Masses of atomic nuclei far from stability

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A.E.L. Dieperink, KVI, The Netherlands

P. Van Isacker, GANIL, France

Introduction: Models for nuclear masses.

Symmetry energy in nuclear matter.

Chaotic mass component?

Mass predictions from pattern recognition.

ALMAS-1, GSI-Darmstadt, October 2006

ECT* doctoral training programme

- Title: “Nuclear structure and reactions” (8th March-8th June 2007, for PhD students).
- Lecture series on shell model, mean-field models, nuclear astrophysics, symmetries in nuclei, reaction theory, exotic nuclei, open quantum systems, fundamental interactions.
- Workshops related to these topics.
- Please:
  - Encourage students to apply;
  - Submit workshop proposals to ECT*.

Nuclear mass formulas

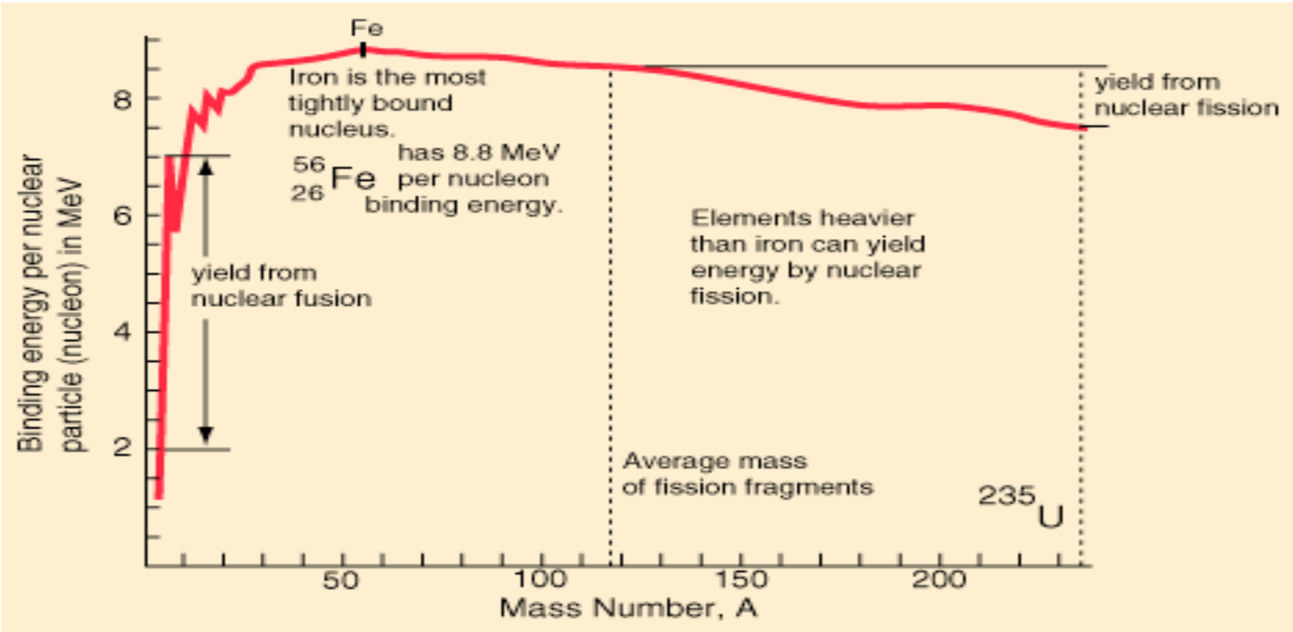
- Global mass formulas:
  - Liquid-drop model (LDM): von Weizsäcker.
  - Macroscopic models with microscopic corrections: FRDM, ...
  - Microscopic models: HFB*n*, RMF, DZ, …
- Local mass formulas:
  - Extrapolations by Wapstra & Audi.
  - IMME, Garvey-Kelson (GK) relations, mass formula of Liran-Zeldes, neural networks, …

D. Lunney *et al.*, Rev. Mod. Phys. **75** (2003) 1021
K. Blaum, Phys. Reports **425** (2006) 1.

Liquid-drop mass formula

- Binding energy of an atomic nucleus:

$$B(N,Z) = a_{\mathrm{vol}} A - a_{\mathrm{sur}} A^{2/3} - a_{\mathrm{cou}} \frac{Z(Z-1)}{A^{1/3}} - a_{\mathrm{sym}} \frac{(N-Z)^2}{A} + a_{\mathrm{pai}} \frac{\delta(N,Z)}{A^{1/2}}$$



