

SHAPE PHASE TRANSITIONS AND NUCLEAR MASSES

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INTRODUCTION

Nuclear structure beyond $A \sim 40$ is dominated by collective features. Properties of the collective features arise from the underlying shell structure, mainly the single particle orbitals that are involved in the collective structure and the effective interactions.

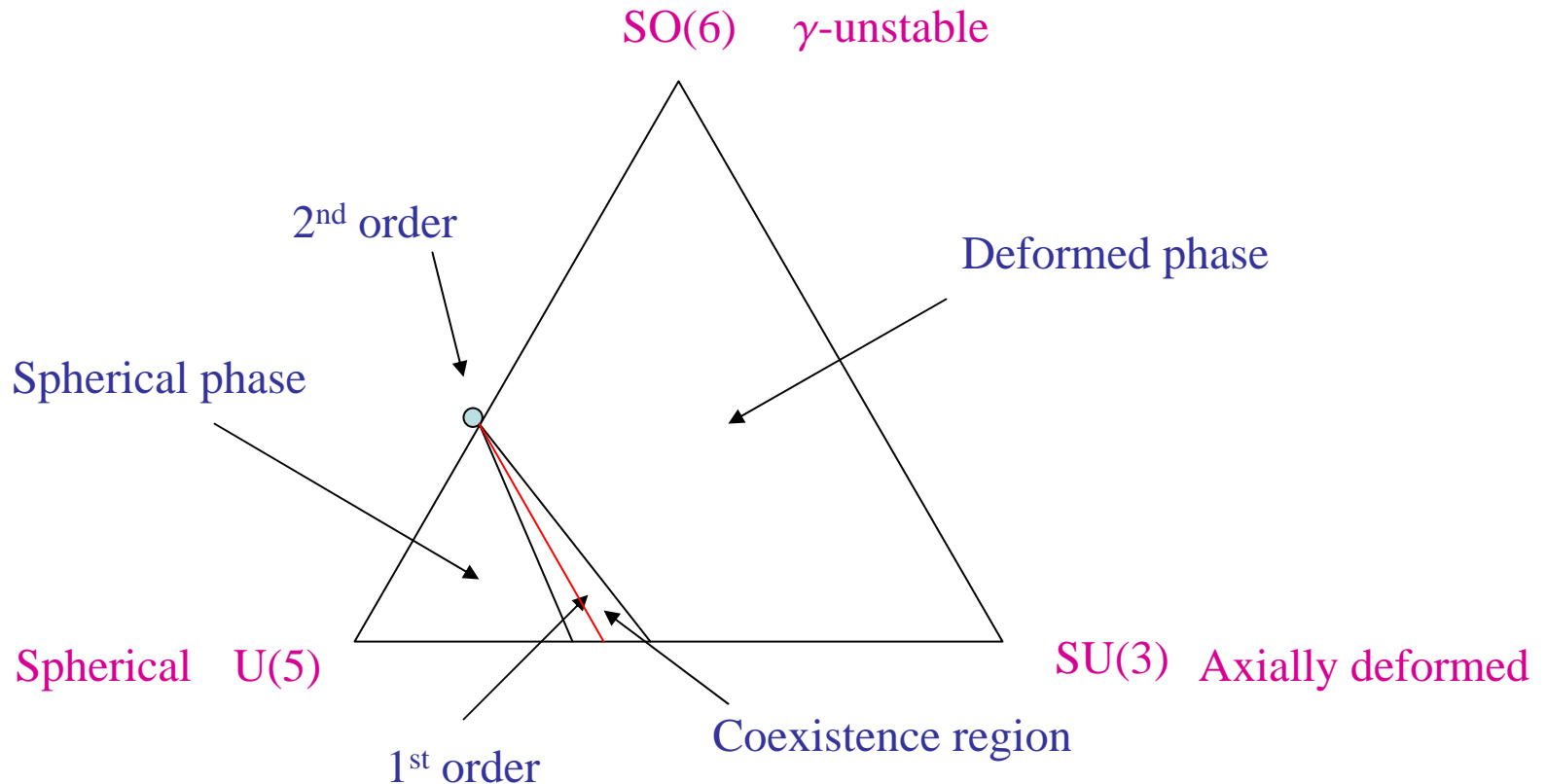
Collective features can be described in terms of **shapes**. A property of collective features in nuclei is the occurrence of several shapes.

