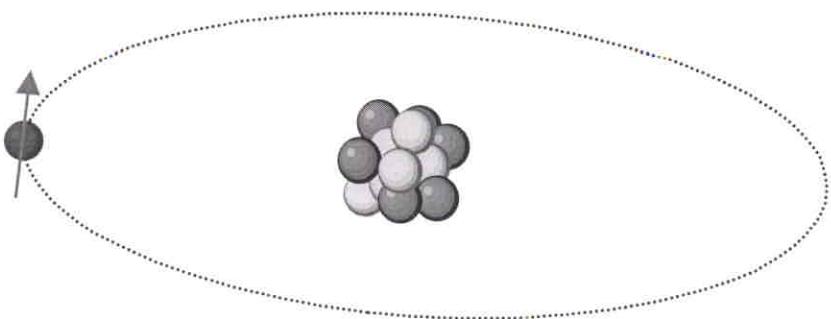


g-factor and mass measurements

Status and perspectives for HITRAP

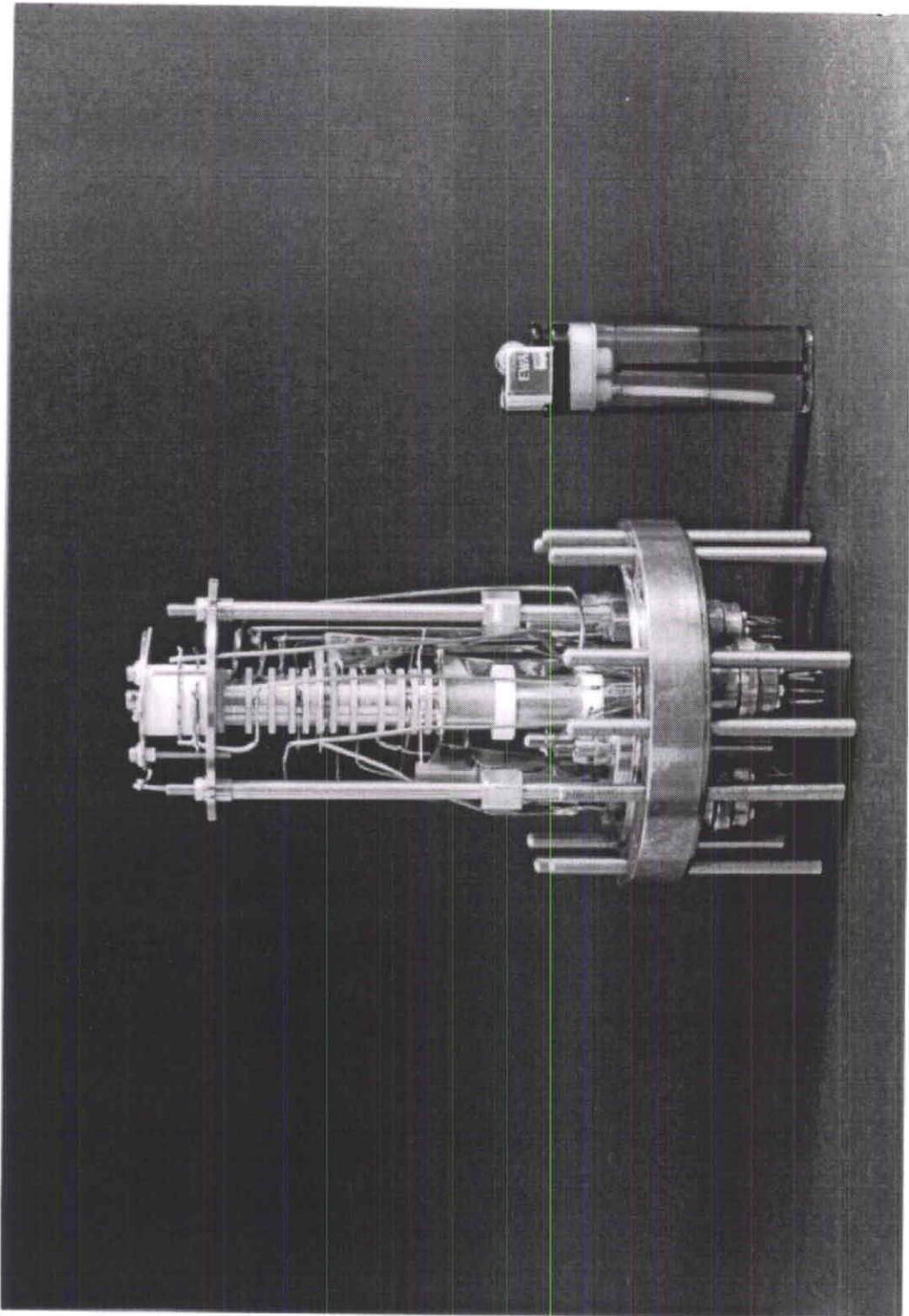
1. g-factors ...
2. ...and what to derive from them
3. Masses and the Lamb shift