C.F. von Weizsäcker, Z. Phys. **96** (1935) 431

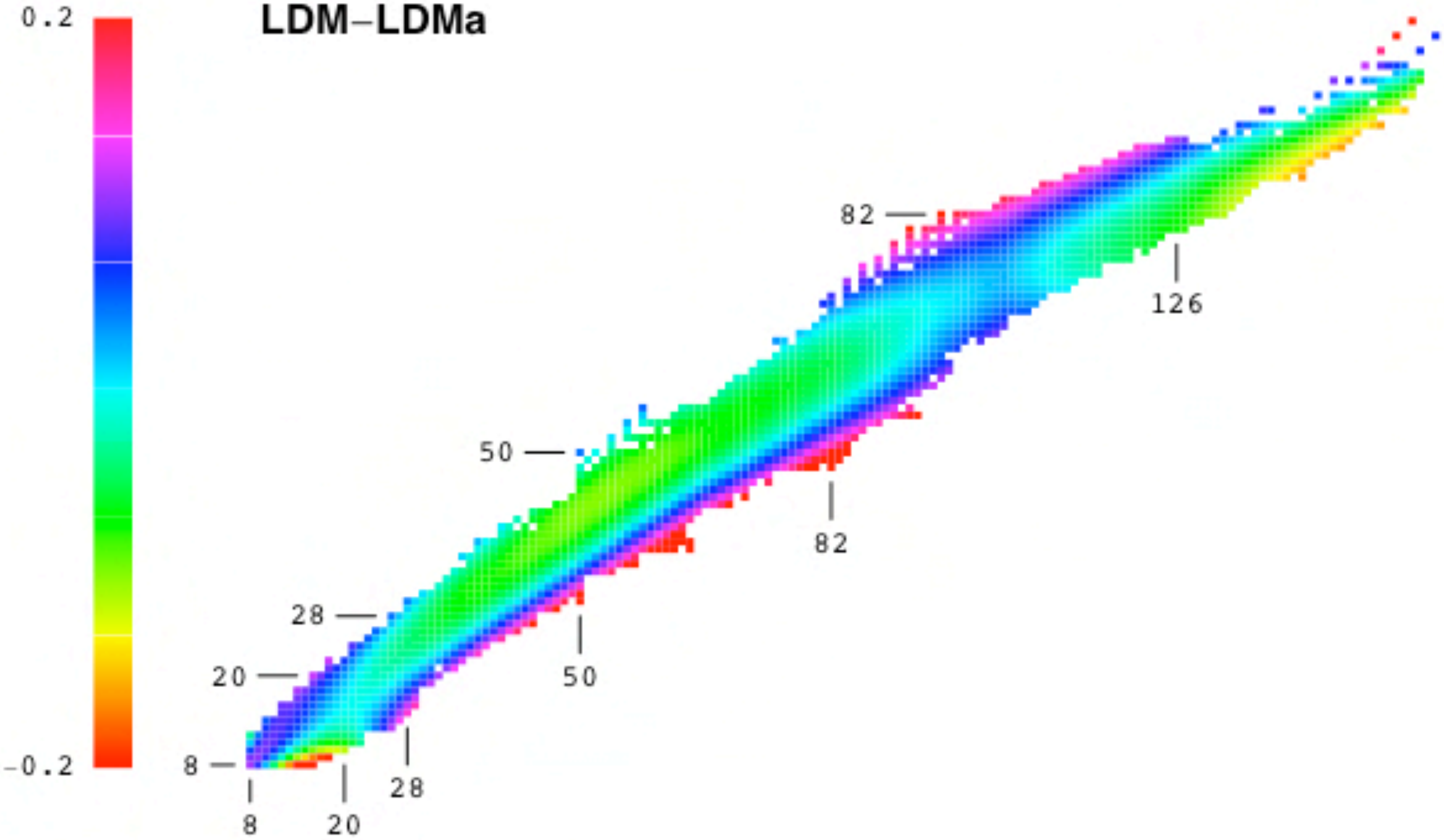
Deficiencies of Weizsäcker formula

- Consistency of the Weizsäcker mass formula requires a *surface-symmetry* term. Derivation relies on the thermodynamics of an asymmetric two-component system:

$$\frac{S_{\mathrm{v}}}{1+S_{\mathrm{v}}A^{-1/3}/S_{\mathrm{s}}}\frac{(N-Z)^2}{A}\approx -a_{\mathrm{vsym}}\frac{(N-Z)^2}{4A}+a_{\mathrm{ssym}}\frac{(N-Z)^2}{4A^{4/3}}$$

W.D. Myers & W.J. Swiatecki, Ann. Phys. **55** (1969) 395
A. Bohr & B.R. Mottelson, *Nuclear Structure II* (1975)
A.W. Steiner *et al.*, Phys. Reports **411** (2005) 325
P. Danielewicz, Nucl. Phys. A **727** (2003) 233

LDM versus LDMa



Symmetry energy

- Energy per particle in nuclear matter:

$$E(\rho,x) = E\left(\rho,x=\tfrac{1}{2}\right) + S(\rho)(1-2x)^2, \quad x = Z/A$$

- Symmetry energy $S(\rho)$ is density dependent:

$$S(\rho) = a_4 + p_0(\rho-\rho_0) + \Delta K(\rho-\rho_0)^2$$

- In Thomas-Fermi approximation (r=0):

$$\frac{S_\mathrm{v}}{S_\mathrm{s}} = \frac{3}{R\rho_0}\int dr\rho(r)\left(\frac{S(\rho_0)}{S(\rho)}-1\right)$$

R. Furnstahl, Nucl. Phys. A **706** (2002) 85

Quantal effects & Wigner cusp

- The $(N\text{-}Z)^2$ dependence of the symmetry term arises in a macroscopic approximation.
- Quantal theories gives rise to
  - $T(T+1)$: isospin SU(2).
  - $T(T+4)$: supermultiplet SU(4).
- This suggests a generalization of the form $T(T+r)$, with r a parameter.

N. Zeldes, Phys. Lett. B **429** (1998) 20
J. Jänecke & T.W. O'Donnell, Phys. Lett. B **605** (2005) 87

Modified mass formula

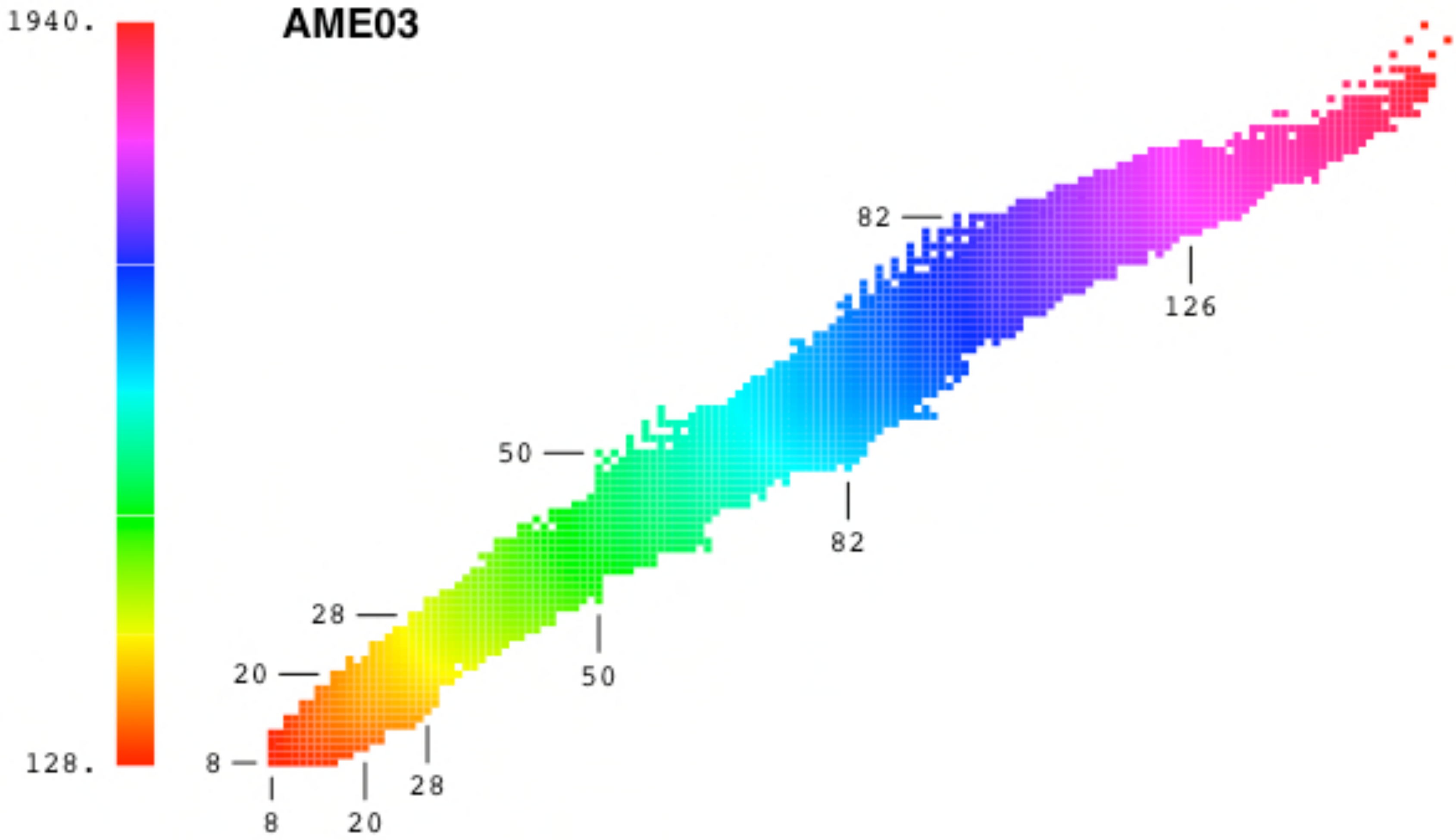
- Add Wigner and surface-symmetry energy:

$$B(N,Z) = a_{\mathrm{vol}}A - a_{\mathrm{sur}}A^{2/3} - a_{\mathrm{cou}}\frac{Z(Z-1)}{A^{1/3}}$$