Nuclear masses are an ideal tool to study shapes and shape changes, since the binding energy is strongly influenced by the shape of the nucleus.

A convenient framework for studying shapes of nuclei, the transitions between them and their connection with the underlying shell structure is provided by the **Interacting Boson Model**.

Shapes and shape changes within this model can be studied by making use of intrinsic or coherent states and an algorithm called “quantum phase transitions” or “ground state phase transitions”.

Phase structure of the interacting boson model



Phase transitions and their order influence all observables, in particular **ground state energies**, and thus nuclear masses.

The binding energy of an even-even nucleus, within the framework of the Interacting Boson Model, can be written as

$$E_B = E_0 + E_D$$

where E_0 is a smooth contribution and E_D is the contribution of deformation.

The smooth contribution is a universal property of a major shell. In the interacting boson model, it is a quadratic function of N (the number of pairs)

$$E_0 = C + AN + B \frac{1}{2} N(N - 1)$$

This contribution can be obtained empirically from semi-magic nuclei.

The empirical analysis is best performed in terms of **two-neutron** (or two-proton) **separation energies**

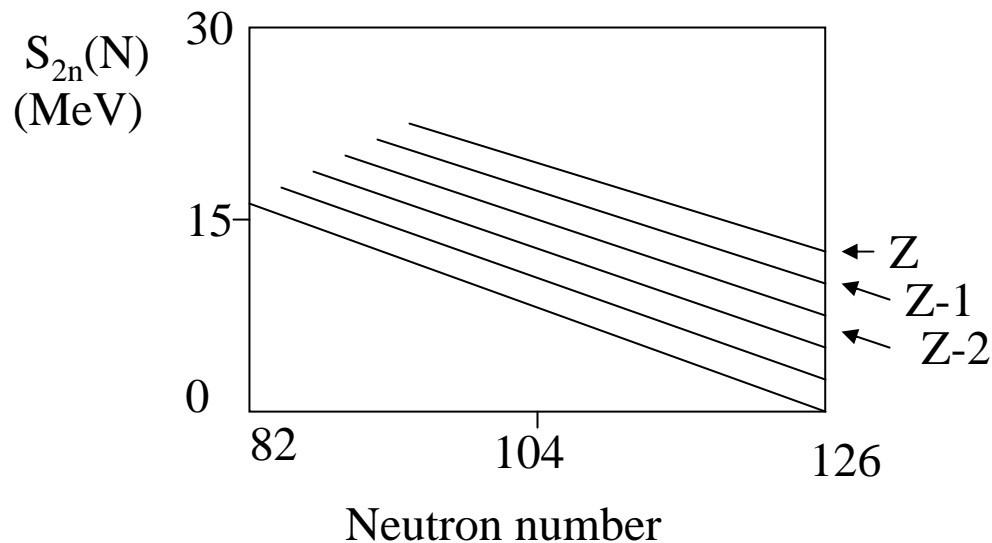
$$S_{2n}(N) = E_B(N+1) - E_B(N)$$

where N is the number of pairs (boson number)

In a plot of $S_{2n}(N)$ versus N , the interacting boson model gives a straight line

$$S_{2n}(N) = A + BN$$

In the absence of deformation effects, the plot of $S_{2n}(N)$ should be a series of parallel lines



SIGNATURES OF SHAPE PHASE TRANSITIONS IN NUCLEAR MASSES

At a zeroth-order transition, the deformation energy E_D is discontinuous

At a first order transition, the derivative of E_D is discontinuous

At a second order transition, the second derivative of E_D is discontinuous

...