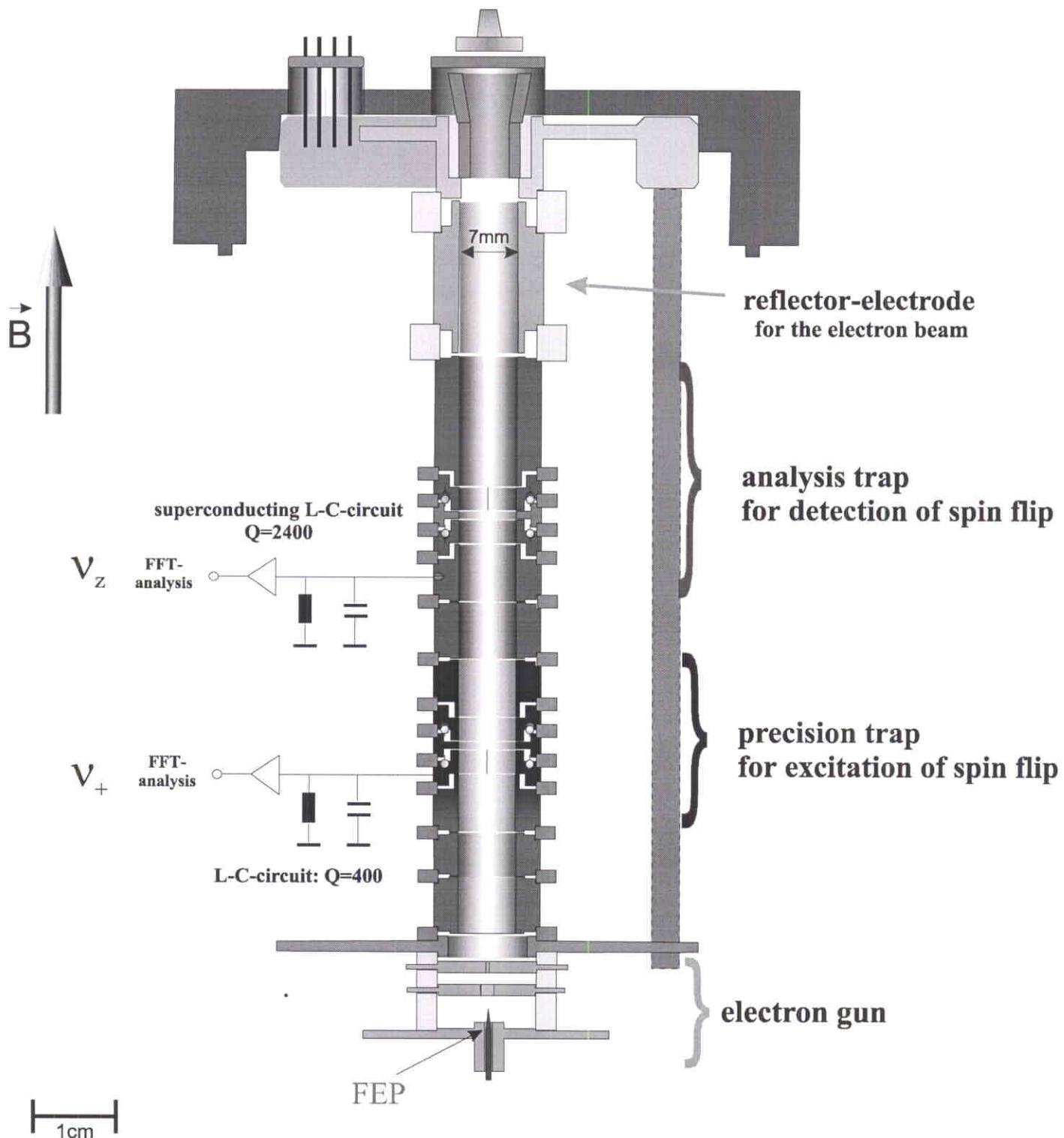
g factor of a bound electron



Why ?

Primary goal:
Investigation of QED in strong fields.

