The Importance of HITRAP for QED Investigations

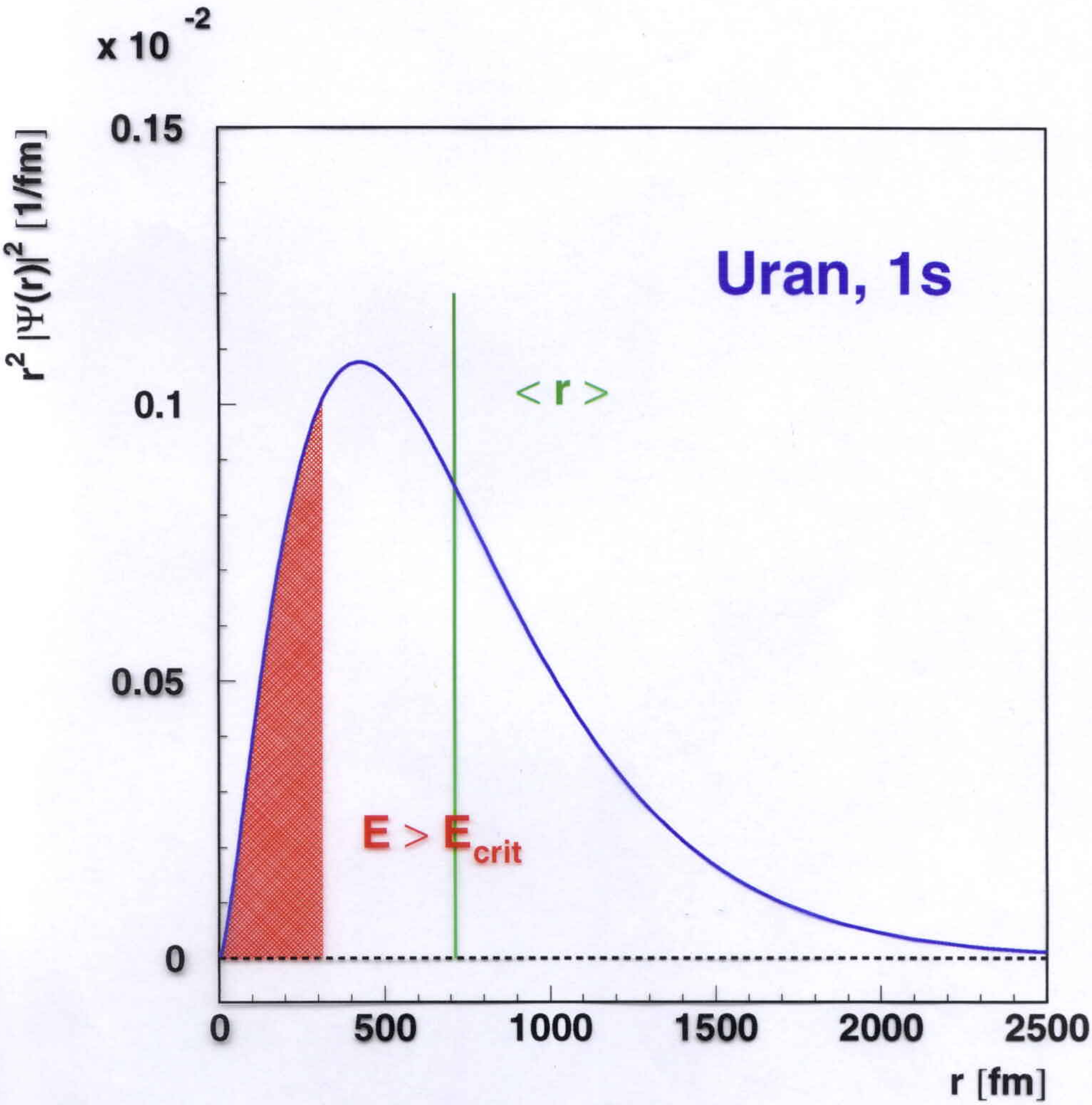
I Motivation: QED of strong fields

II The Lamb shift

III The *g*-factor and hyperfine splitting

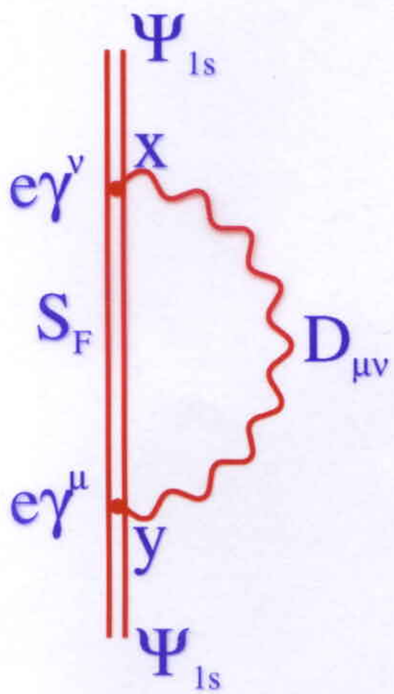
IV Violation of fundamental symmetries

V Conclusions

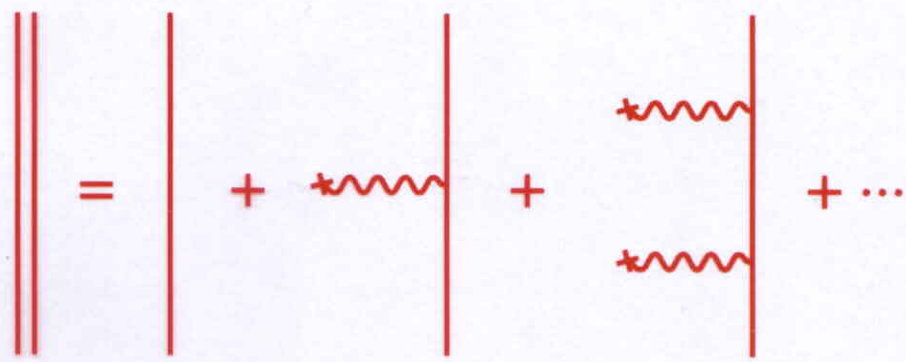


x 10 -2
0.15
0.1
0.05
0
$r^2 \, |\Psi(r)|^2$ [1/fm]
Uran, 1s
< r >
$E > E_{crit}$
0
500
1000
1500
2000
2500
r [fm]

Self Energy



