Isochronous Ring at RIKEN

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Objective

- to Establish

A New Scheme of Precise Mass Measurement

- For Energetic RI Beam

From RIBF at RIKEN

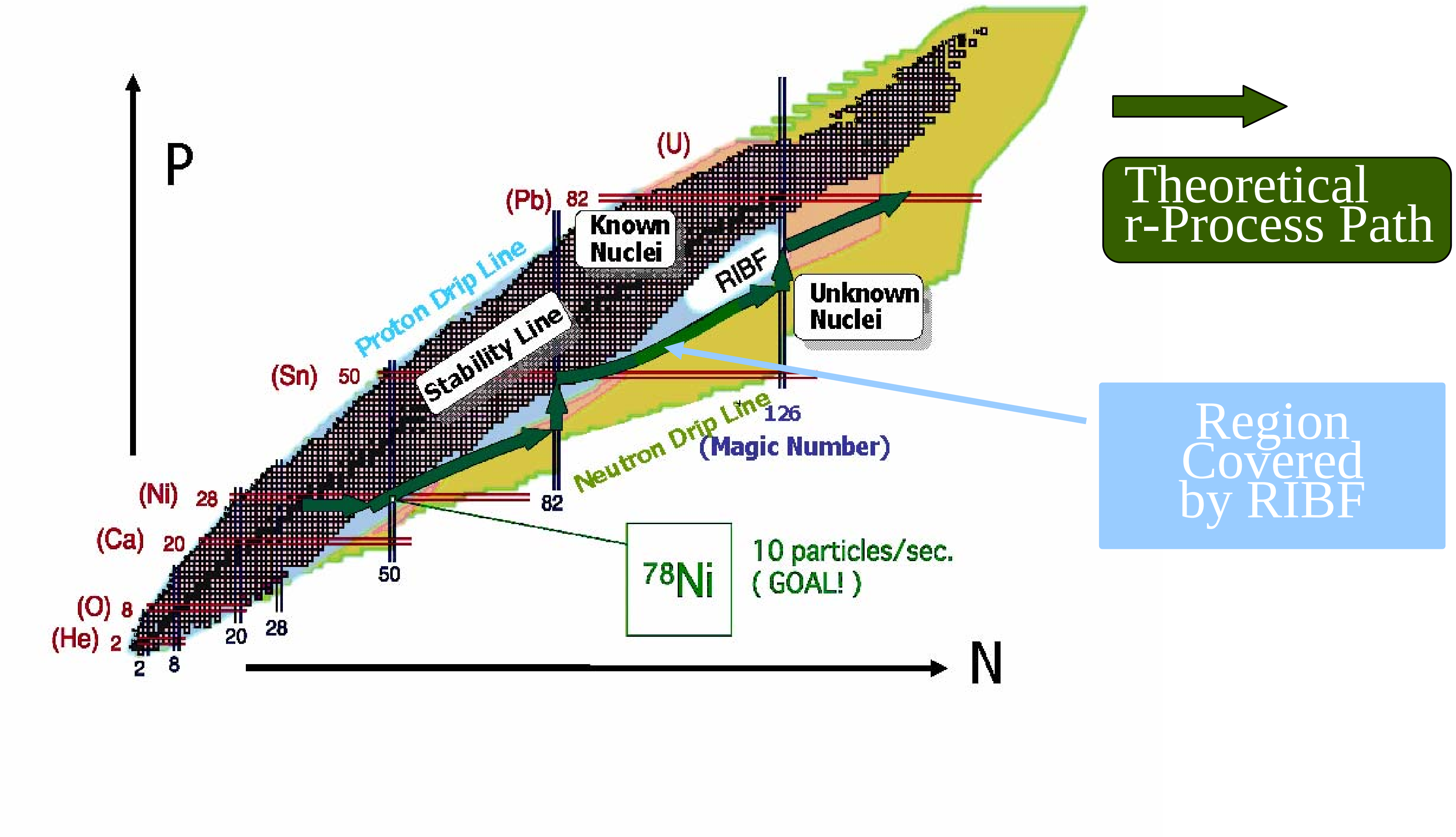
200 A MeV, m/q~3.0

Physical Interests

to Study

- Nuclei Far from Stability
  - ✓ “Exotic“ Nuclear Structure
- Astrophysical r-Process Path
  - ✓ Super Nova Nucleo-Syntheses

Nuclei Far from stability

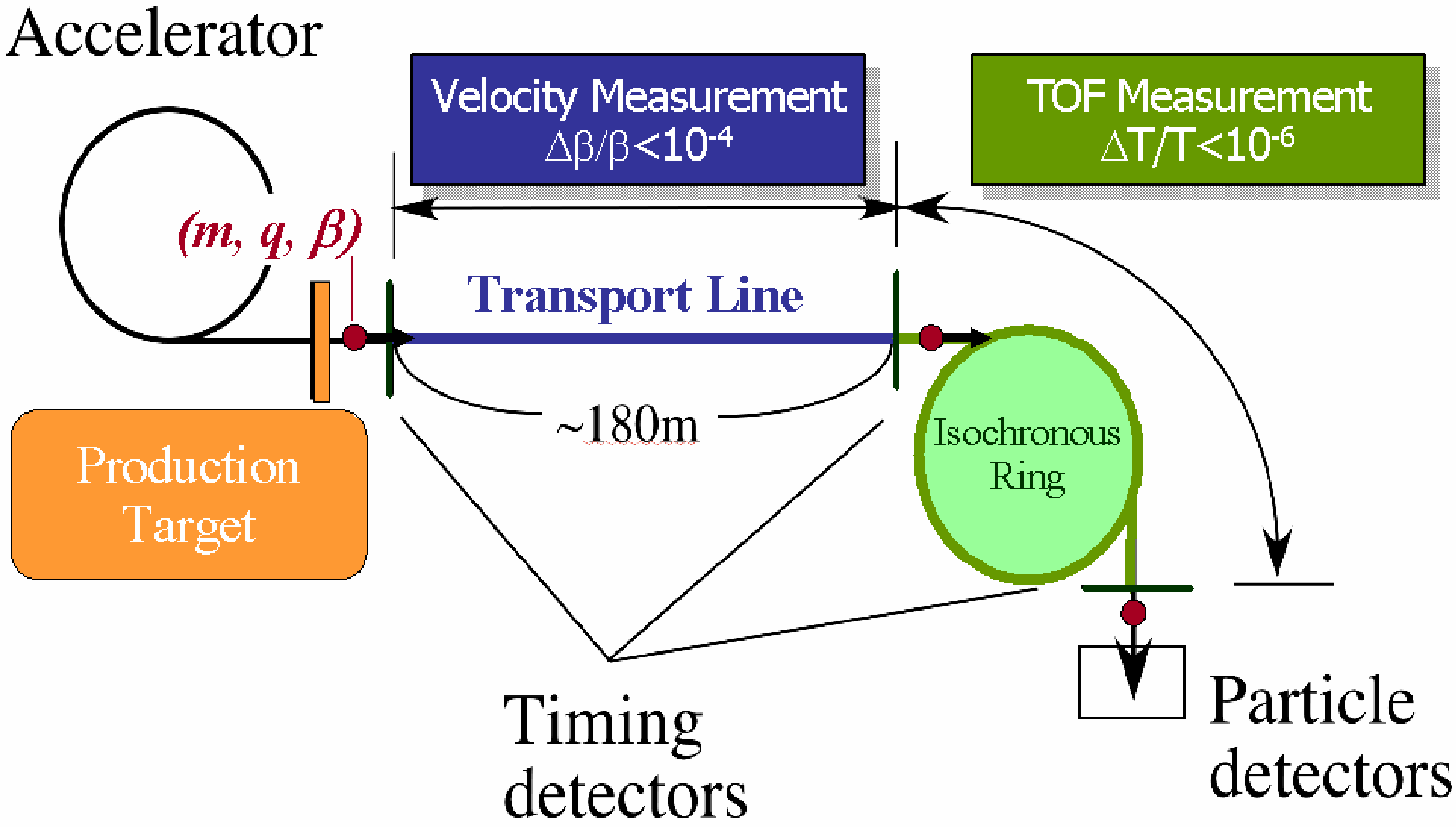


What is New Scheme ?