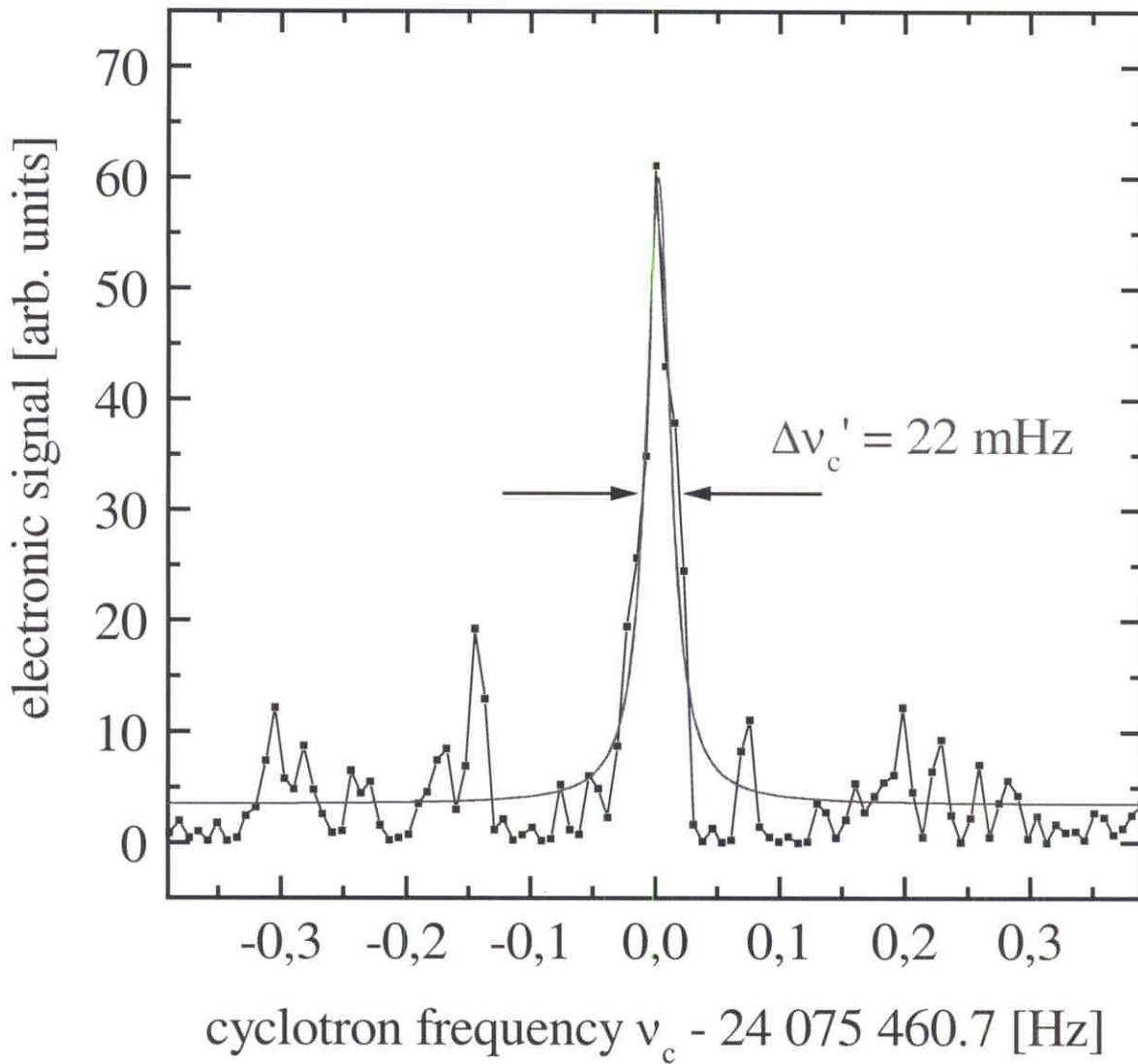




Cyclotron Frequency Spectrum of a Single C⁵⁺-Ion in a Penning Trap

- single ion at T = 4 Kelvin

- linewidth: $\Delta v_c / v_c = 9 \cdot 10^{-10}$