Hence, first order shape phase transitions appear as discontinuities in

$$S_{2n}(N)$$

Second order transitions as discontinuities in the double separation energies

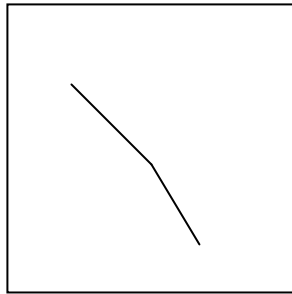
$$S'_{2n}(N) = S_{2n}(N+1) - S_{2n}(N)$$

[The smooth contribution does not have discontinuities.]

Expected behavior of $S_{2n}(N)$ at a phase transition

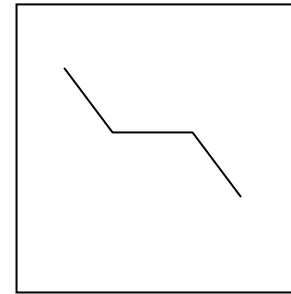
2nd order

$$\frac{\partial E}{\partial g}$$



1st order

$$\frac{\partial E}{\partial g}$$



$$g \propto N$$

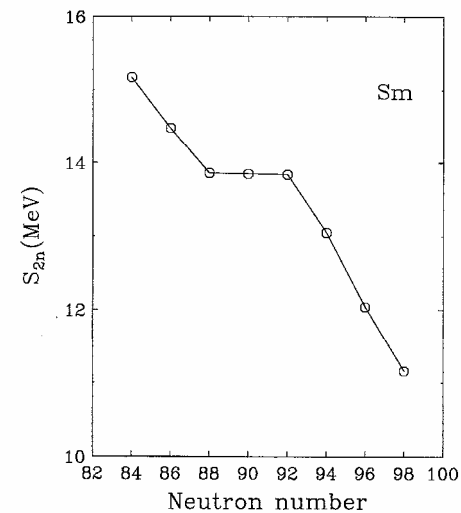
$$\frac{\partial E}{\partial g} \propto S_{2n}(N)$$

Control parameter, g

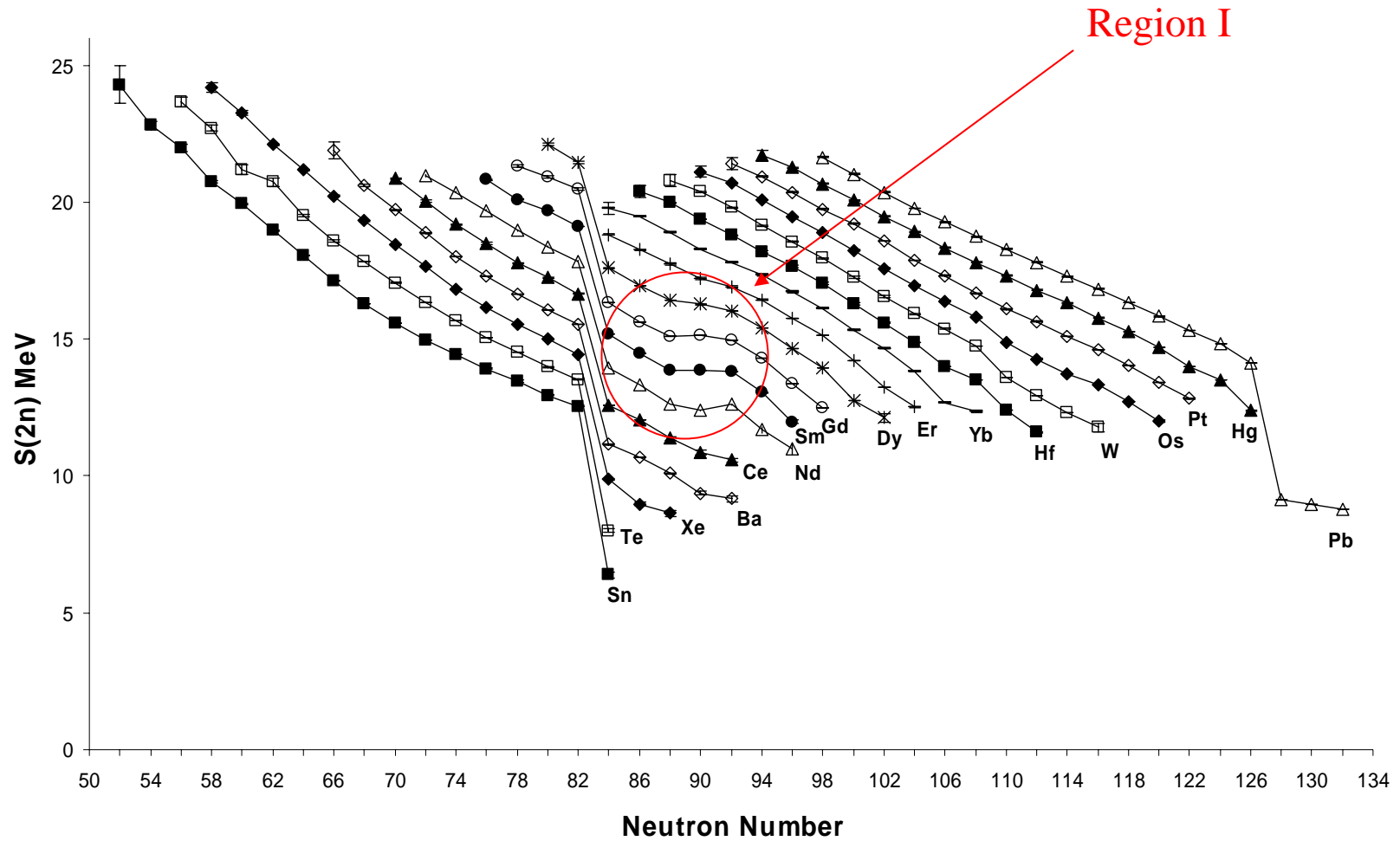
Two regions of 1st order transitions have been identified