Combination of

- Velocity Measurement
  in Transport Line
- TOF Measurement
  in Isochronous Ring

Proposed Apparatus



Principle of Mass Measurement

- For Known Nucleus, $$T_0 = 2\pi \frac{m_0\gamma_0}{qB}$$

  T_0, m_0, q, γ_0: Fixed

- For Unknown Nucleus, $$T_1 = 2\pi \frac{m_1\gamma_1}{qB}$$

  T_1, γ_1: Measured

- Then, ***m_1: <u>Determined</u>***

$$\frac{m_1}{q} = \left(\frac{m_0}{q}\right)\frac{T_1}{T_0}\frac{\gamma_0}{\gamma_1} = \left(\frac{m_0}{q}\right)\frac{T_1}{T_0}\sqrt{\frac{1-\beta_1{}^2}{1-\left(\frac{T_1}{T_0}\beta_1\right)^2}}$$

$$\boxed{\gamma = \frac{1}{\sqrt{1-\beta^2}}}$$

Accuracy of Measurement

- Expected Accuracy for Mass

$$\left(\frac{\delta m_1}{m_1}\right)^2 = \left(\frac{\delta T_1}{T_1}\right)^2 + \underbrace{\left(\frac{2\beta_1^{\ 2}}{\left(1-\beta_1^{\ 2}\right)^2}\frac{T_1 - T_0}{T_0}\right)}_{\sim 10^{-4} \text{ for } \sim 1\% \ TOF \text{ Difference}}\left(\frac{\delta\beta_1}{\beta_1}\right)^2$$

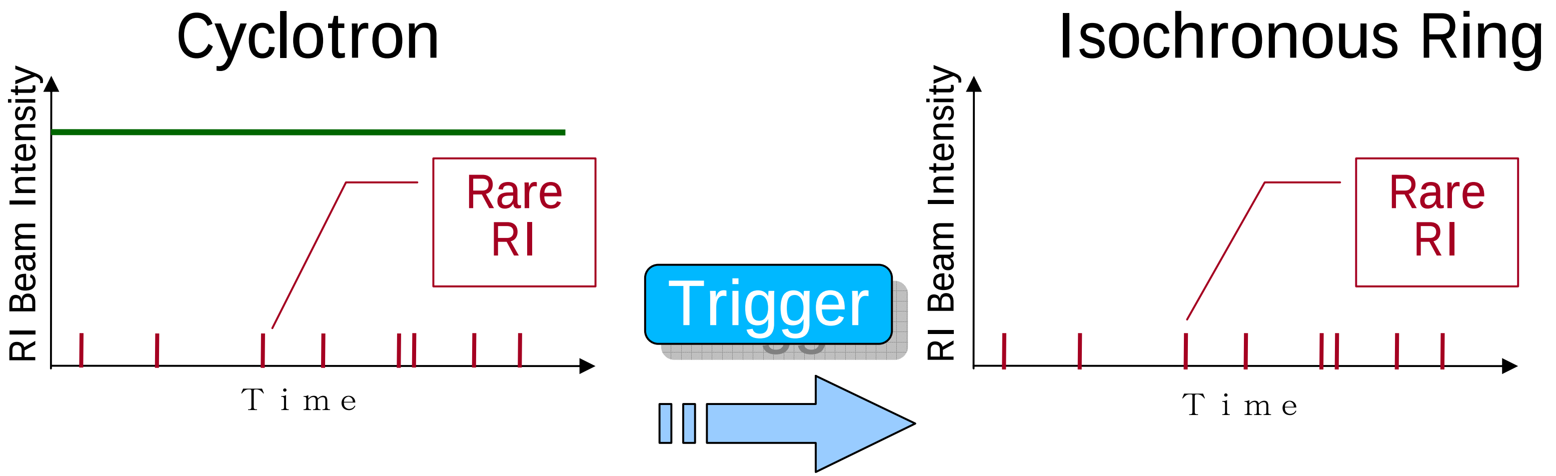
- Mass Resolution of $\sim 10^{-6}$ is Achieved under Feasible Condition

$$\frac{\delta T_1}{T_1} \sim 10^{-6}, \quad \frac{\delta \beta_1}{\beta_1} \sim 10^{-4}$$

Individual Injection

Triggered Injection

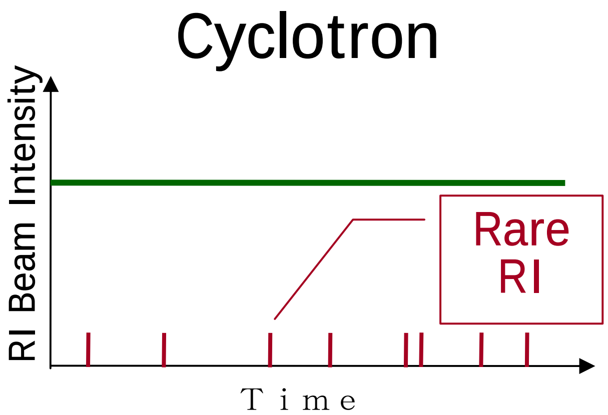
- Sampling on Trigger
- Accepts Only Interested Particles
- 100% Efficiency in Principle

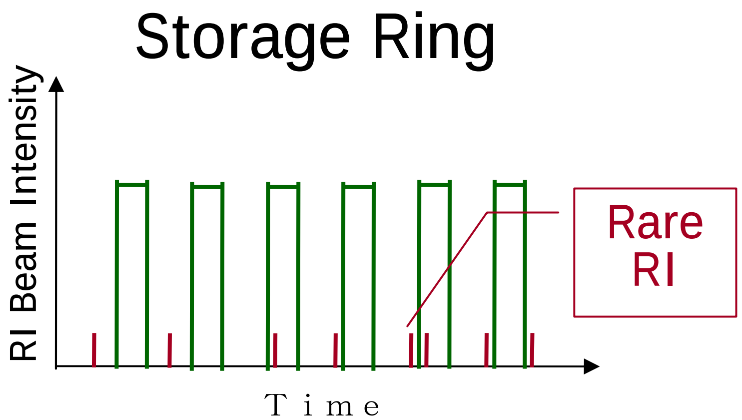


Conventional Method

Cyclic Injection

- Sampling of Beam Particles
  Synchronized with Operation of Storage Ring
- Low Efficiency