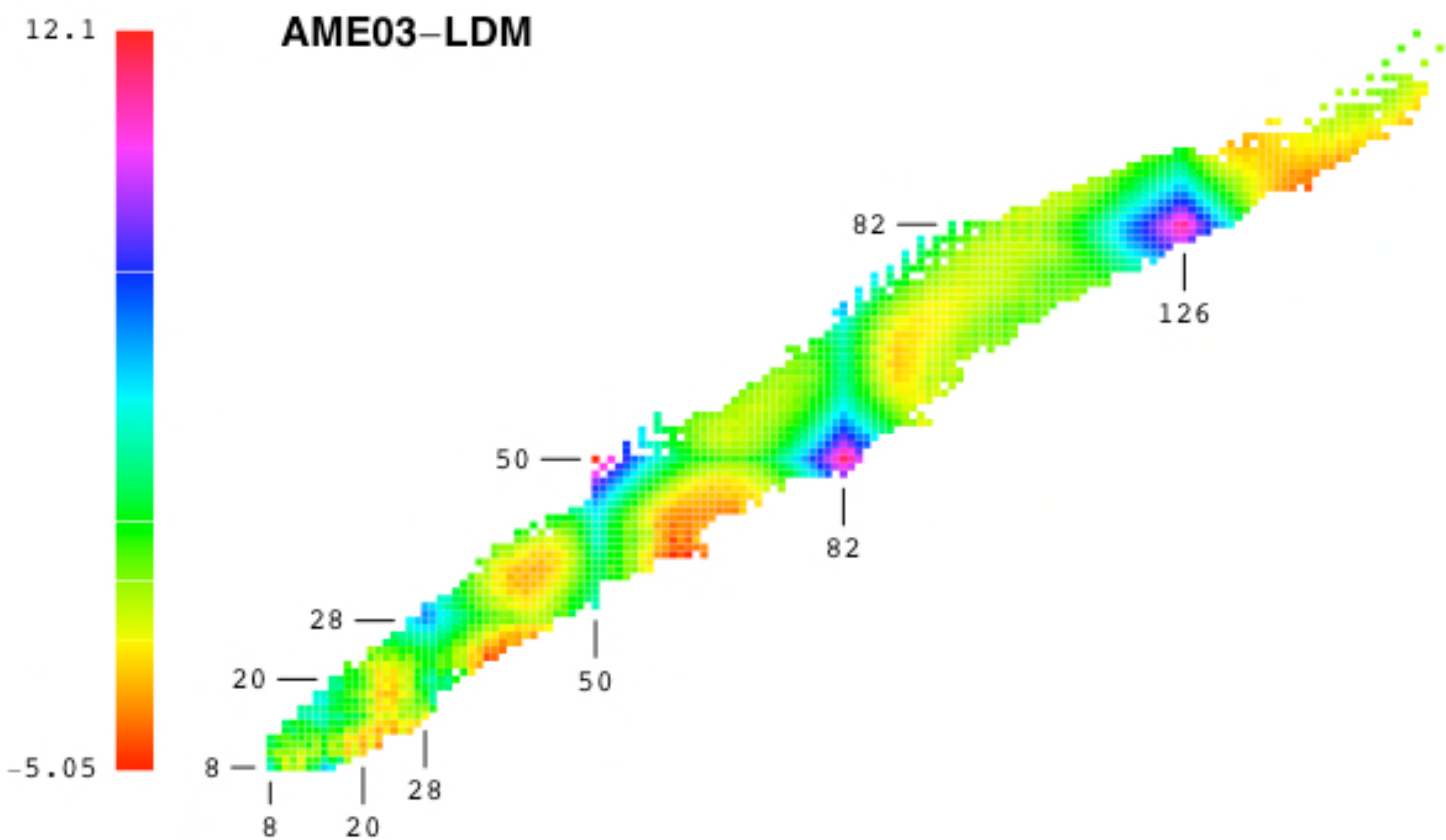
$$-\frac{S_{\mathrm{v}}}{1+S_{\mathrm{v}}A^{-1/3}/S_{\mathrm{s}}}\frac{T(T+r)}{A} + a_{\mathrm{pai}}\frac{\delta(N,Z)}{A^{1/2}}$$

- Fit to AME03: $\sigma_{\mathrm{rms}} \approx 2.4$ MeV.