Expansion in $Z\alpha$



Energy shift

$$\Delta E = 4\pi i\alpha \int dt\, d^3x\, d^3y\, \overline{\Psi}_{1s}(y)\gamma^{\nu} S_F(x,y)\gamma^{\mu}\Psi_{1s}(x) D_{\mu\nu}(x,y)$$

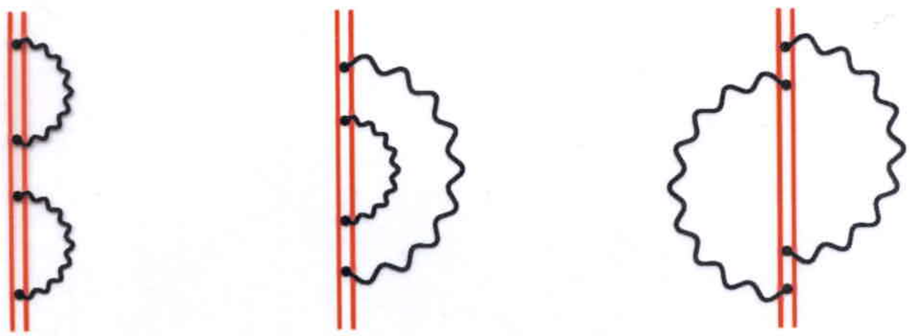
$$-\delta m \int d^3x\, \overline{\Psi}_{1s}(x)\, \Psi(x)$$

Propagator

$$[\gamma(\hat{p} - eA^e) - m]_x\, S_F(x,y) = \delta(x-y)$$

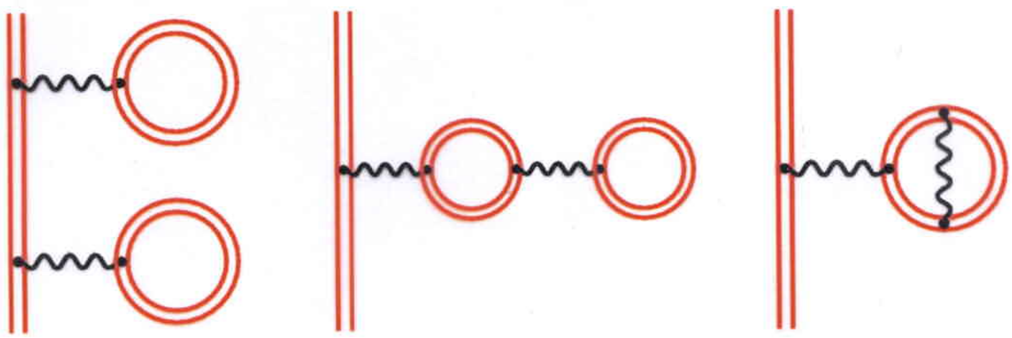
$$\Delta E_{1s}({}^{238}_{92}U) = 355.0432 \text{ eV}$$