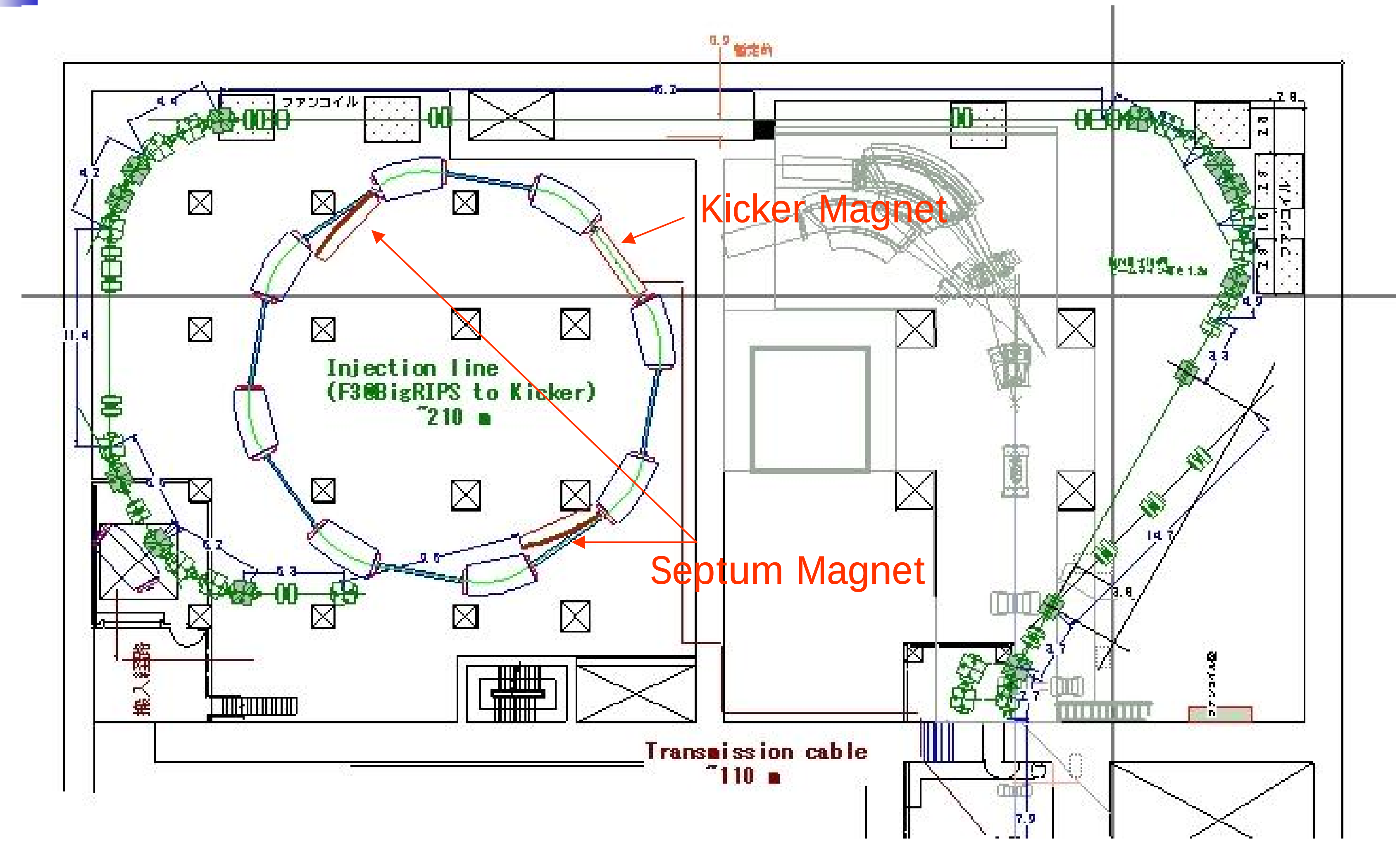
Dead Time/Maximum Rate

- Dead Time ~ **400** ***μsec***
  - ✓Beam Transport
  - ✓for 1,000 turns Circulation in Isochronous Ring

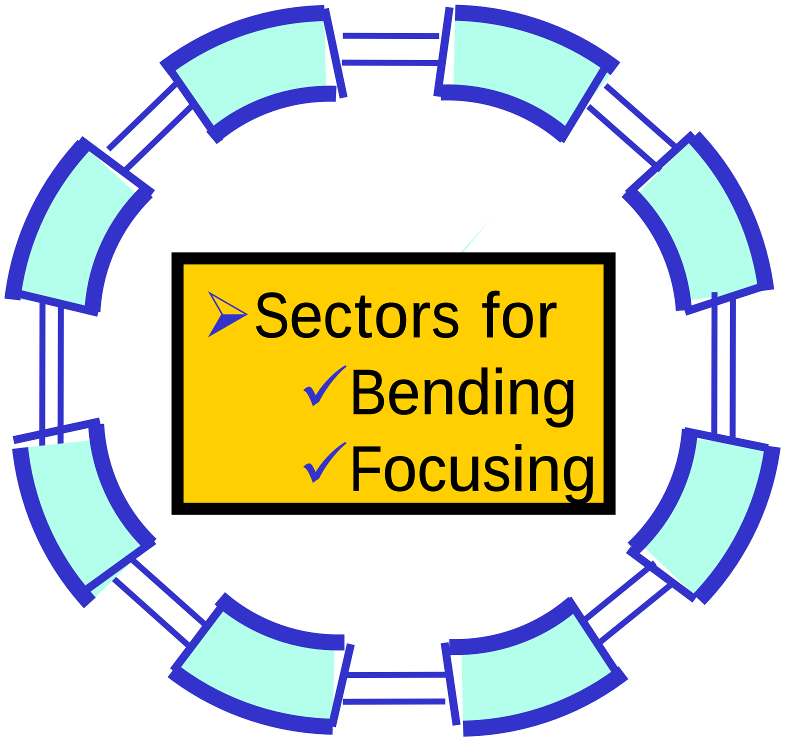
  ⇒ ~ **2,500** ***particles /sec*** **at max.**
- Expected Rate of Rare RI around ^{78}Ni

  ~ **10** ***particles/sec***

Layout of Experimental Apparatus at RIBF(Plan View)



Configuration

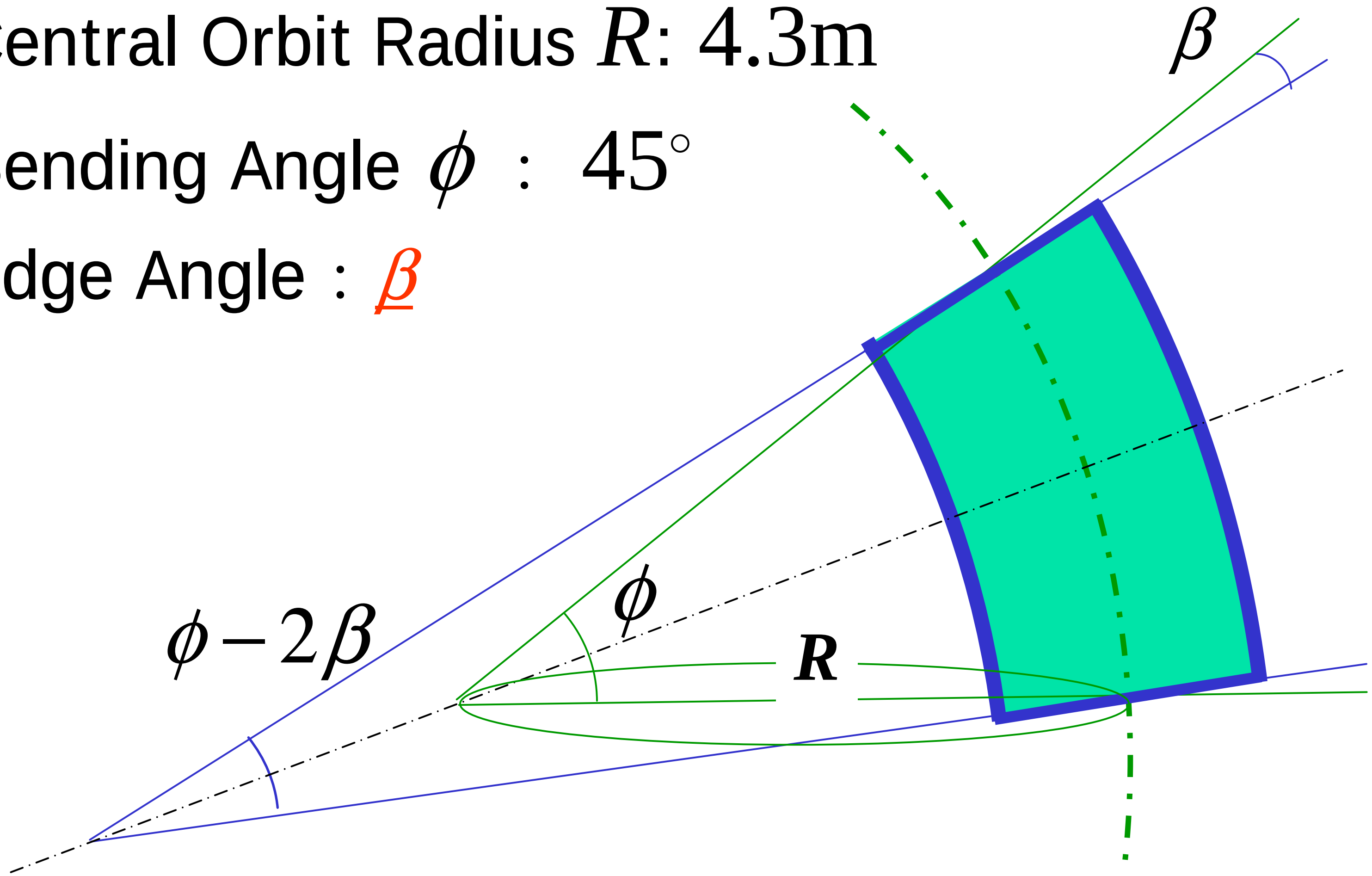


- Separate Sector Ring:
  - ✓8 Sectors
  - ✓8 Straight Sections
- Circumference : 67.36 m
- Circulation : 1,000 Turns
- Isochronisity: $\frac{\Delta T}{T} \leq 10^{-6}$
- Momentum Range:

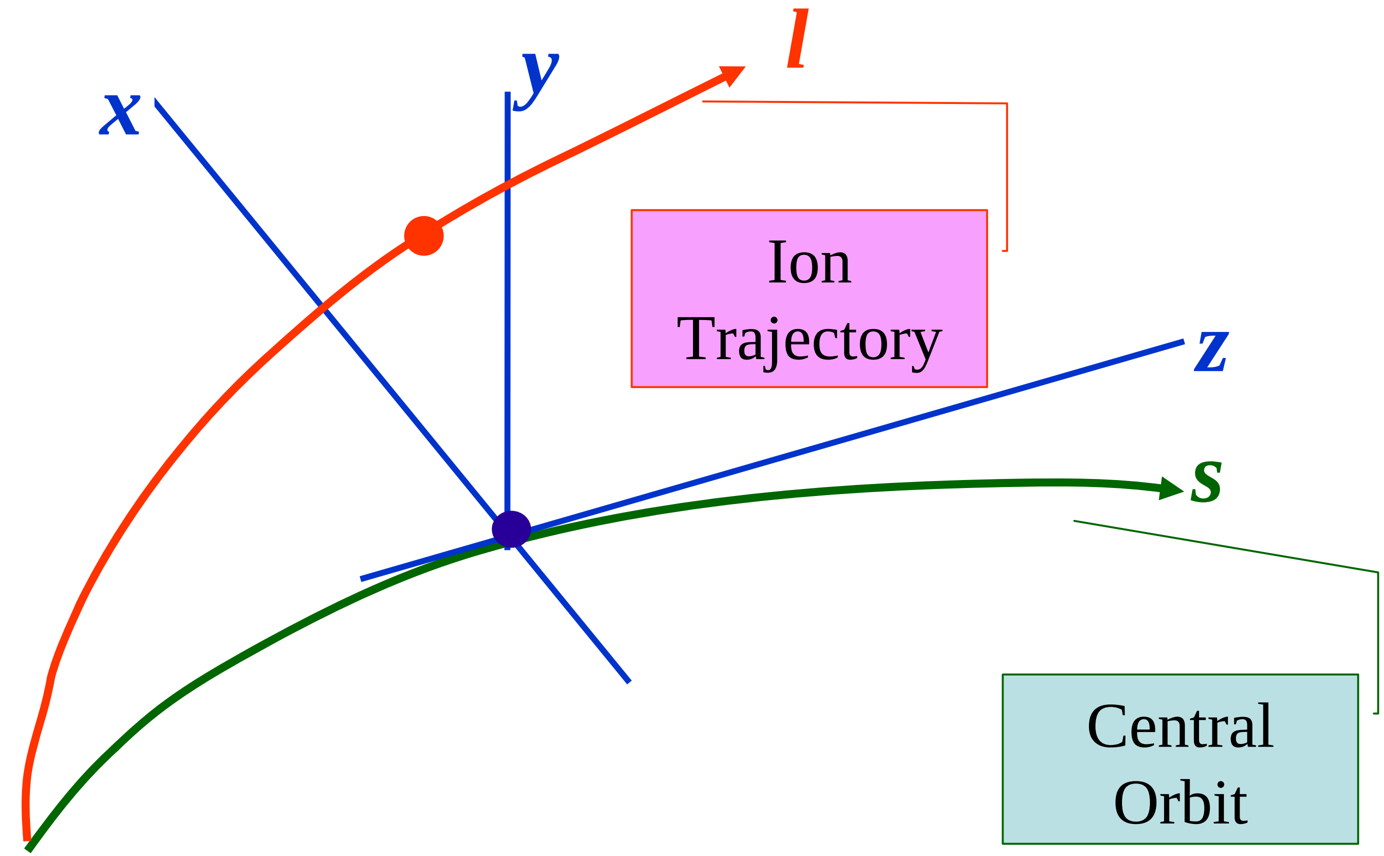
$$-0.01 \leq \frac{\Delta P}{P} \leq +0.01$$

Flat-Edged Sector

- Central Orbit Radius R: 4.3m
- Bending Angle ϕ : 45°
- Edge Angle : β



Coordinate System



Edge Angle β

Needed for

- Vertical Focusing

$$\Delta y' = -\tan\beta \times \frac{y}{R\gamma}$$

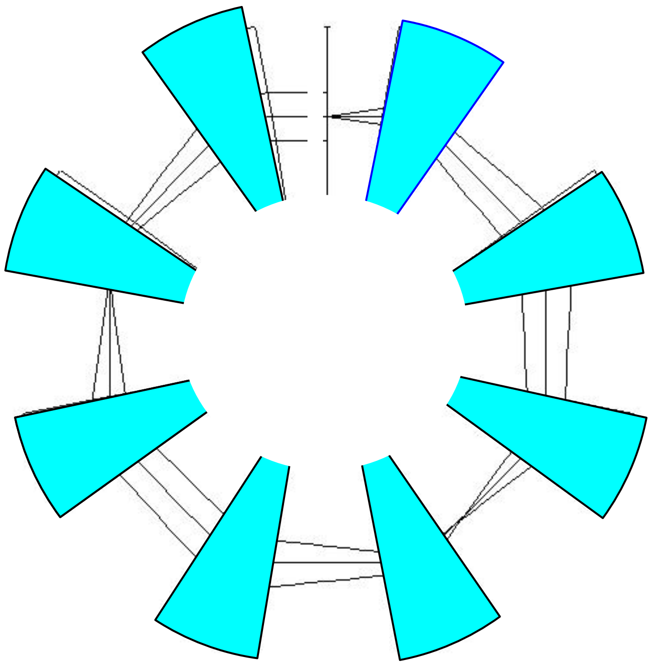
(in Hard Edge Approx.)

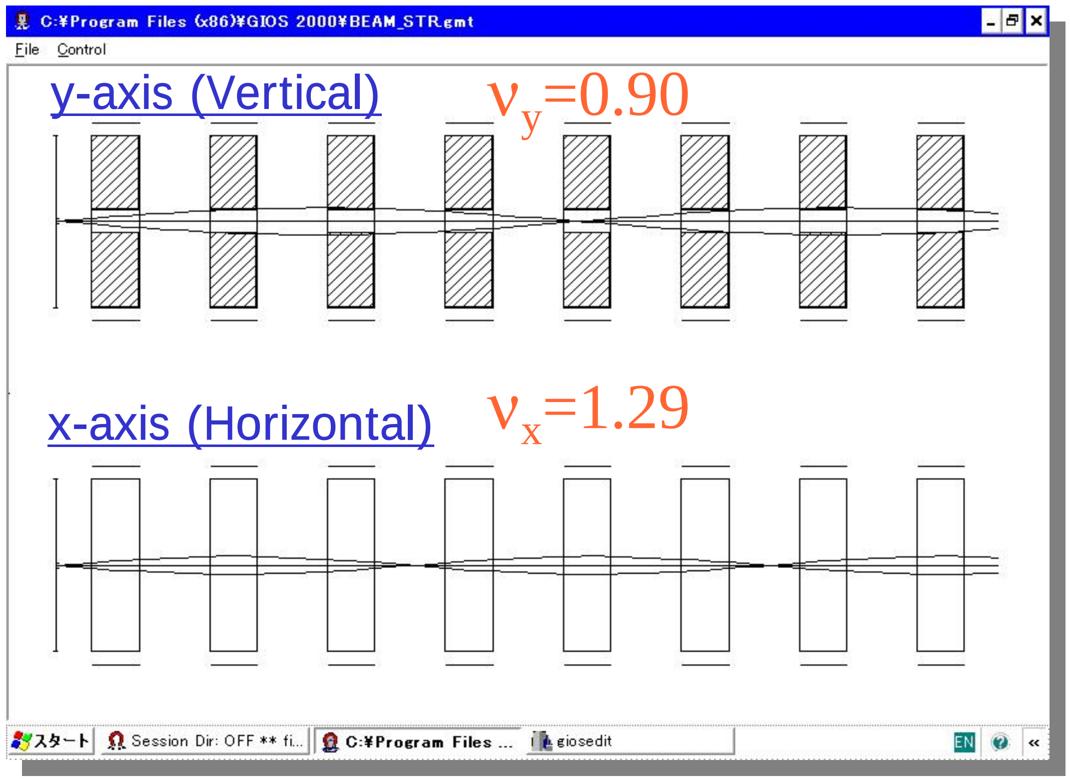
- Horizontal Tune
  - ✓ Designed Value is $\nu_y \approx 0.9$

Edge Angle for ν_y=0.9

with Use of GIOS Simulator

$$\beta = 7.6^\circ$$





Isochronisity

Keep the Same TOF for Particles with Different Momenta by

- Applying Non-Uniform Magnetic Field
  - ✓Shifts the Trajectory along x-Axis
  - ✓Changes the Curvature of Trajectory