The nuclear mass surface



The ‘unfolding’ of the mass surface

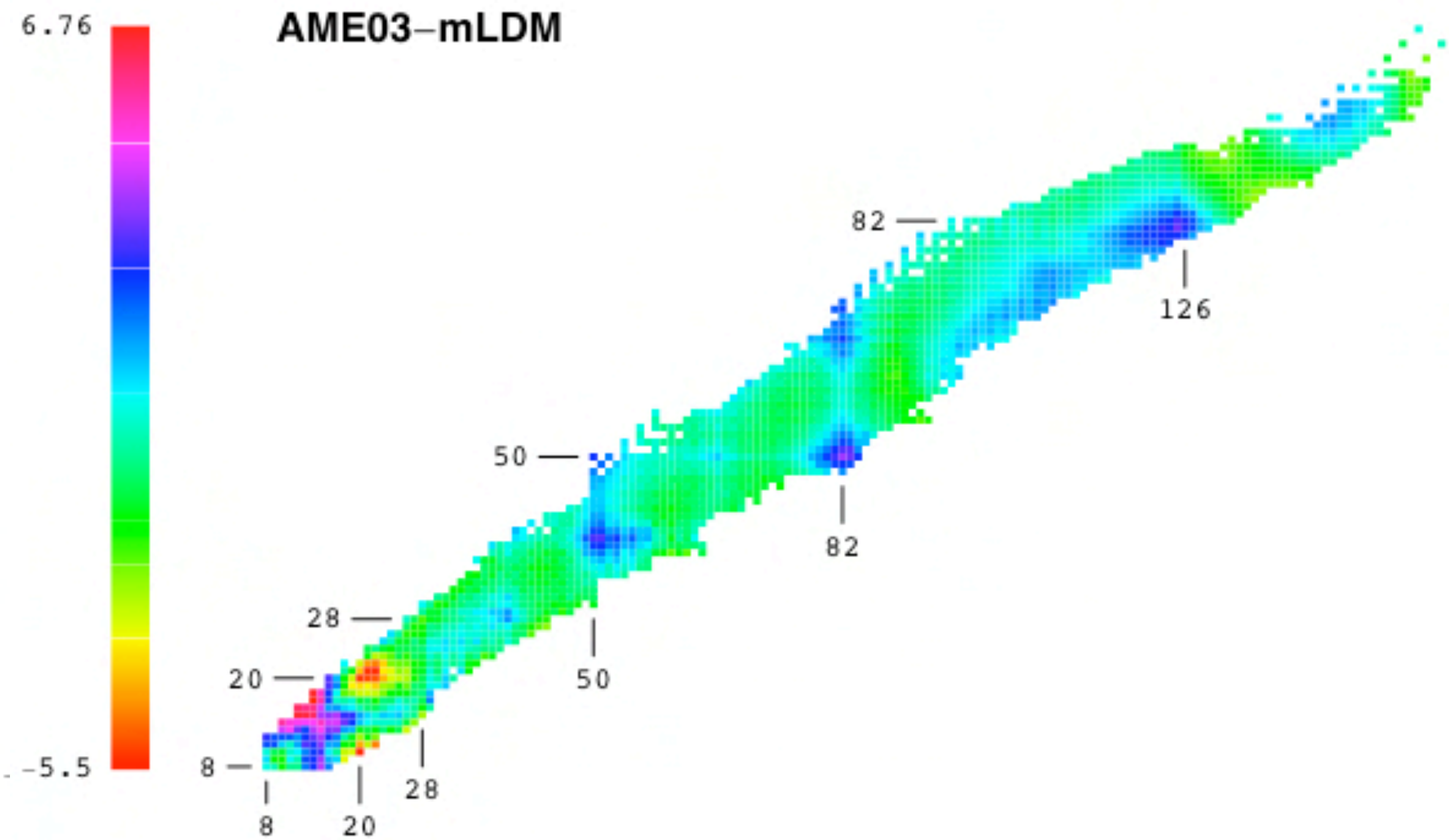


Shell corrections

- Observed deviations suggest shell corrections depending on $n_\nu+n_\pi$, the total number of valence neutrons + protons (particles or holes).
- A simple parametrisation consists of two terms, linear and quadratic in $F_{\mathrm{max}}=(n_\nu+n_\pi)/2$.

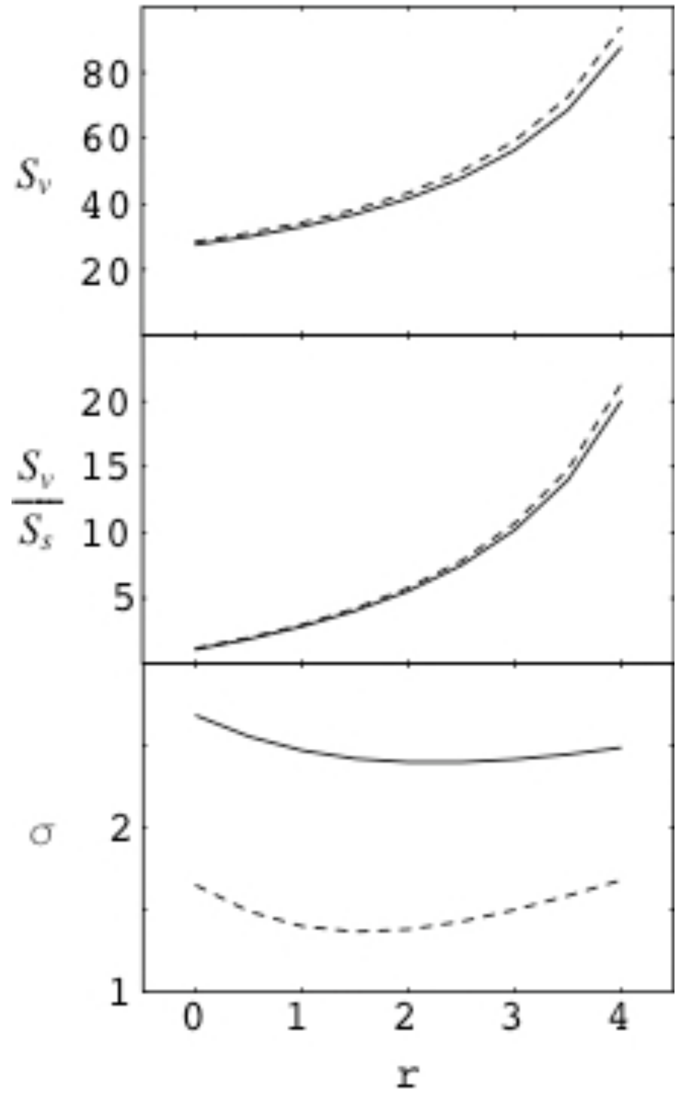
E.D. Davis *et al.*, Phys. Rev. C **44** (1991) 1655

Shell-corrected LDM



Dependence on r

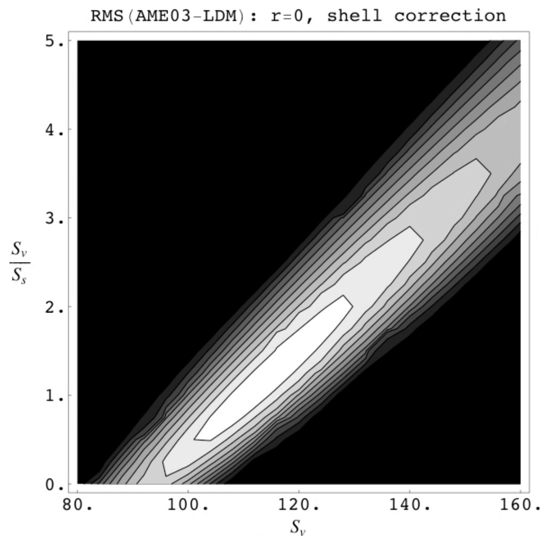
- Rms deviation σ decreases significantly with shell corrections.
- Rms deviation σ has shallow minimum in r.
- Both S_v and S_v/S_s are ill determined.

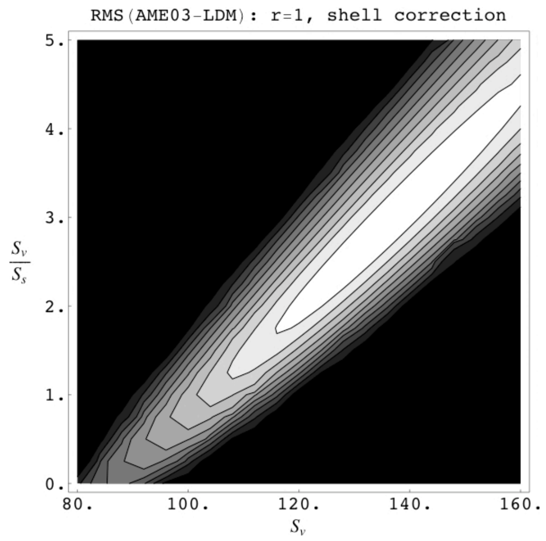


A.E.L. Dieperink and P. Van Isacker, to be published

Correlations

- Volume- and surface-symmetry terms are correlated.
- Correlation depends strongly on *r*.
- What nuclear properties are needed to determine S_v and S_s?
- How can we fix *r*?