QED corrections of second order in α



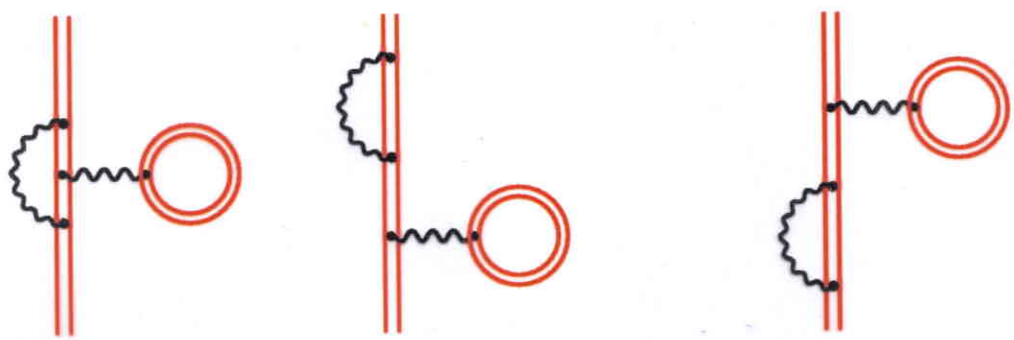
a) b) c)

SESE



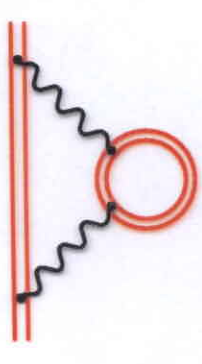
a) b) c)

VPVP



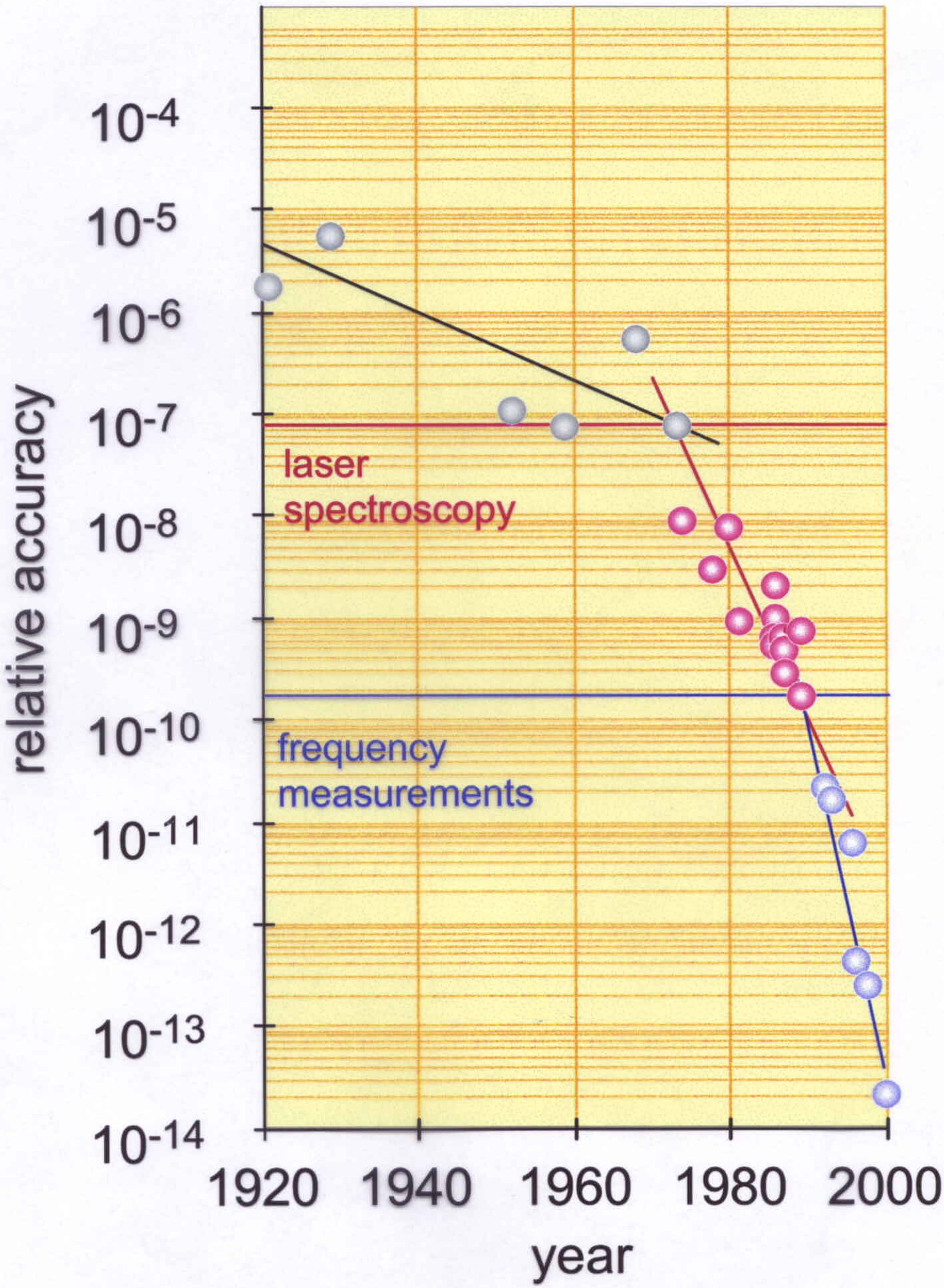
a) b) c)