Region I: Nd-Sm-Gd-Dy

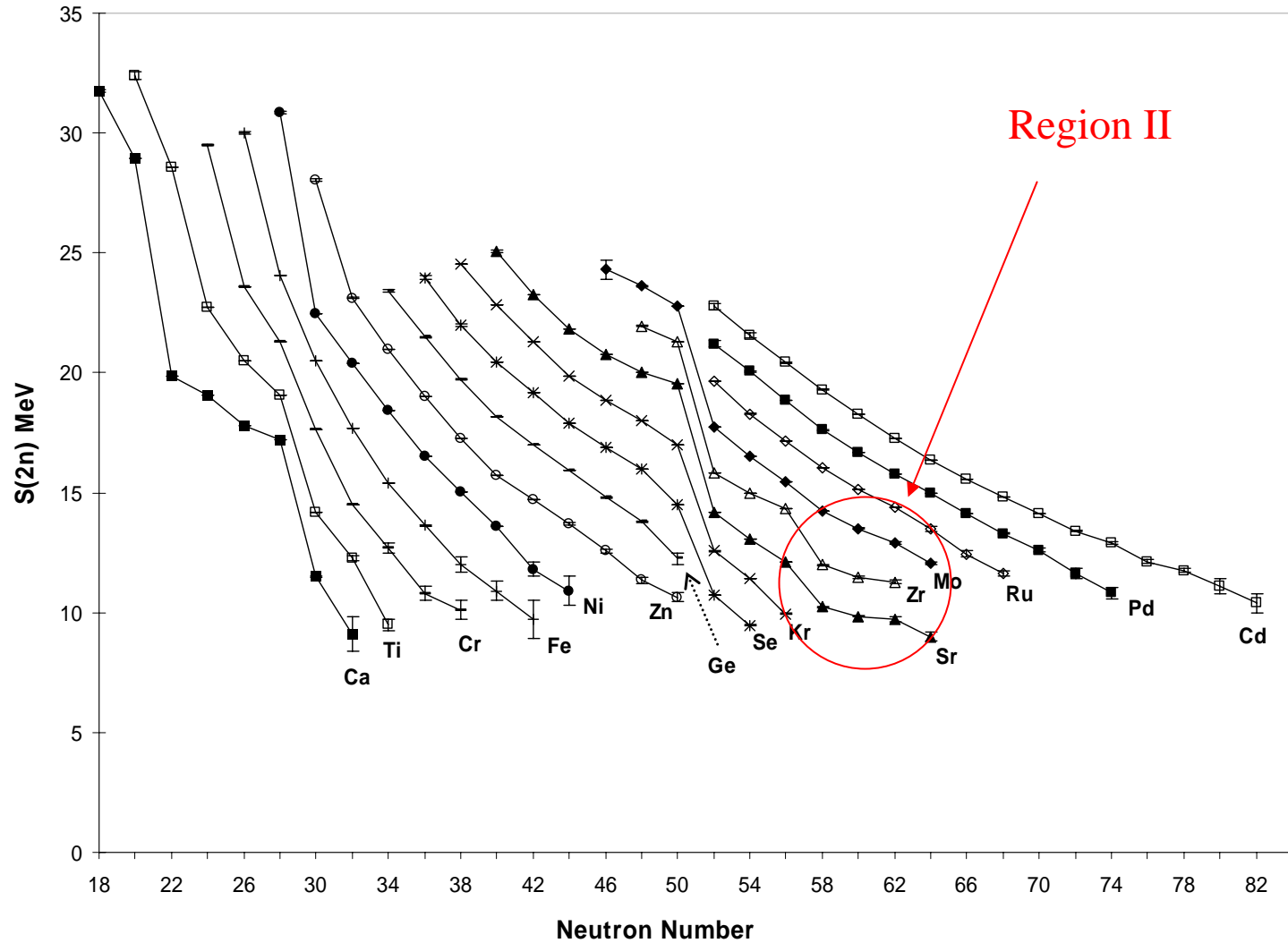
Region II: Zr-Sr



Evidence of 1st order phase transitions in the masses of Nd-Sm-Gd-Dy isotopes at neutron number 88-90



Evidence of 1st order phase transitions in the masses of Sr-Zr isotopes at neutron number 58-60



ROLE OF GSI-FAIR

The experimental information on the neutron rich side of