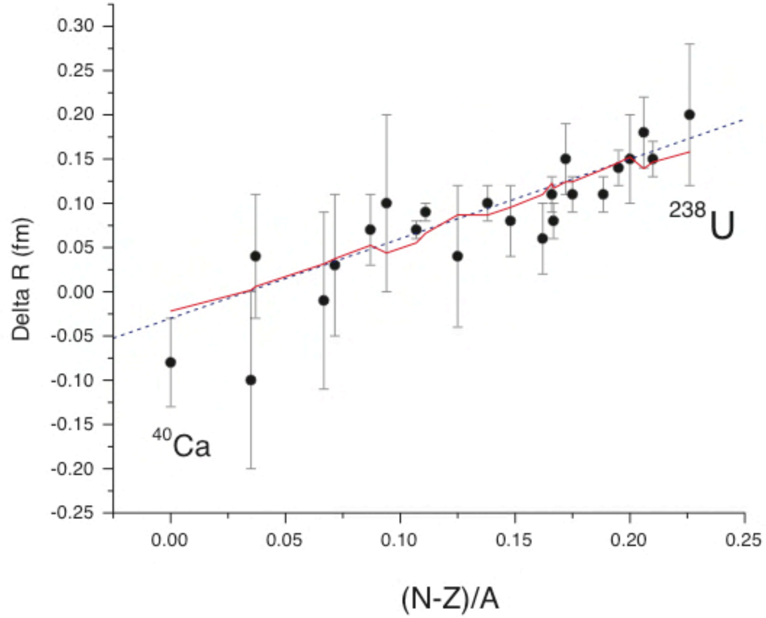




S_v and S_s from neutron skins

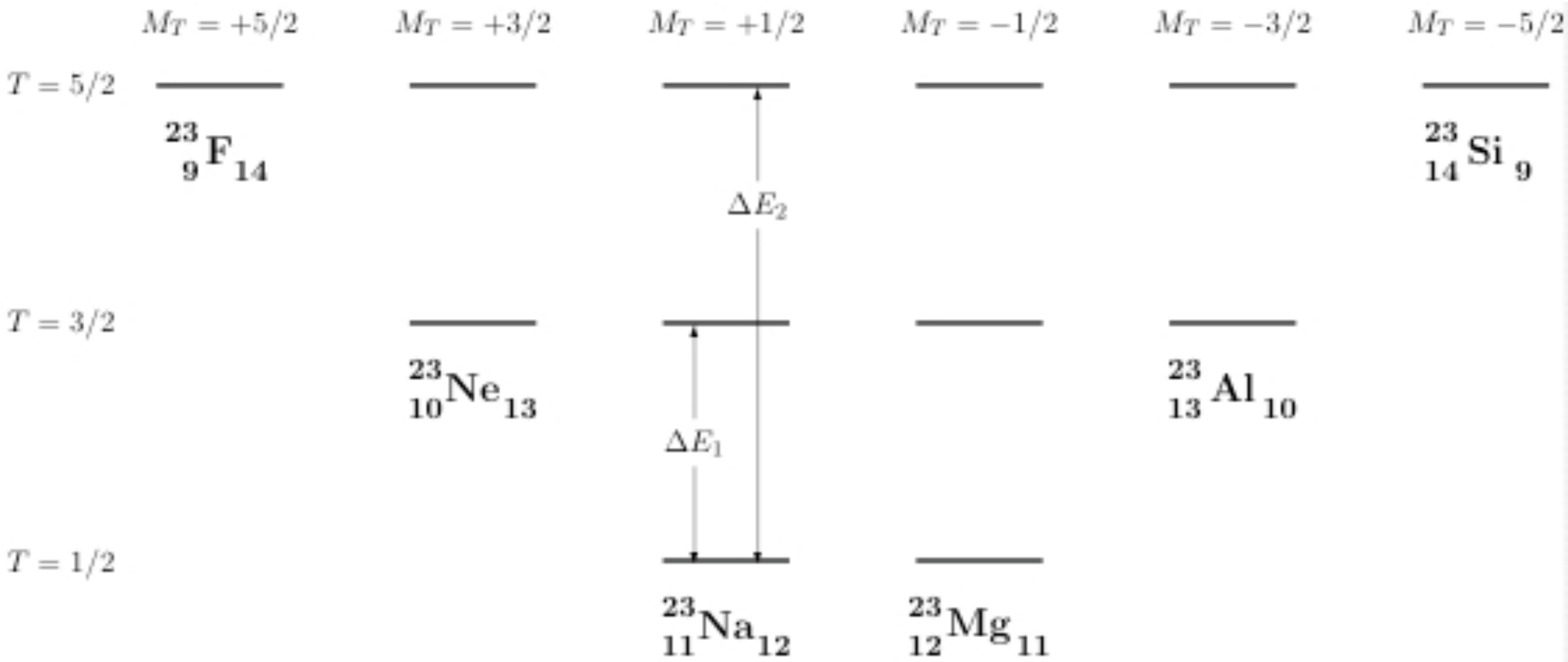
- Dependence of neutron skin on S_v and S_s (hard-sphere approximation):

$$\frac{R_n - R_p}{R} = \frac{A}{6NZ}\frac{N - Z - a_{cou}A^{2/3}/(12S_v)}{1 + S_v A^{1/3}/S_s}$$