SEVP



S(VP)E

		$1S_{1/2}$ (in eV)
Binding energy E_B for point nucleus:		-132279.96
Correction	Order	
finite size		198.81 (.38)
– Uehling		-93.58
– WK		4.99
total VP	$m\alpha(\alpha Z)^4$	-88.60
SE	$m\alpha(\alpha Z)^4$	355.05
SESE a,b,c	$m\alpha^2(\alpha Z)^4$	new: -1.87 (.10)
VPVP a	$m\alpha^2(\alpha Z)^5$	-0.22
VPVP b	$m\alpha^2(\alpha Z)^5$	-0.15
VPVP c	$m\alpha^2(\alpha Z)^4$	-0.60 (.30)
SEVP a,b,c	$m\alpha^2(\alpha Z)^5$	1.14
S(VP)E	$m\alpha^2(\alpha Z)^5$	0.13 (.06)
Recoil	$m\frac{m}{M}(\alpha Z)^2$	0.46 (.01)
Nuclear pol.	$m\frac{m}{M}(\alpha Z)^2$	-0.20 (.10)
Total binding energy		-131816.01
Sum of corrections (Theory)		463.95 (.50)
Lamb Shift (Exp.)		$468. \pm 13.$



relative accuracy
10-4
10-5
10-6
10-7
10-8
10-9
10-10
10-11
10-12
10-13
10-14
laser
spectroscopy
frequency
measurements
1920
1940
1960
1980
2000
year

Hydrogen

Self energy

$$\Delta E = 10.316793659(1)\,\frac{\alpha}{\pi}\,\alpha^4 mc^2$$

Hänsch et al.

$$\nu_{2S\text{-}1S} = 2\,466\,061\,413\,187\,103\,(46)\ \mathrm{Hz}$$

$$R_\infty = 10\,973\,731.568\,549\,(83)\ \mathrm{m}^{-1}$$

$$R_\infty \sim \alpha^2$$

Time-dependence of α?

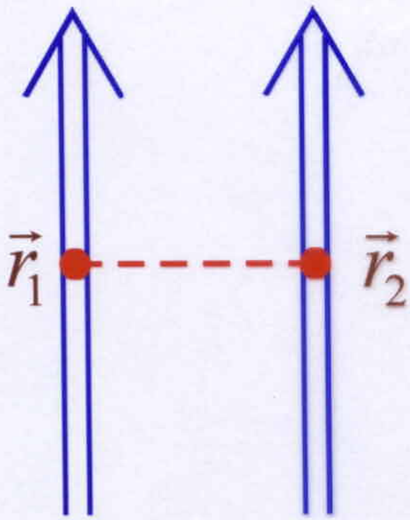
Asymmetry of the natural line profile for the hydrogen atom

There are non-resonant corrections to the line profile which cause deviations from a Lorentz profile.

These corrections depend on the particular measurement process and thus indicate the limit up to which the concept of the energy of an excited atomic state has a physical meaning.