the phase transition is incomplete. To complete this information and to better understand how shape phase transitions occur in nuclei, one needs

Region I

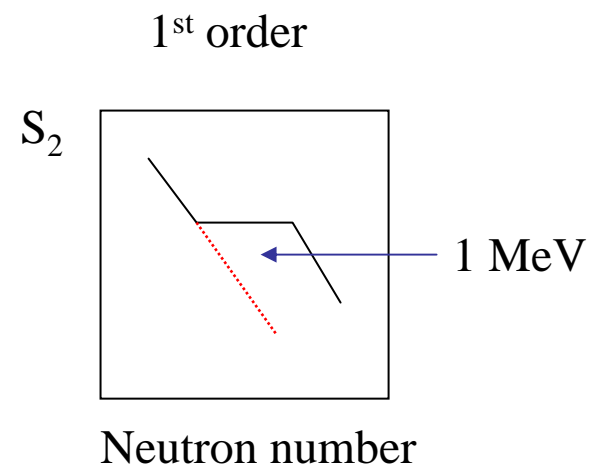
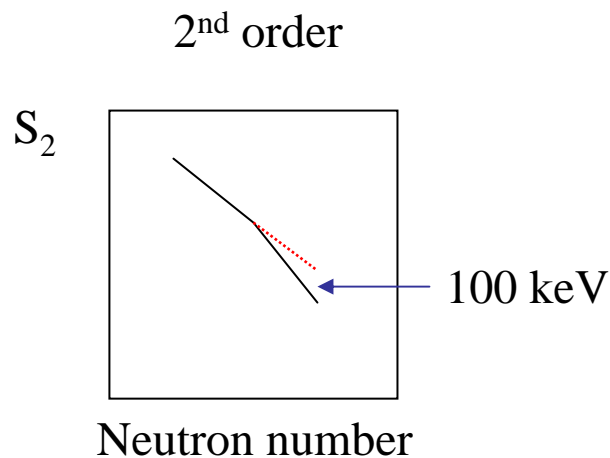
Extend measurements of $2n$ separation energies in Sn-Te-Xe-Ba-Ce as far as possible in the neutron rich region with $N > 82$