P. Danielewicz, Nucl. Phys. A **727** (2003) 233
A.E.L. Dieperink & P. Van Isacker, to be published

r from isobaric multiplets



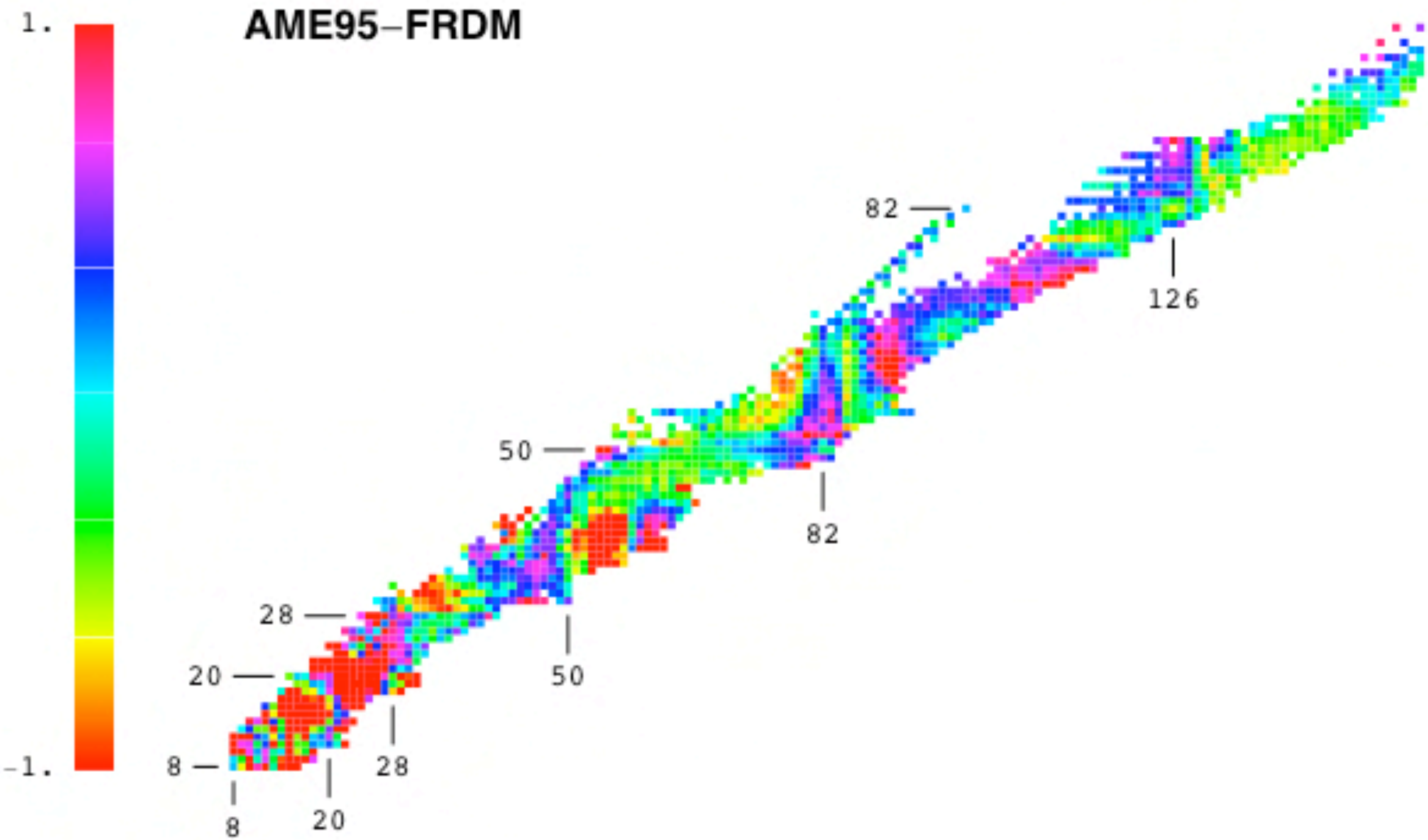
- From T=3/2 and T=5/2 states in M_T=±1/2 nuclei:

$$\frac{\Delta E_2}{\Delta E_1} = \frac{2(r+3)}{r+2} \Rightarrow r = \frac{6\Delta E_1 - 2\Delta E_2}{\Delta E_2 - 2\Delta E_1}$$

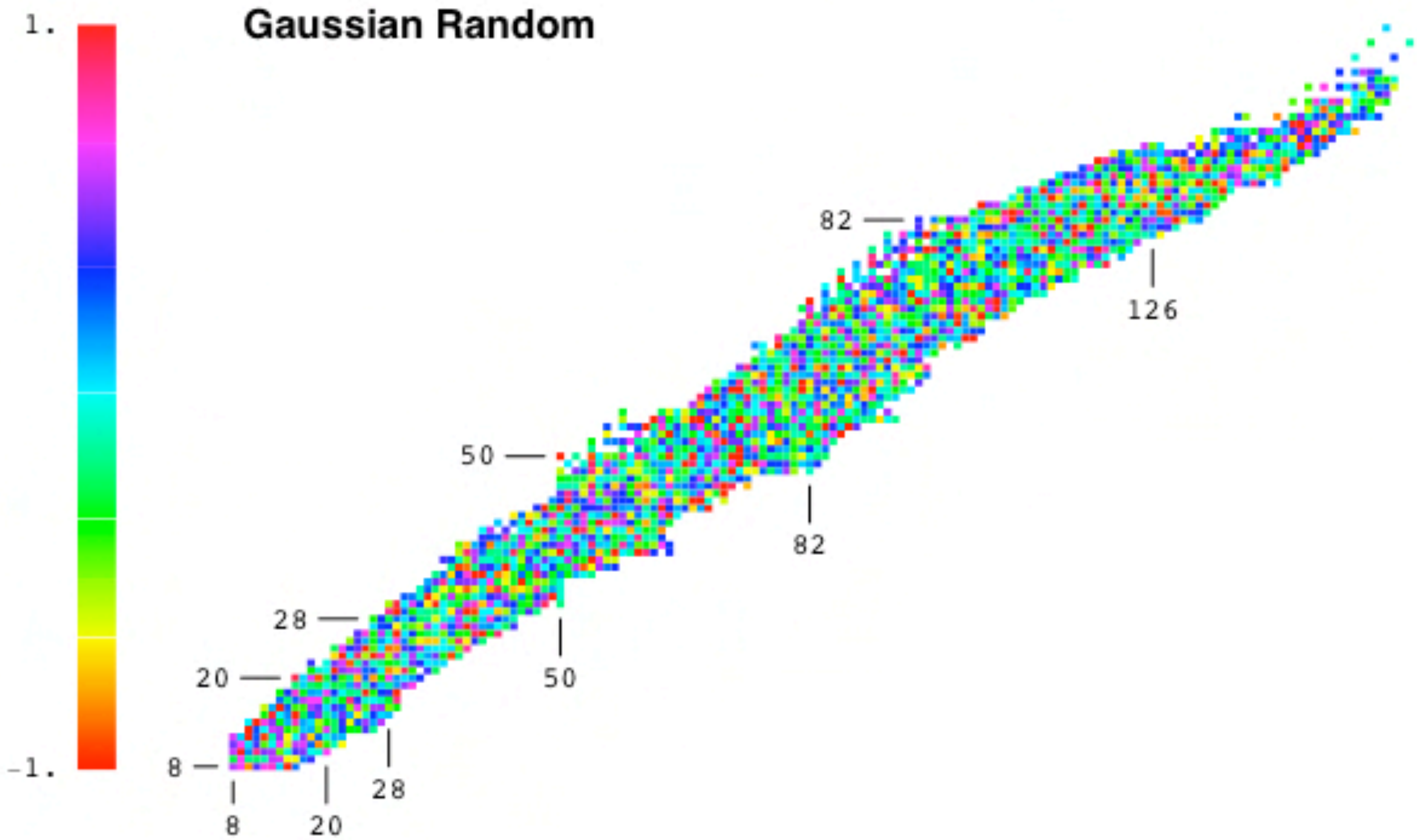
- In ^{23}Na: ΔE_1=7.891 & ΔE_2=19.586 ⇒ r=2.15