$\Delta E \sim 3\,\mathrm{Hz}$ **for the 1s-2p transition in H.**
Compton scattering

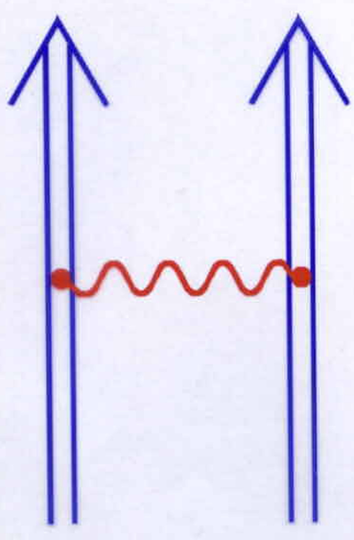
e⁻ e⁻ interaction



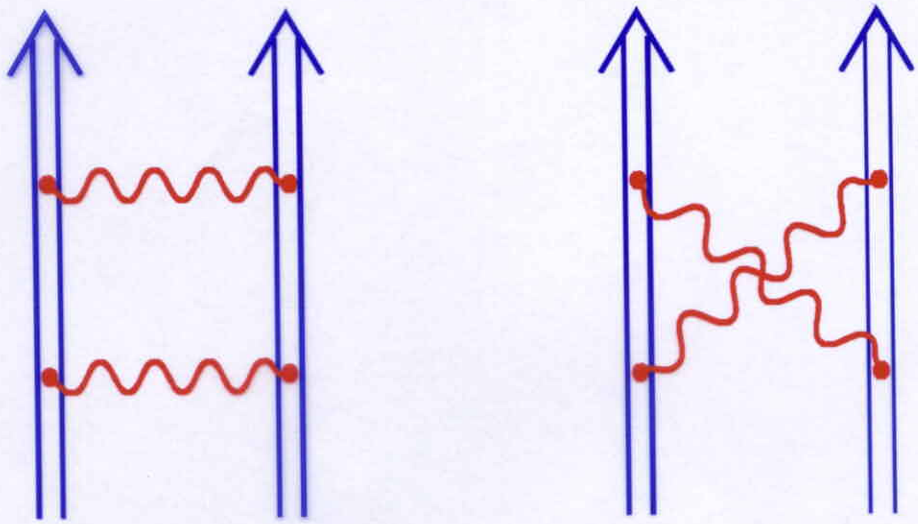
$$\frac{e^2}{\left|\vec{r}_1 - \vec{r}_2\right|}$$

Instaneous Coulomb interaction

Breit interaction: **Magnetic interaction and retardation**



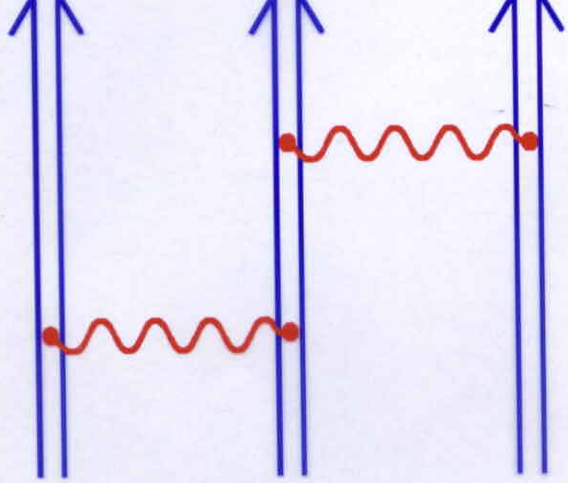
Two-photon exchange



Electron propagator

$$S\left(x_1, x_2\right) = \frac{1}{2\pi i}\int\limits_{-\infty}^{+\infty} d\omega\, e^{i\omega\left(t_1 - t_2\right)} \sum_n \frac{\Psi_n\left(\vec{r}_1\right)\overline{\Psi}_n\left(\vec{r}_2\right)}{\varepsilon_n\left(1 - i0\right) + \omega}$$