How to measure the g factor ?

$$\hbar\omega_L = g \frac{e\hbar}{2m_e} B$$

ω_L : Larmor precession frequency

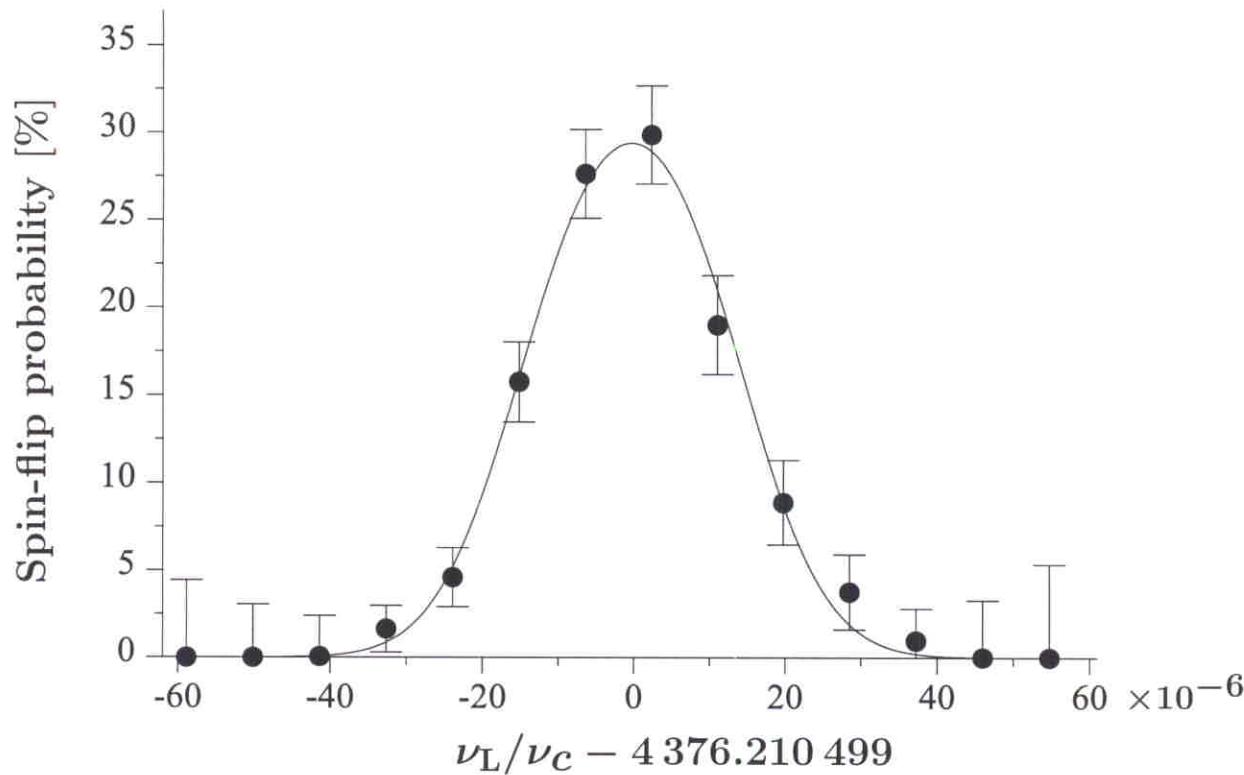
The B field is also required !