Both result the Change of Path Length

Magnetic Field

Combination of

- Main Field: B_0

  Giving a Storage Force

- Trim Field: $B_T(x)$

  Keeping an Isochronicity

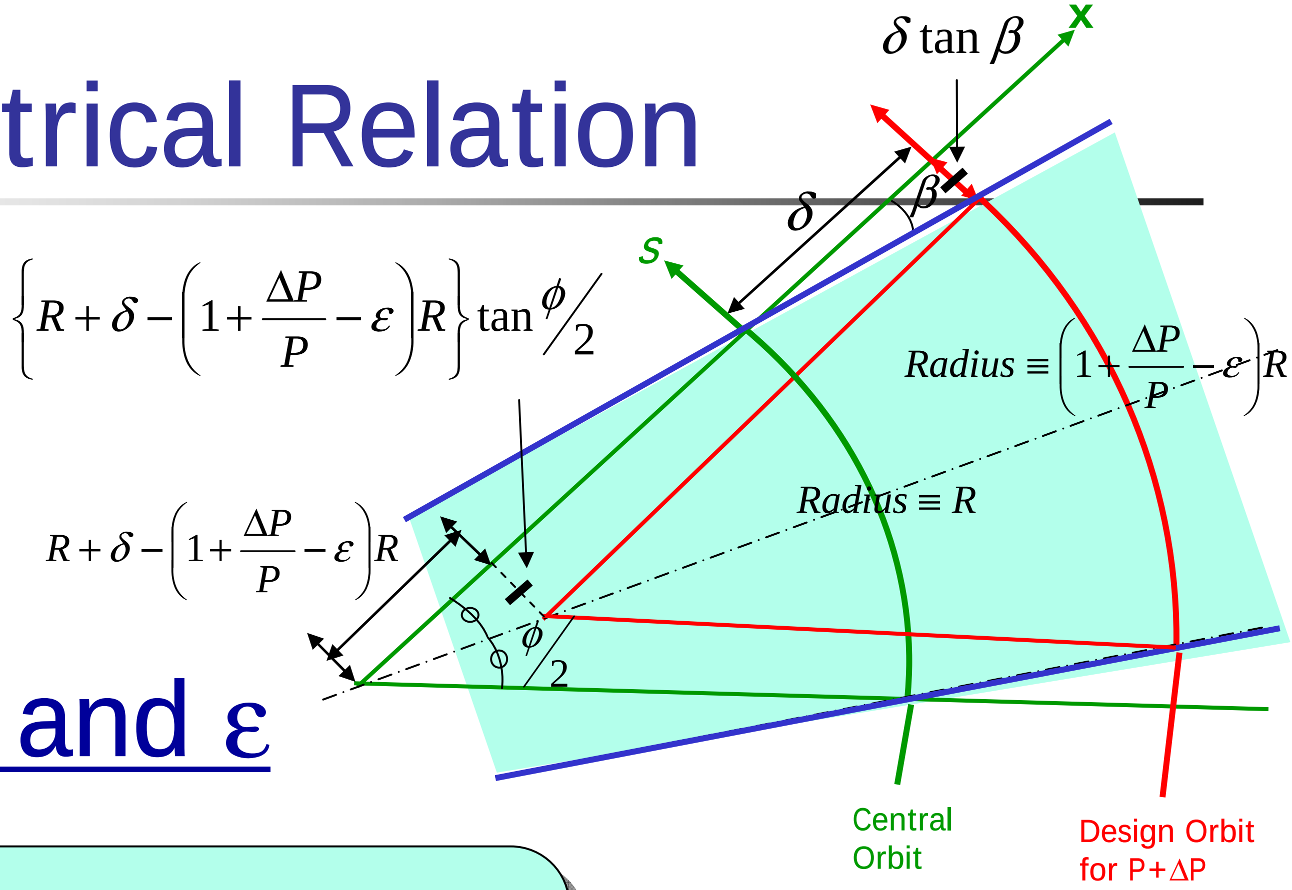
$$B_T(x) = B_0\left(Ax + Bx^2\right)$$

Adjusting of Trim Field

- Step1: First Order Solution in $\Delta p/p$
  - Obtained under 1)Geometrical Relation and 2)Isochronisity Condition
  - Gives 1)Desirable Value of A and 2)Momentum Dispersion ξ

- Step2: Further Fine Tuning for A and Determination of Parameter B
  - By Hand
  - using Newly Developed Simulator

Geometrical Relation



Between δ and ε

$$\left\{R+\delta-\left(1+\frac{\Delta P}{P}-\varepsilon\right)R\right\}\tan\frac{\phi}{2}=\delta\tan\beta$$

ε: Fraction of Trim Field

Isochronisity Condition

Same TOF among Different Momenta

$$\frac{(\gamma+\Delta\gamma)\left\{R\phi\left(1+\frac{\Delta P}{P}-\varepsilon\right)+S+2\delta\tan\beta\right\}}{P+\Delta P}=\frac{\gamma\left(R\phi+S\right)}{P}$$

TOF of Design Orbit
for p+Δp

TOF of
Central
Orbit

for Single Unit Segment

First Order Solution in $\Delta p/p$

$$\delta = \xi \frac{\Delta p}{p}, \quad \varepsilon = \left(1 - \frac{\xi}{R} \frac{\tan \phi/2 - \tan\beta}{\tan \phi/2}\right) \frac{\Delta P}{P}$$

ξ : Momentum Dispersion

$$\xi = \frac{\partial x}{\partial \left(\Delta P/P\right)}$$

$$= \frac{\tan \phi/2}{\tan \phi/2 - \tan\beta} \frac{\phi + S/R}{\phi + \dfrac{2\tan \phi/2 \tan\beta}{\tan \phi/2 - \tan\beta}} \frac{1}{\gamma^2} R$$

ε as a function of δ

Using δ/ξ in place of ΔP/P

$$\varepsilon = \left(1 - \frac{\xi}{R}\frac{\tan{\phi}/{2} - \tan\beta}{\tan{\phi}/{2}}\right)\frac{\Delta P}{P}$$

$$= \left(1 - \frac{\xi}{R}\frac{\tan{\phi}/{2} - \tan\beta}{\tan{\phi}/{2}}\right)\frac{\delta}{\xi}$$

Estimate of Parameter A

- Under the Approximation $\delta \fallingdotseq x$

$$\varepsilon = \left(1 - \frac{\xi}{R}\frac{\tan{\phi/2} - \tan\beta}{\tan{\phi/2}}\right)\frac{1}{\xi}x \quad (\equiv Ax)$$

- Parameter A

$$A = \left(1 - \frac{\xi}{R}\frac{\tan{\phi/2} - \tan\beta}{\tan{\phi/2}}\right)\frac{1}{\xi}$$

Newly Developed Simulator

- Magnetic Sector
  - ✓Divided into *M* Small Sub-sectors with Bending Angle

$$\delta\phi = \frac{\phi - 2\beta}{M}$$

- Orbit in Sub-sector
  - ✓Circular Orbit According to Local Field Strength
- 4th Order Runge-Kutta Method

4th Order Runge-Kutta Method

Integration

Integration	Using	To Evaluate
Step.1	$B(s_i, x_i)$	(s_1, x_1, x_1')
Step.2	$B((s_i + s_1)/2, (x_i + x_1)/2)$	(s_2, x_2, x_2')
Step.3	$B((s_i + s_2)/2, (x_i + x_2)/2)$	(s_3, x_3, x_3')
Step.4	$B(s_3, x_3)$	(s_4, x_4, x_4')

$$s_e = \frac{1}{6}(s_1 + 2s_2 + 2s_3 + s_4) \quad , \quad x_e' = \frac{1}{6}(x_1' + 2x_2' + 2x_3' + x_4')$$