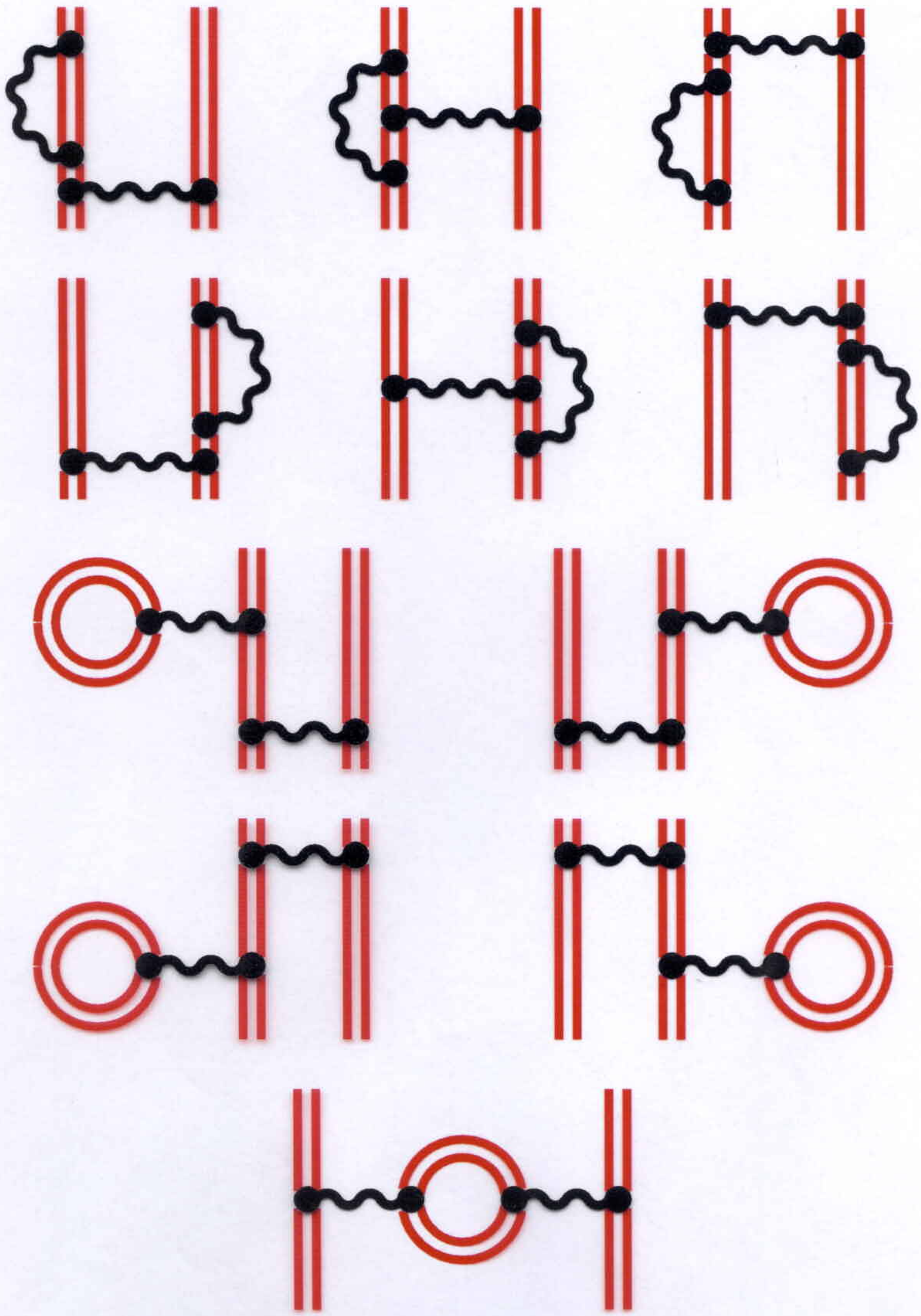
Three-electron atoms



What is the advantage of the QED formalism compared with atomic many-particle methods (RMBPT)?

1. Intermediate states incorporate states of negative energies
2. Crossed-photon interaction
3. Exact treatment of retardation
4. Gauge invariance and radiative corrections

Two electron QED screening corrections of first order:



$2p_{1/2} - 2s$ transition energy (eV)

	Z = 30	Z = 92
Finite nucleus	-0.014	-33.35
1 photon	62.147	368.83
2 ph (RMBPT)	-3.890	-13.55
Δ 2 ph (QED)	0.001	0.17
3 photon	-0.029	0.17
SE + VP	-0.906	-42.93
Scr. SE	0.100	1.52
Scr. VP	-0.006	-0.36
Nuclear recoil	-0.016	-0.07
Theory	57.384(4)	280.48(11)
Experiment	57.384(3)	280.59(9)

Hyperfine structure splitting for $^{209}Bi^{82+}$

rms radius: 5.519 fm
magnetic moment: $\mu = 4.1106(2)\ \mu_N$

	Δ E [eV]
Point nucleus	5.8395
Finite size	–0.6473
Bohr-Weisskopf (DCM)	–0.107(7)
Bohr-Weisskopf (SP)	–0.061(27)
Vacuum polarization	
Magnetic loop correction	0.0062
Correction of wave function	0.0233
Self energy	–0.0593
Sum of QED corrections	–0.0298
Total (DCM)	5.0554(80)
Total (SP)	5.101(27)
Experiment	5.0840(8)