$$\omega_C = \frac{q_{\text{ion}}}{M_{\text{ion}}} B$$

ω_C : Cyclotron frequency of the ion

$$g = 2 \left(\frac{\omega_L}{\omega_C} \right) \frac{\left(\frac{q}{M} \right)_{\text{ion}}}{\left(\frac{e}{m} \right)_e}$$

Larmor resonance of the electron bound in hydrogenlike carbon



$$\frac{\nu_L}{\nu_c} = 4\ 376.210\ 498\ 9(19)(13)$$

$$\begin{aligned}
 g &= 2 \left(\frac{\nu_l}{\nu_c} \right) \frac{m_e}{M(^{12}\text{C}^{5+})} \frac{q(C^{5+})}{e} \\
 &= 2.001\ 041\ 596\ 3(10)(44)
 \end{aligned}$$

Here,

$$\begin{aligned}
 m_e &= 0.000\ 548\ 579\ 911\ 1(12) \text{ u} \\
 M(^{12}\text{C}^{5+}) &= (12.0 - 0.000\ 000\ 579\ 837) \text{ u}
 \end{aligned}$$

Contribution to g_J

numerical value (in 10^{-9})

$^{12}\text{C}^{5+}$

$^{16}\text{O}^{7+}$

	$^{12}\text{C}^{5+}$	$^{16}\text{O}^{7+}$
from Dirac eq.	1 998 721 354.4	1 997 726 003.1
fin. nuc. size	0.4	1.6
recoil	87.6	117.0
QED, $\sim (\alpha/\pi)$	2 323 664.0(1)	2 324 415.7(1)
QED, $\sim (\alpha/\pi)^2$		
(and higher)	-3 516.2(3)	-3 517.1(6)