$$x_e = \frac{1}{6}(x_1 + 2x_2 + 2x_3 + x_4)$$

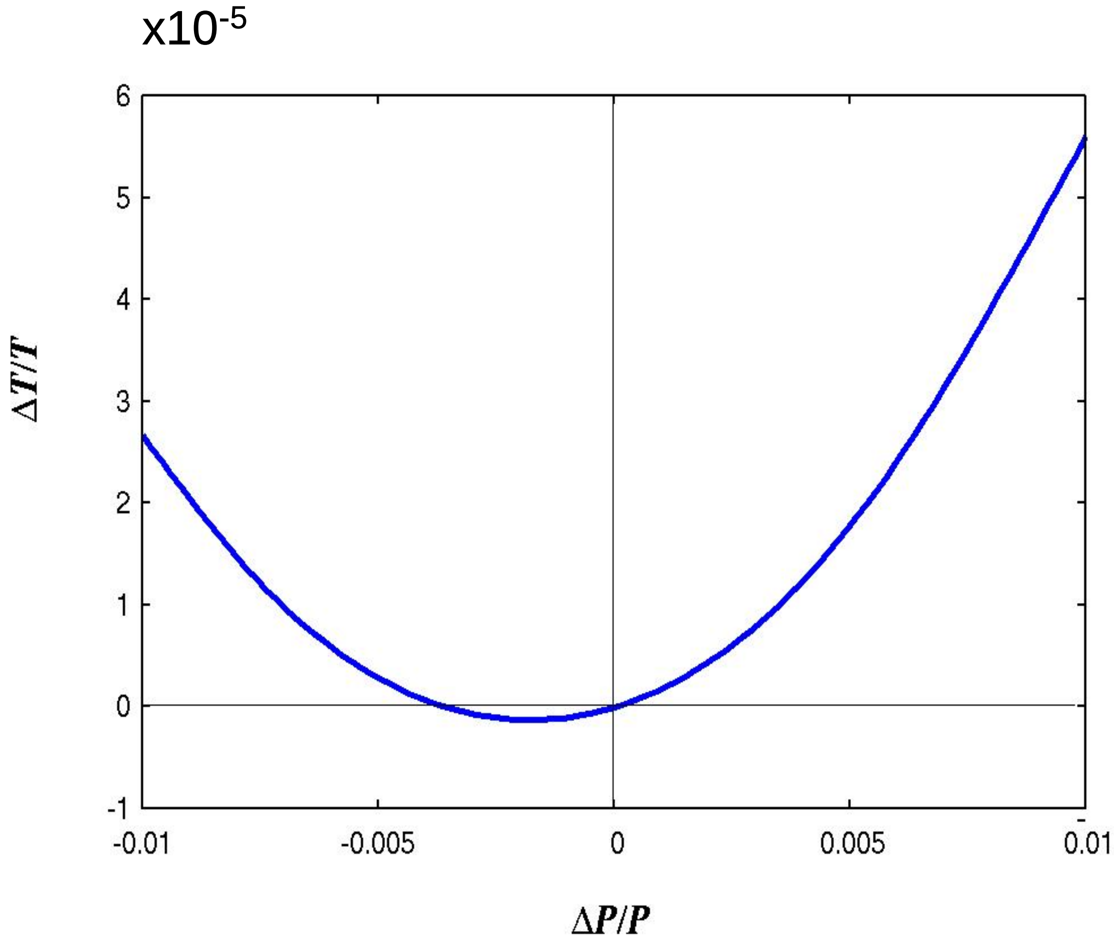
First Order Solution

Achieves

$|\Delta T/T| \leq 6\text{x}10^{-5}$

for

$|\Delta P/P| \leq 1\%$



Second Order Solution

Fine Tuning of A

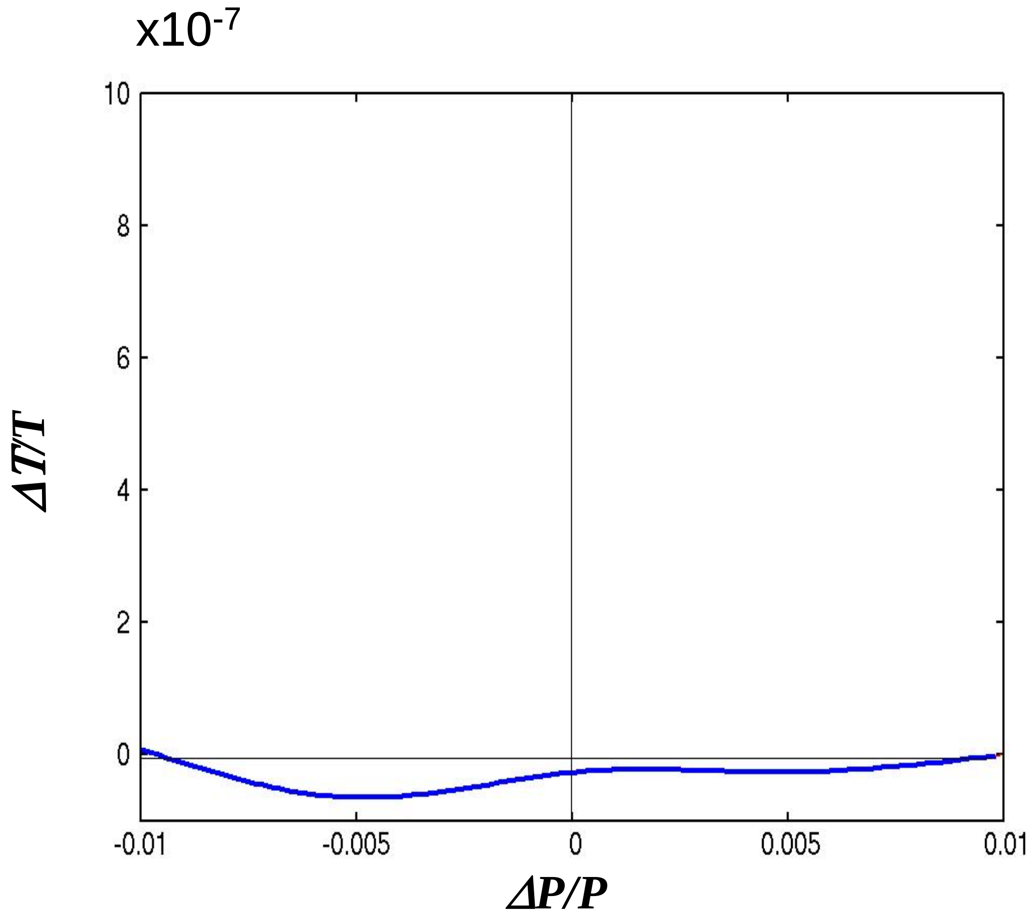
Determination of B

Achieves

$|\Delta T/T| \leq 5 \times 10^{-8}$

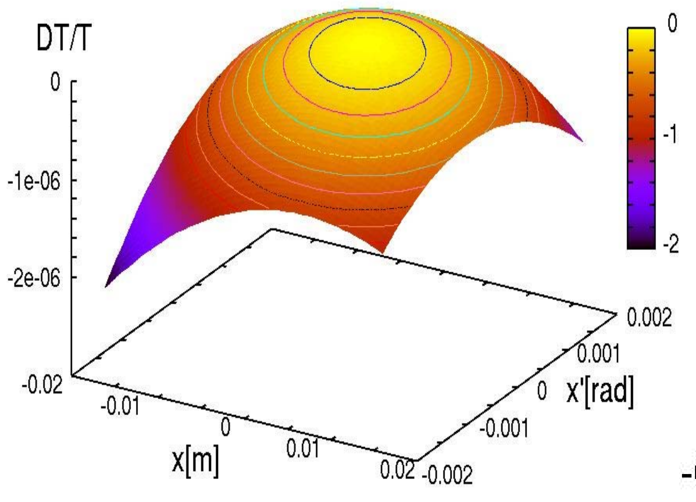
for

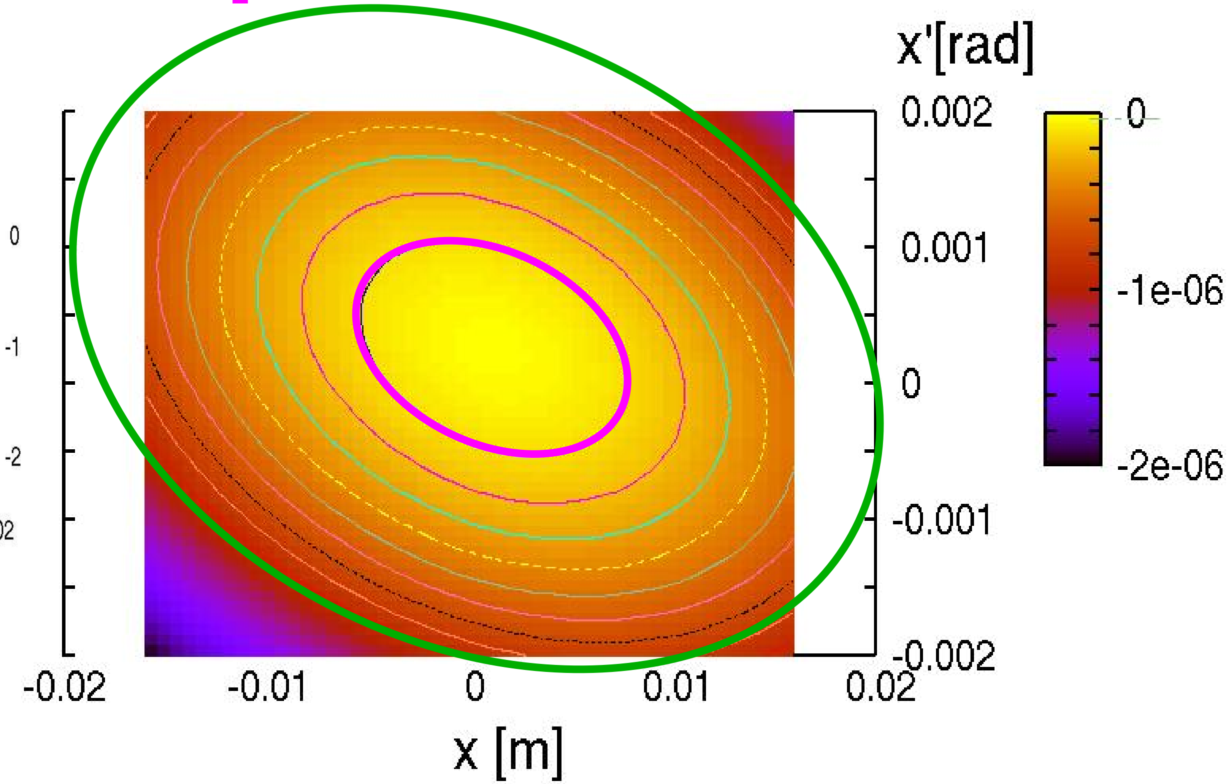