Fundamental constants from g-factor experiments

- g factor of the free electron

Experiment: **2.002 319 304 376 6 (84)**
Theory: **2.002 319 304 402(54)**

$\Longrightarrow$ **most precise value for α**
$$\alpha = 1/137.035\,999\,58\,(52)$$

- Electronic g factor in $^{12}C^{5+}$

Experiment: **2.001 041 596 3 (10)(44)**
Theory (2002): **2.001 041 590 1(3)**
[(1999): **2.001 041 589 8(26)]**

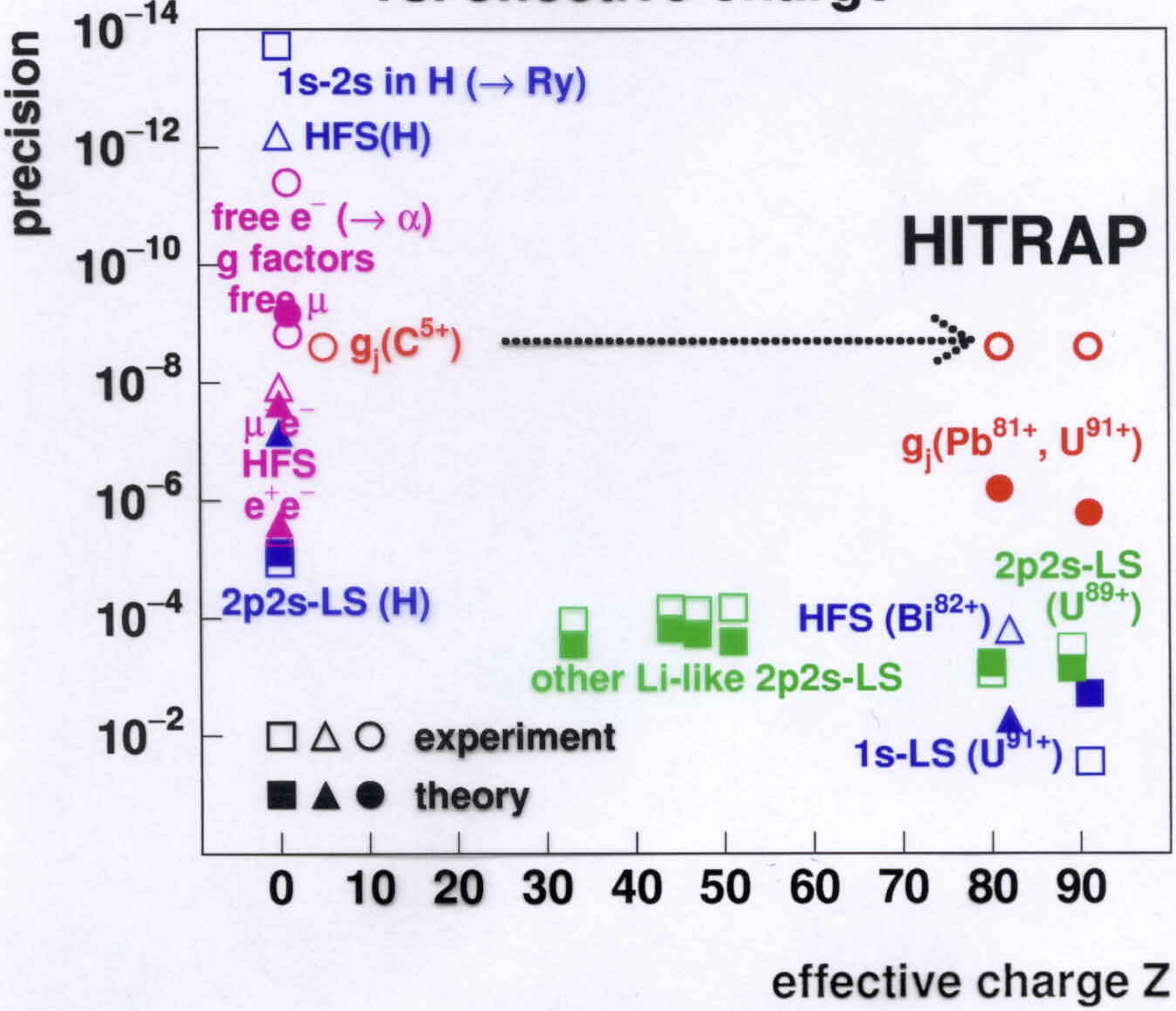
$\Longrightarrow$ **most precise value for m_e**
$$m_e = 0.000\,548\,579\,909\,3\,(3)\mathrm{u}$$
[1998 CODATA: $m_e = 0.000\,548\,579\,911\,0\,(12)\mathrm{u}$]