J. Jänecke & T.W. O'Donnell, Phys. Lett. B **605** (2005) 87

Deviations from FRDM

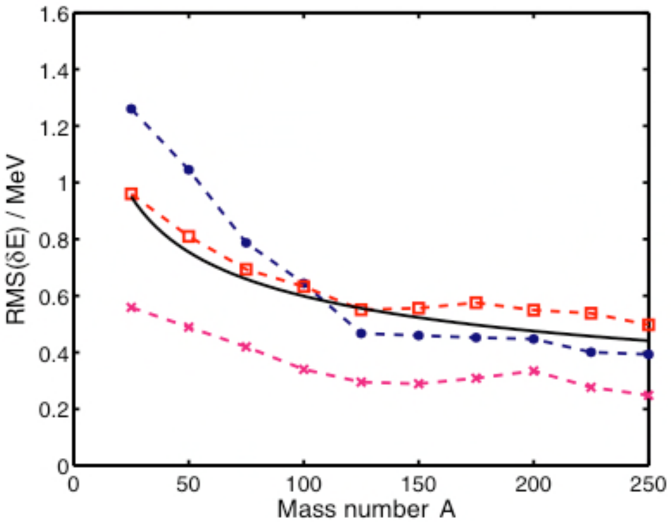


Uncorrelated deviations



Nuclear masses and chaos

- Precision of actual mass formulas is ~ 0.5 MeV.
- Bohigas & Leboeuf: Current mass formulas assume regular dynamics and neglect a ‘chaotic’ deviation.
- Åberg: […] it will certainly be extremely hard to improve the theoretical description of nuclear masses.



$$\sigma_{\text{chaos}}(N,Z) = \frac{2.78}{A^{1/3}}\ \text{MeV}$$

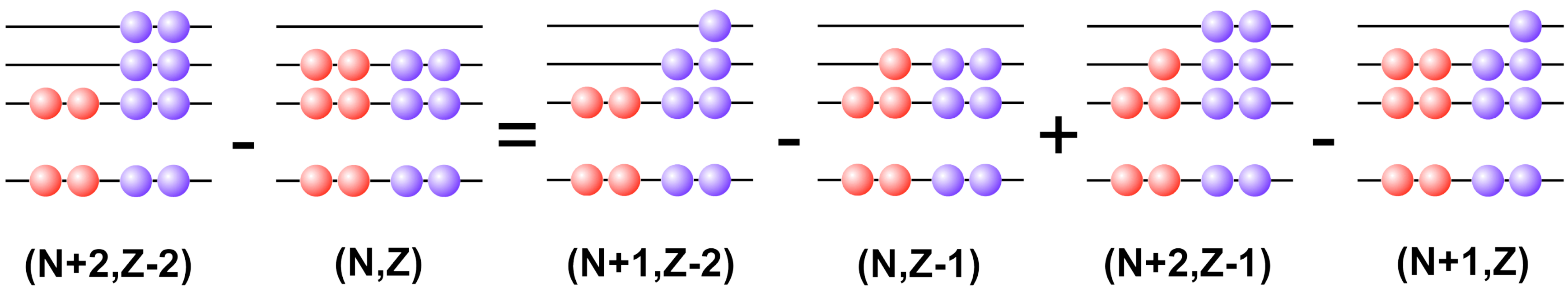
O. Bohigas & P. Leboeuf, Phys. Rev. Lett. **88** (2002) 092502
S. Åberg, Nature **417** (2002) 499

Garvey-Kelson relations

- Relations between masses of neighbouring nuclei:

$$-B(N+1,Z-2)+B(N+1,Z)-B(N+2,Z-1)$$
$$+B(N+2,Z-2)-B(N,Z)+B(N,Z-1)=0$$

$$B(N+2,Z)-B(N,Z-2)+B(N+1,Z-2)$$
$$-B(N+2,Z-1)+B(N,Z-1)-B(N+1,Z)=0$$



Extended Garvey-Kelson relations

- The mass of every nucleus can be "predicted" on the basis of its neighbours in 12 different ways. The average gives:

	$N-2$	$N-1$	N	$N+1$	$N+2$
$Z-2$	$+1$	-2	$+2$	-2	$+1$
$Z-1$	-2		$+4$		-2
Z	$+2$	$+4$	-12	$+4$	$+2$
$Z+1$	-2		$+4$		-2
$Z+2$	$+1$	-2	$+2$	-2	$+1$

Mass deviations and errors

- Deviations $\sigma_{\rm rms}$ and errors $\sigma_{\rm err}$:

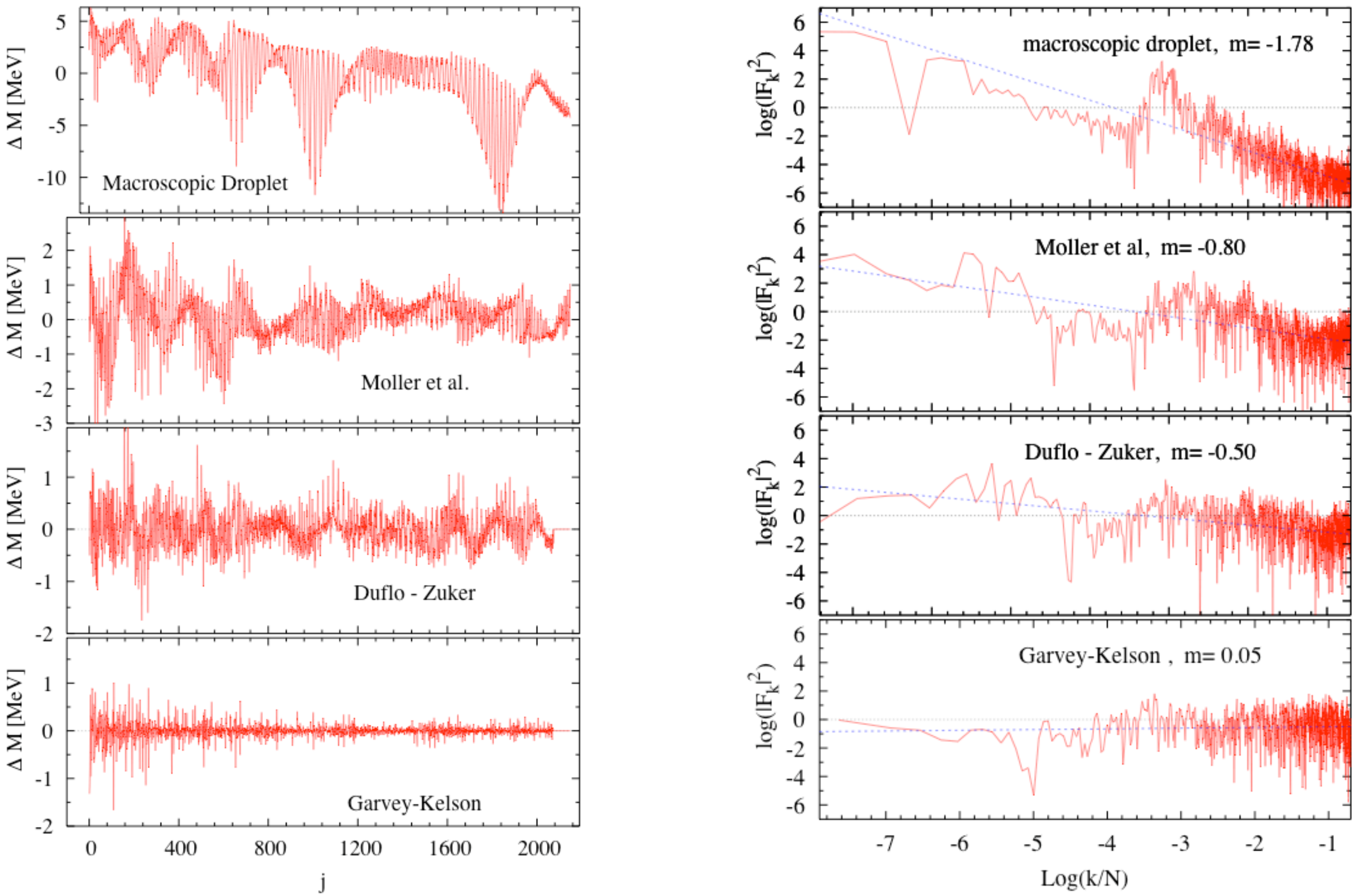
$$\sigma_{\rm rms} = \sqrt{\frac{1}{M}\sum_{j=1}^{M}\left(B_j^{\rm expt} - B_j^{\rm theo}\right)^2}, \quad \sigma_{\rm err} = \sqrt{\frac{1}{M}\sum_{j=1}^{M}\left(\Delta B_j^{\rm expt}\right)^2}$$