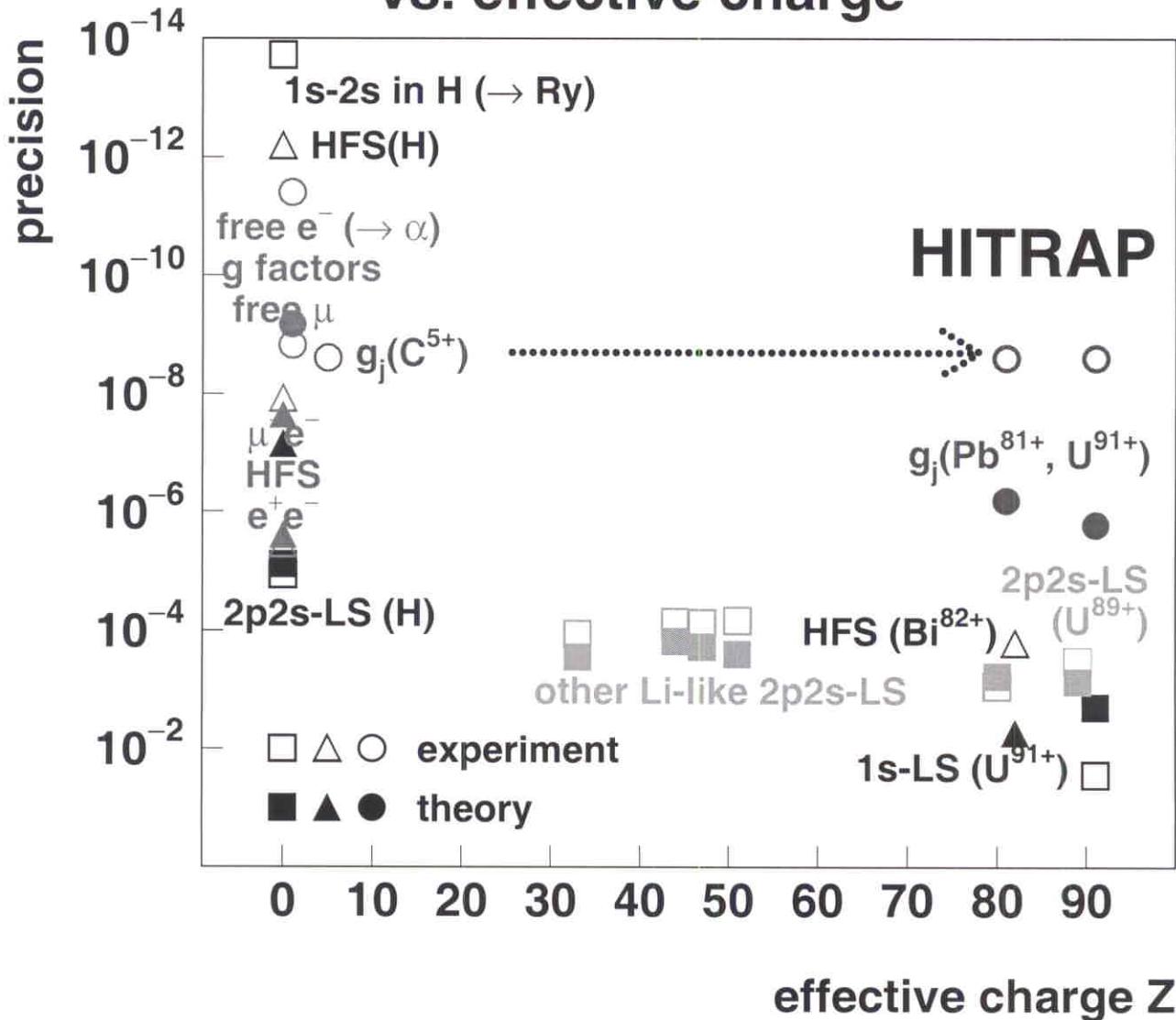
total:	2 001 041 590.1(3)	2 000 047 020.2(6)
Experiment:	2 001 041 596.3(10)(44)	2 000 047 026.8(16)(44)

Contribution to g_J

	numerical value (in 10^{-9})
	$^{12}\text{C}^{5+}$
	$^{16}\text{O}^{7+}$

from Dirac eq.	1 998 721 354.4	1 997 726 003.1
fin. nuc. size	0.4	1.6
recoil	87.6	117.0
QED, $\sim (\alpha/\pi)$	2 323 664.0(1)	2 321 017.2(1)
QED, $\sim (\alpha/\pi)^2$		
(and higher)	-3 516.2(3)	-3 517.1(6)
total:	2 001 041 590.1(3)	2 000 047 020.2(6)
Experiment:	2 001 041 596.3(10)(44)	2 000 047 026.8(16)(44)
\implies		
[van Dyck et al., 1995:	$m_e = 0.000\,548\,579\,909\,3(3)$ u	
[Codata 2000:	$m_e = 0.000\,548\,579\,911\,1(12)$ u	
	$m_e = 0.000\,548\,579\,911\,0(12)$ u]	
V. A. Yerokhin, P. Indelicato, V. M. Shabaev, Phys. Rev. Lett. 89 (2002) 143001,		
T. Beier et al. Phys. Rev. Lett. 88 (2002) 011603.		

QED: theor./exp. precision vs. effective charge



What else is possible ?

- We have seen:

Determination of the mass of the electron from $^{12}\text{C}^5+$.