$$\frac{m_p}{m_e} = 1836.152\,673\,1(10)$$

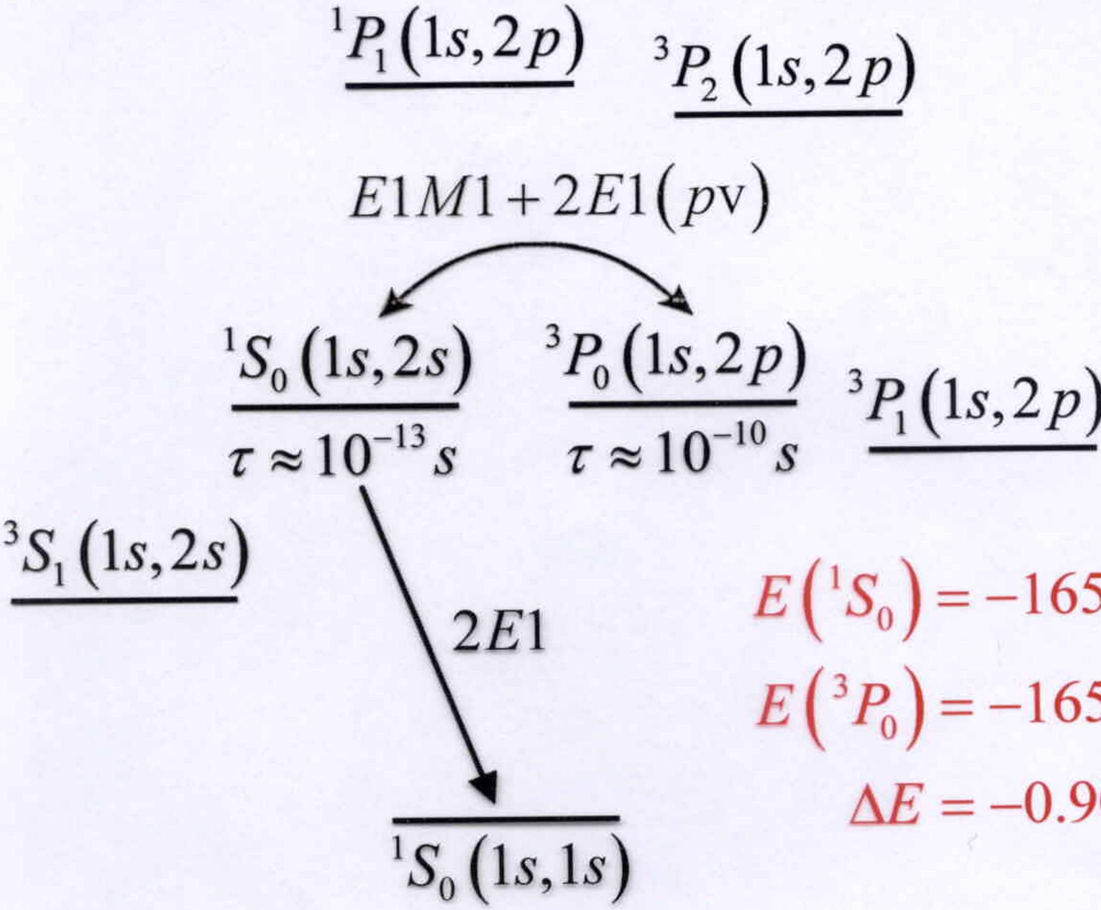
[1998 CODATA:]

$$\frac{m_p}{m_e} = 1836.152\,667\,5(39)$$

QED: theor./exp. precision
vs. effective charge



Helium-like Uranium



$$E(^1S_0) = -165\,114.406 \text{ eV}$$
$$E(^3P_0) = -165\,113.5 \text{ eV}$$
$$\Delta E = -0.906 \text{ eV}$$

Parity admixture

$$\eta = \frac{\left\langle 2\,^3P_0 \right| \frac{G_F}{2\sqrt{2}}\left(1 - 4\sin^2\Theta_w - \frac{N}{Z}\right)\rho_{\text{el}}\gamma_5 \left| 2\,^1S_0 \right\rangle}{E(2\,^3P_0) - E(2\,^1S_0)}$$

G_F: Fermi constant,
N: neutron number,
ρ_{el}: electric charge density

Θ_w: Weinberg angle
Z: proton number

$$|\eta| = \mathbf{5 \cdot 10^{-6}}$$