- Deviation and errors (in MeV) for $N,Z \geq 8$:

	mLDM	FRDM	HFB9	DZ	GK1	GK12
M	2149	2149	2149	2149	2066	1008
$\sigma_{\rm rms}$	1.187	0.650	0.733	0.360	0.155	0.087
$\sigma_{\rm err}$	0.121	0.121	0.121	0.121	0.111	0.034

J. Barea *et al.*, Phys. Rev. Lett. **94** (2005) 102501

LDM, FRDM, DZ, GK fluctuations



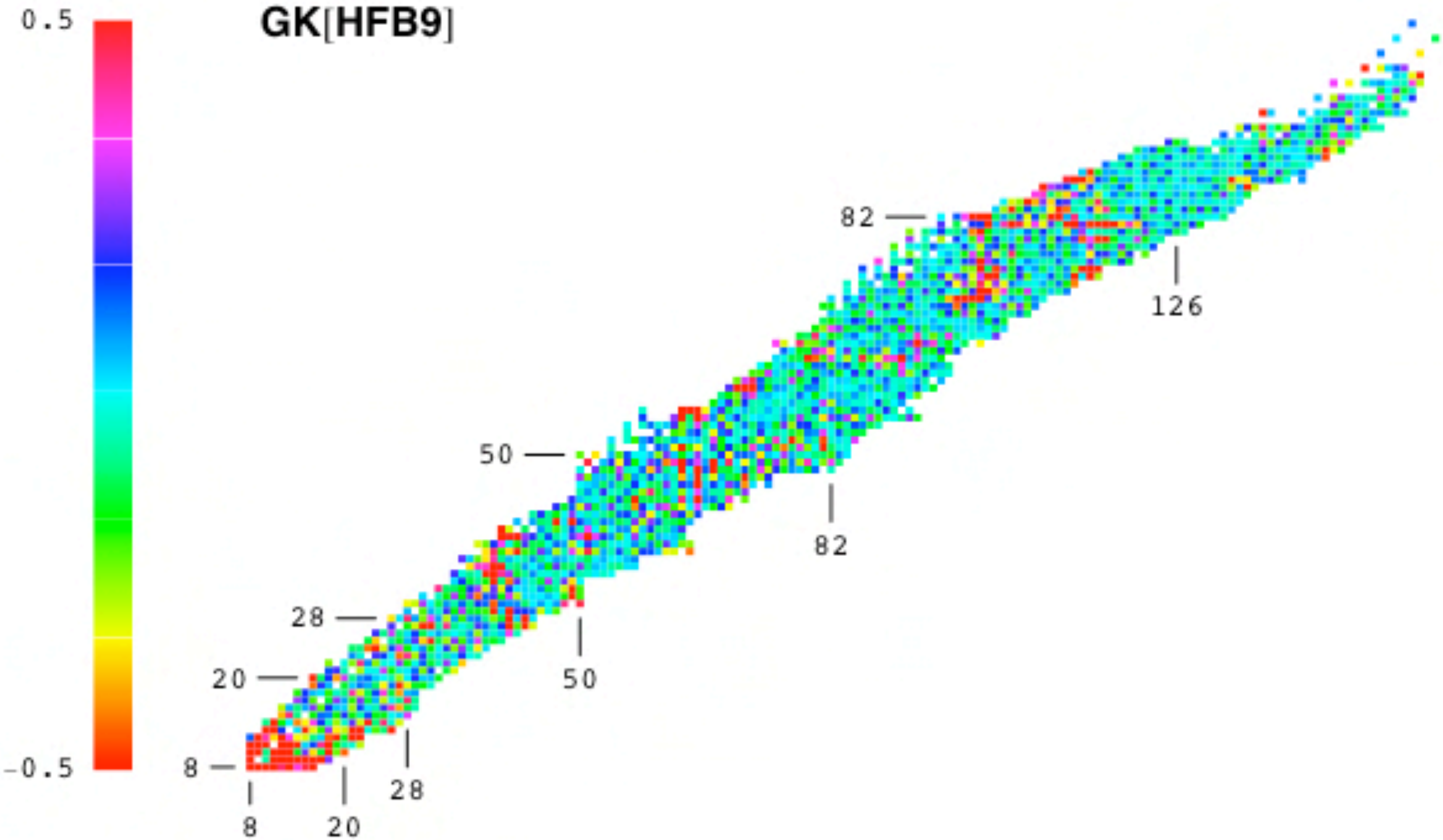
Correlated chaos

- The Garvey-Kelson relations provide an example of uncorrelated deviations (white noise) with $\sigma_{rms} \approx 100$ keV.
- The chaotic mass component is not entirely uncorrelated and is, at least partially, predictable.

"It's chaos but organized chaos"

Charles Mingus

Tests of mass predictions from GK



Test of mass predictions from GK

- Do mass predictions satisfy the GK relations? Example: How well is GK12 satisfied?

	All in	nuclei AME95	"All"	nuclei	"All" not in	nuclei AME95
	M	σ_{rms}	M	σ_{rms}	M	σ_{rms}
Expt	570	0.098	$--$	$--$	$--$	$--$
FRDM	1555	0.125	7223	0.280	5668	0.309
HFB9	1555	0.212	6286	0.339	4731	0.372
DZ	1555	0.037	7188	0.045	5633	0.047

Test of mass predictions from GK

- Do mass predictions satisfy the GK relations? Example: How well is GK12 satisfied?

	All in	nuclei AME03	"All"	nuclei	"All" not in	nuclei AME03
	M	σ_{rms}	M	σ_{rms}	M	σ_{rms}
Expt	1008	0.087	$--$	$--$	$--$	$--$
FRDM	1909	0.126	7223	0.280	5314	0.318
HFB9	1918	0.226	6286	0.339	4368	0.379
DZ	1918	0.037	7188	0.045	5270	0.047

Conclusions

- Consider surface *and* Wigner corrections in the liquid-drop mass formula to determine the symmetry energy in nuclei.
- Chaotic mass component is ***not*** unpredictable and can be (at least partially) calculated.
- Measured masses do satisfy GK relations; several existing mass formulas do not.