- Fine structure constant α :

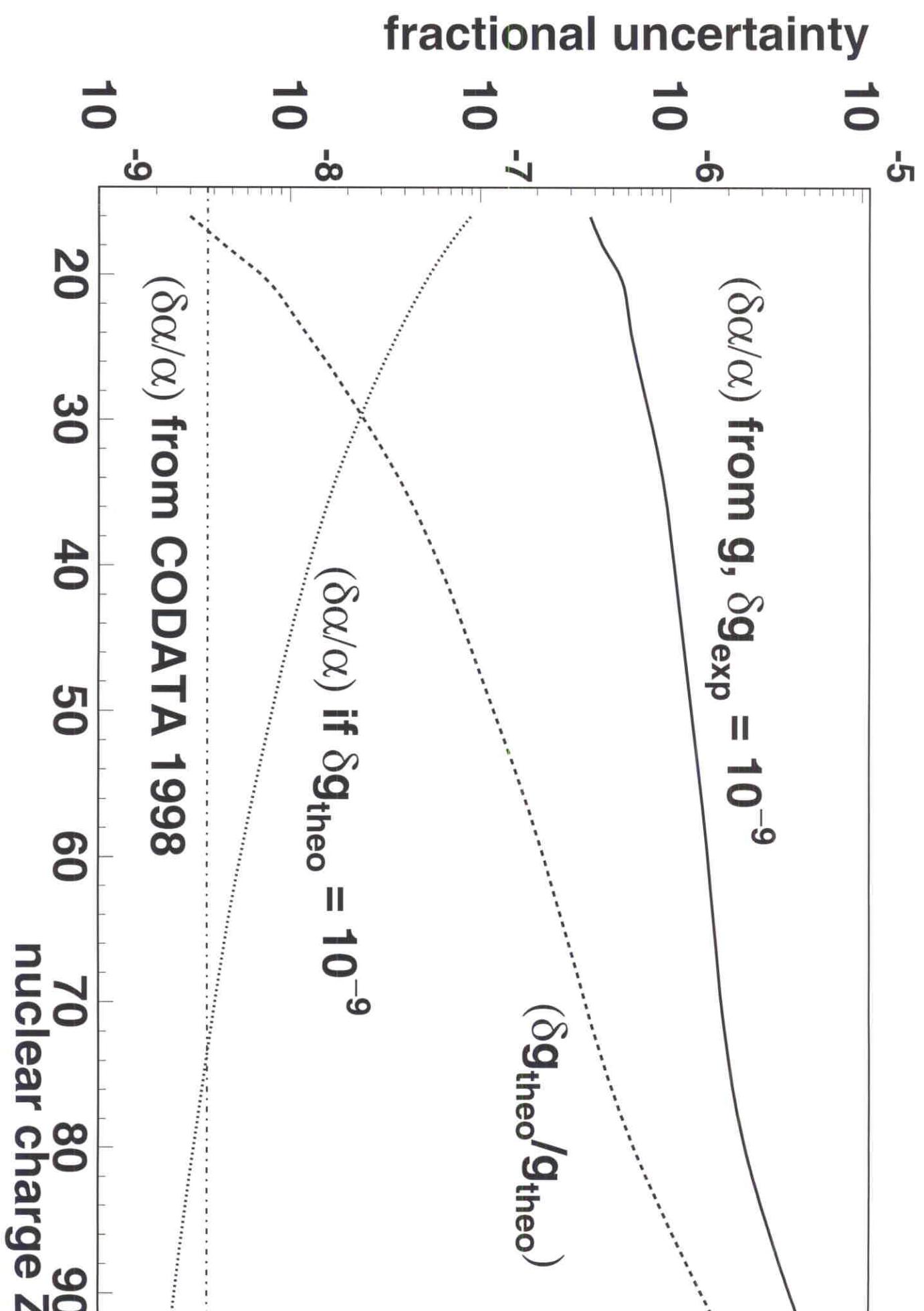
$$\frac{\delta\alpha}{\alpha} \sim \frac{1}{(Z\alpha)^2} \frac{\delta g}{g}$$

only $\sim 10^{-3}$ for carbon,
but $\sim 10^{-9}$ for uranium (!).

For an experiment on H-like Ca, a value of

$$\frac{\delta\alpha}{\alpha} \sim 10^{-8}$$

is possible !



