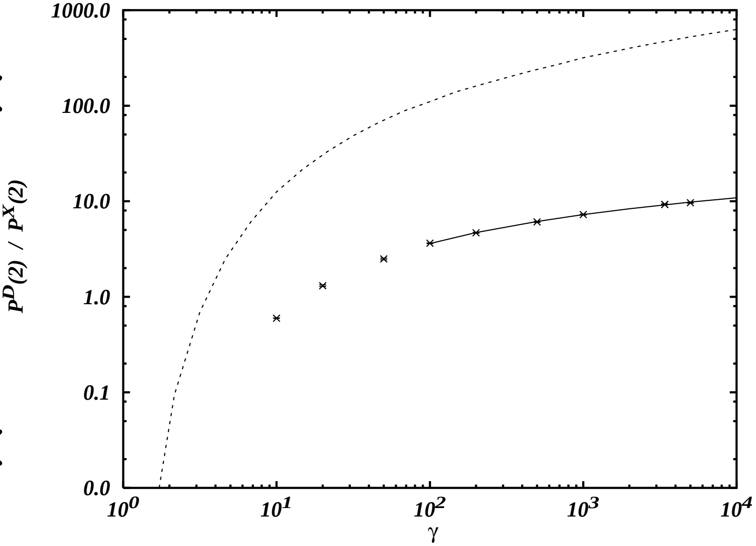
Summary

- Free pair production: Coulomb corrections and multiple pair production.
- Following perturbation theory for “large” γ:
- Differential cross sections and probabilities in lowest order.
- Multiple pair production cross section large.
- For total cross section: Bethe Maximon Davis corrections.
- High energy “eikonal” approach a possible second way.
- Coulomb corrections not completely understood, especially for small b.
- Bound-free pair production:
  Cross section is of importance for lead-beams at the LHC.

Deviation from Poisson

- Application of Poisson distribution has been criticized.
- Deviation from Poisson calculated for $b \approx 0$:





- e^+e^- pairs behave as uncorrelated quasi-bosons.
  Large space of “quantum numbers”
- Of course: Search for deviations, correlations possible and interesting.

Comparison with literature

- Agreement with older (approximate) calculations:

RHIC,Au	LHC,Pb	
$\gamma_{coll} = 100$	2957	
94.9	225	Meier et al. (2001)
93	226	Anholt&Becker87, Becker87 (ex.)
37	86	Bertulani&Baur88 (ap.)
229	562	Bertulani&Dolci01 (ap.)
89	206 (U) 195 (Au)	Baltz&Rhoades-Brown (ex.) &Weneser 91,93,94
72	—	Rhoades-Brown&Bottcher&Strayer 89
90	222 ($\gamma_{coll} = 3400$)	Aste&Hencken&Trautmann94 (EPA)
77	($b_{min} = 2\lambda_c$)	Ågger&Sørensen97
85	($b_{min} = \lambda_c$)	(EPA)
94	204 (expol.)	Exp.: Grafstrøm et al. 99
		(all final states).