$|\Delta P/P| \leq 1\%$



Emittance at $\Delta P/P=0$

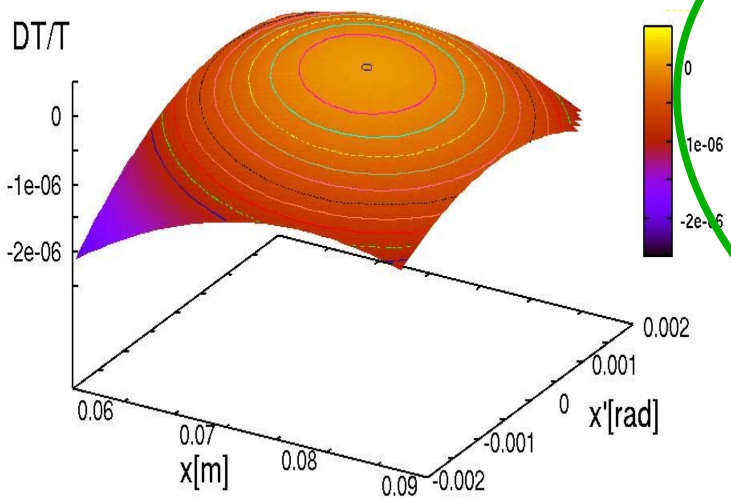
- 47 [π mm mrad] for $\Delta T/T<10^{-6}$
- 5.2 [π mm mrad] for $\Delta T/T<10^{-7}$

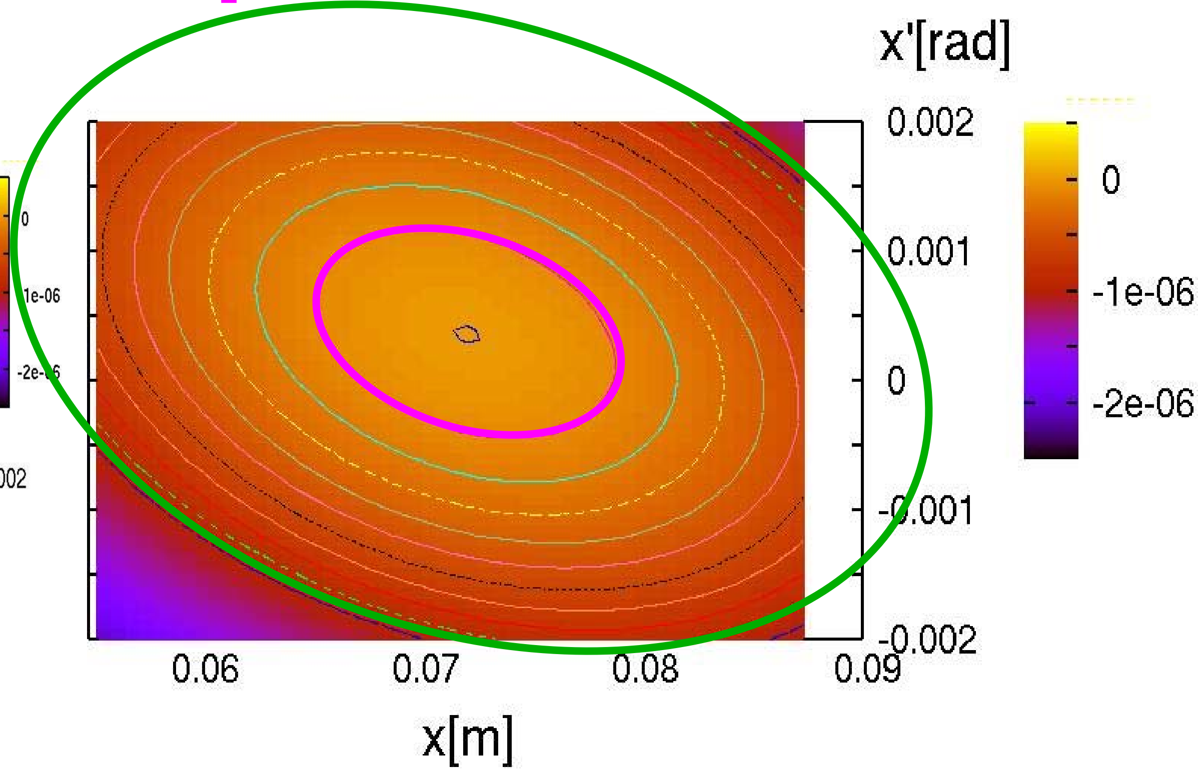




Emittance at $\Delta P/P=+0.01$

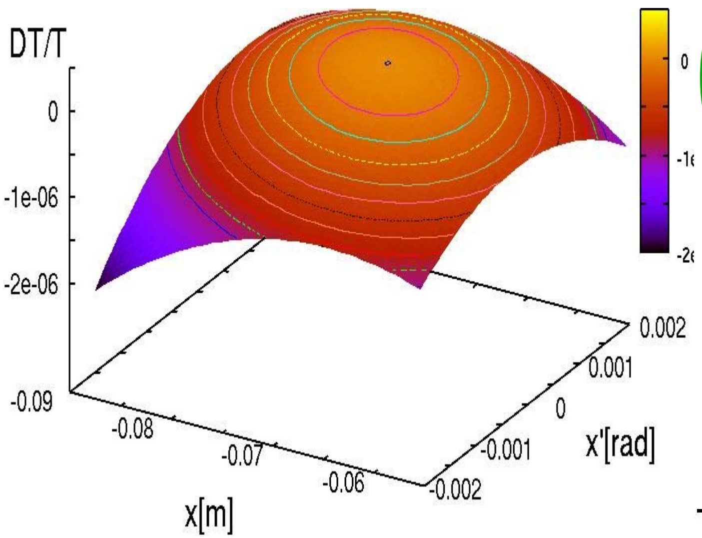
- 48 [π mm mrad] for $\Delta T/T<10^{-6}$
- 5.3 [π mm mrad] for $\Delta T/T<10^{-7}$