Nuclear magnetic moments:

$$g_{\text{atom}} = g^{(e)} \frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)} - \frac{m_e}{m_p} g^{(\text{N})} \frac{F(F+1) + I(I+1) - J(J+1)}{2F(F+1)}$$

With

$$\frac{m_e}{m_p} \sim \frac{1}{1800}$$

and

$$\frac{\delta g_{\text{atom}}}{g_{\text{atom}}} \sim 10^{-9} \quad (\text{from experiment})$$

$$\frac{\delta g^{(e)}}{g^{(e)}} \sim 10^{-9} \quad (\text{from theory})$$

$$\Rightarrow \frac{\delta g^{(\text{N})}}{g^{(\text{N})}} \sim 10^{-6} \text{ possible.}$$

Advantage:

No diamagnetic shielding corrections.

No chemical shift.

Nuclear RMS radii (for isotopes):

$$g_1^{(e)} - g_2^{(e)} \simeq A(Z\alpha) \Delta \langle r^2 \rangle^{1/2}$$

To lowest orders,

$$\begin{aligned} \Delta g &= \frac{8}{3} (\alpha Z)^4 \langle (r/\lambda_C)^2 \rangle \\ &\times \left[1 + (\alpha Z)^2 \left(2 - \Gamma - \frac{\langle r^2 \log(2\alpha Z r/\lambda_C) \rangle}{\langle r^2 \rangle} \right) \right], \end{aligned}$$

(Glazov and Shabaev, 2002)

Exact calculations are also possible !

The currently known rms radius of ${}^{238}\text{U}$,
 5.8604 ± 0.0023 fm (Zumbro *et al.*, 1984)
limits the theoretical prediction for
 $g^{(e)}(\text{U}^{91+})$ to $\sim 10^{-7}$ (!)

Mass spectrometry in a Penning trap:

Measurement of cyclotron frequencies

$$\omega_{C_1} = \frac{Q_1}{M_1} B \quad \omega_{C_2} = \frac{Q_2}{M_2} B$$



$$\boxed{\frac{\omega_{C_1}}{\omega_{C_2}} = \frac{Q_1}{Q_2} \frac{M_2}{M_1}}$$



Measurement of mass ratios

HITRAP: Mass spectrometry with

Highly Charged Ions in a Penning trap:

$$\omega_C = \frac{Q}{M_{\text{total}}} B$$

$$M_{\text{total}} = M_{\text{nucleus}} + N_e m_e - E_{\text{binding}}$$

- Mass measurements in different charge states
- Determination of atomic binding energies (ionization potentials)
- Investigation of highly charged One-, Few-, and Many-electron systems

HITRAP: Lamb shift determination in hydrogenlike uranium

$$\Delta\omega_C/\omega_C = 10^{-11}$$

M_{total}	$\approx 230\ 000\ 000\ 000.00$	eV	(= 230 GeV)
E_{binding}	$= 131\ 816.01(.50)$	eV	(≈ 132 keV)
Lamb shift	$= 463.95(.50)$	eV	(Theory)
exp. Error:	13	eV	(at present)
$\Delta M_t/M_t$	$= 10^{-11}$	$\hat{=}$	2.3 eV (would be nice)

HITRAP: Elimination of nuclear size and structure effects

At present:

- Nuclear size and polarization correction limits theoretical g-factor prediction in U^{91+}
- Bohr-Weisskopf effect limits theoretical hyperfine structure splitting prediction in Bi^{82+}
- Nuclear polarization limits theoretical Lamb shift prediction in U^{91+}

All these limitations can be overcome by measurements on the H-like and the corresponding Li-like ion. (V. M. Shabaev et al.)

- In turn, this would allow for a more precise determination of the corresponding nuclear quantities

Therefore:

- HITRAP will allow high-Z QED investigations with yet unmatched precision.
- HITRAP will allow to drive high-Z QED theory to its limits.
- HITRAP (in combination with QED theory) will give access to nuclear parameters of high-Z nuclei at surpassing precision.