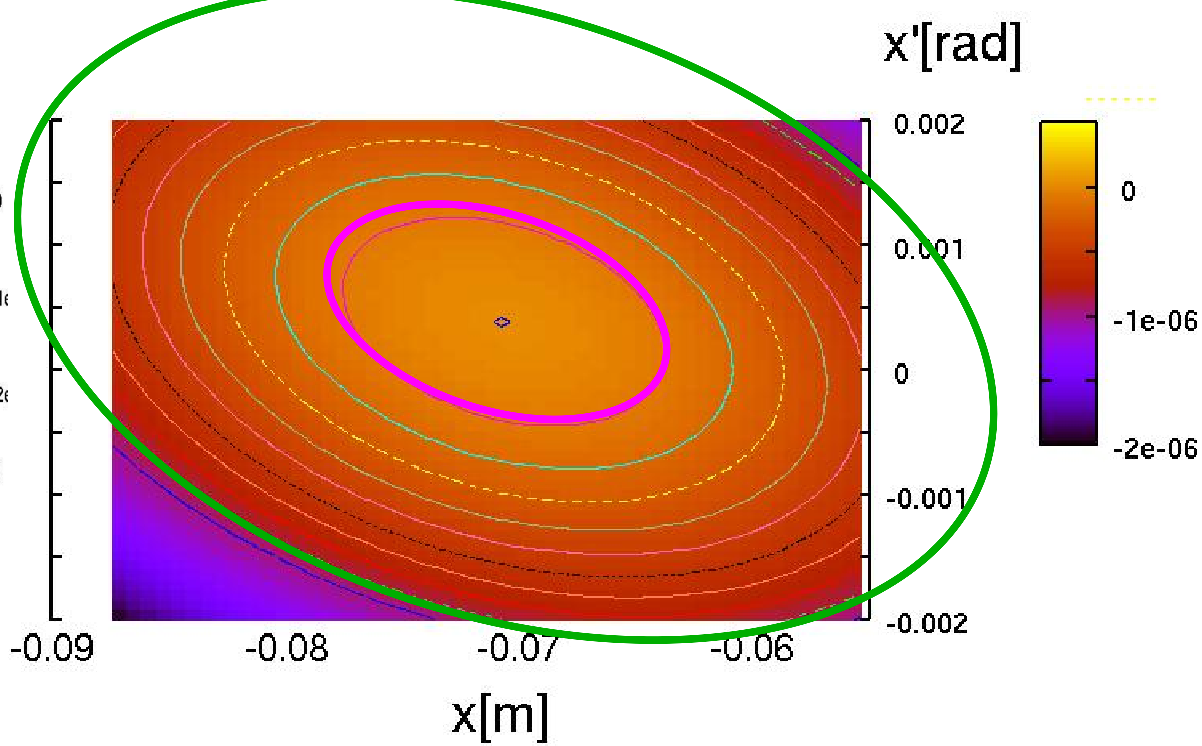




Emittance at $\Delta P/P=-0.01$

- 49 [π mm mrad] for $\Delta T/T<10^{-6}$
- 5.6 [π mm mrad] for $\Delta T/T<10^{-7}$





Emittance -Analytical Evaluation

- After Fine Tuning, Main Contribution for ΔT Comes from Betatron Oscillation
- ΔT is Evaluated by Integrating Flight Length Directly
- Using Different Metrics for
  - ✓Orbit in Magnetic Sector
  - ✓Orbit in Straight Section

Betatron Oscillation

- Equation of Motion

$$x = a \sin \frac{\nu_x}{\rho}(s - \delta)$$

$$y = b \sin \frac{\nu_z}{\rho}(s - \varepsilon)$$

- Parameters a, b, δ and ε
  - ✓Determined by Initial Condition

Metrics for *(dx, dy, ds)*

- Magnetic Sector

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1+\dfrac{x}{R} \end{pmatrix}$$

- Straight Section

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Flight Length in Magnetic Sector

- Line Element

$$dl = \sqrt{\left(\frac{dx}{ds}\right)^2 + \left(\frac{dy}{ds}\right)^2 + \left(1 + \frac{x}{\rho}\right)^2}\ ds$$

- Integration up to 2nd Order

$$\int dl \cong \sqrt{1 + \left(\frac{\nu_x a}{\rho}\right)^2 + \left(\frac{\nu_y b}{\rho}\right)^2} \int_0^{NL} ds + \frac{1}{2\sqrt{1 + \left(\frac{\nu_x a}{\rho}\right)^2 + \left(\frac{\nu_y b}{\rho}\right)^2}} \int_0^{NL} ds \left(-\frac{(1+\nu_x{}^2)x^2 + \nu_y{}^2 y^2}{\rho^2} + \frac{2x}{\rho} \right)$$

Flight Length in Straight Section

- Line Element

$$dl = \sqrt{\left(\frac{dx}{ds}\right)^2 + \left(\frac{dy}{ds}\right)^2 + 1}\, ds$$

$\nu_x{}^2$

- Integration up to 2nd Order

$$\int dl \cong \sqrt{1 + \left(\frac{\nu_x a}{\rho}\right)^2 + \left(\frac{\nu_y b}{\rho}\right)^2} \int_0^{NL} ds$$

$$+ \frac{1}{2\sqrt{1 + \left(\frac{\nu_x a}{\rho}\right)^2 + \left(\frac{\nu_y b}{\rho}\right)^2}} \int_0^{NL} ds \left(- \frac{\nu_x{}^2 x^2 + \nu_y{}^2 y^2}{\rho^2} \right)$$

Emittance -Analytical Evaluation

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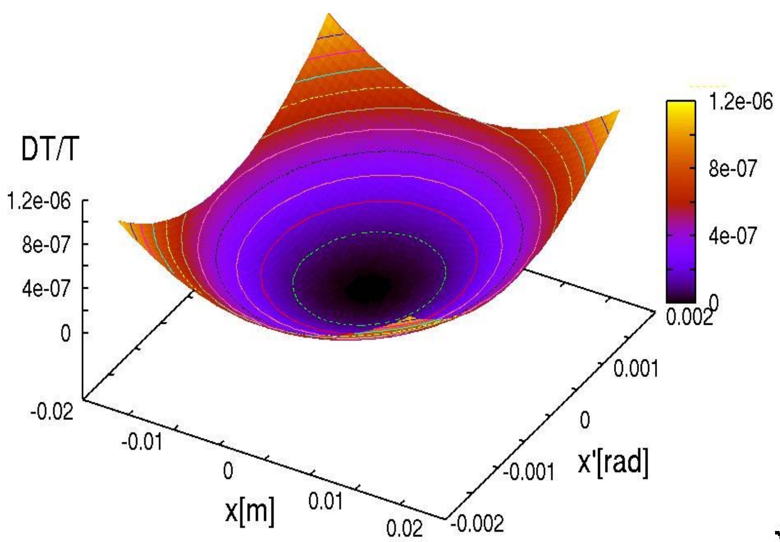

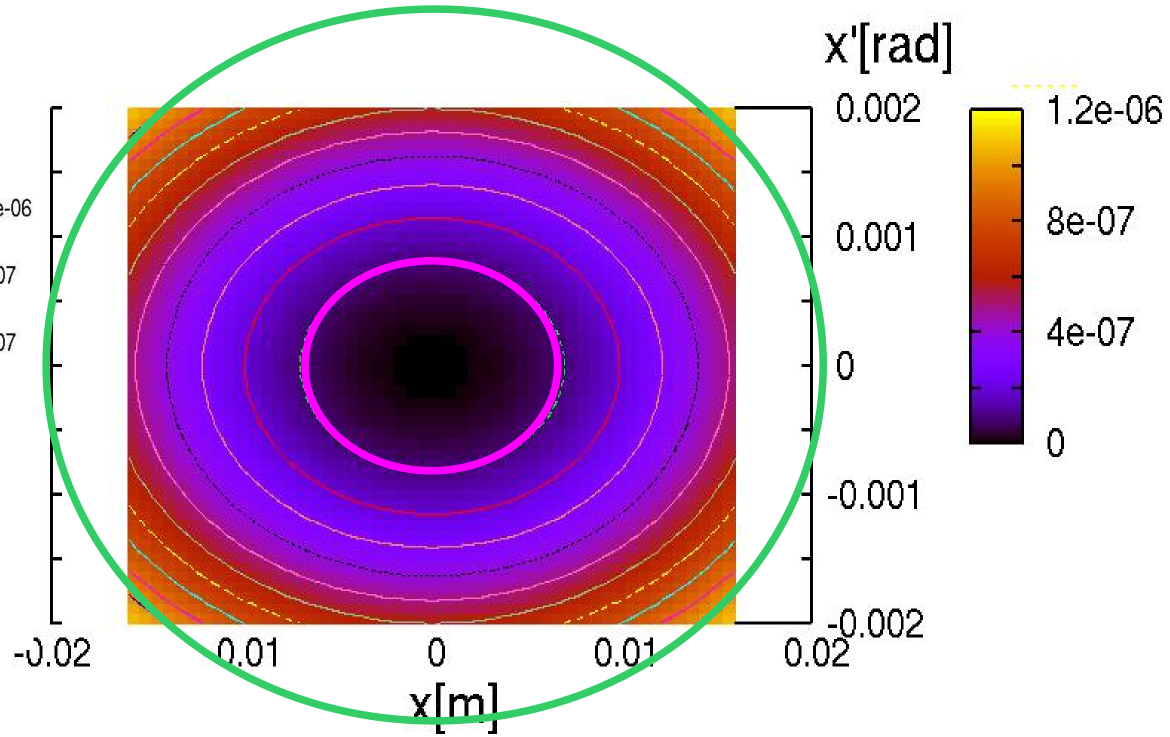

Summary

- New Scheme of Precise Mass Measurement
- to Study
  - ✓ Nuclei Far from Stability
  - ✓ Astrophysical r-Process Path
- Preliminary Design Study Shows

Promising!