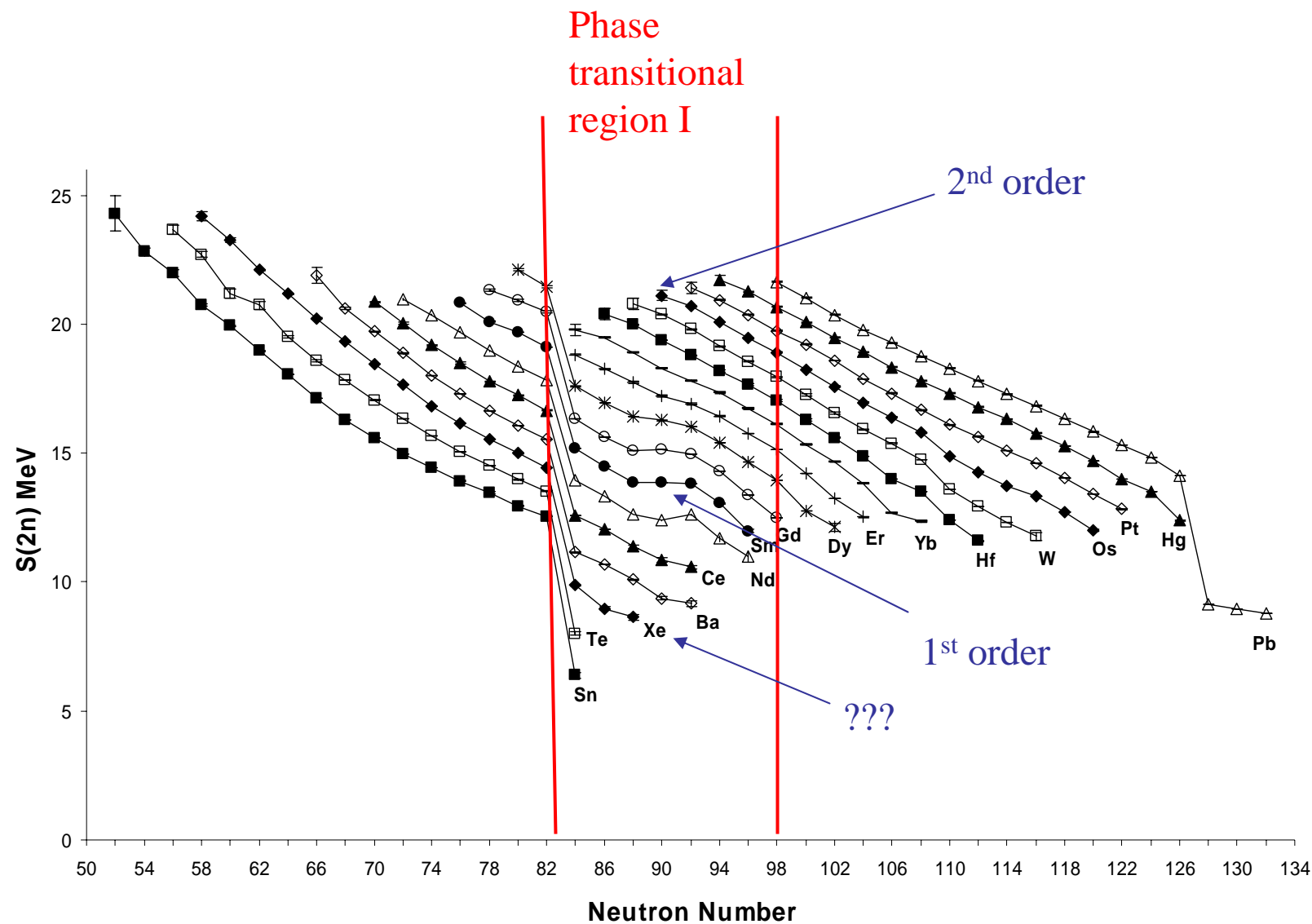
Region II

Same as above for Ge-Se-Kr-Sr-Zr-Mo with $N > 50$

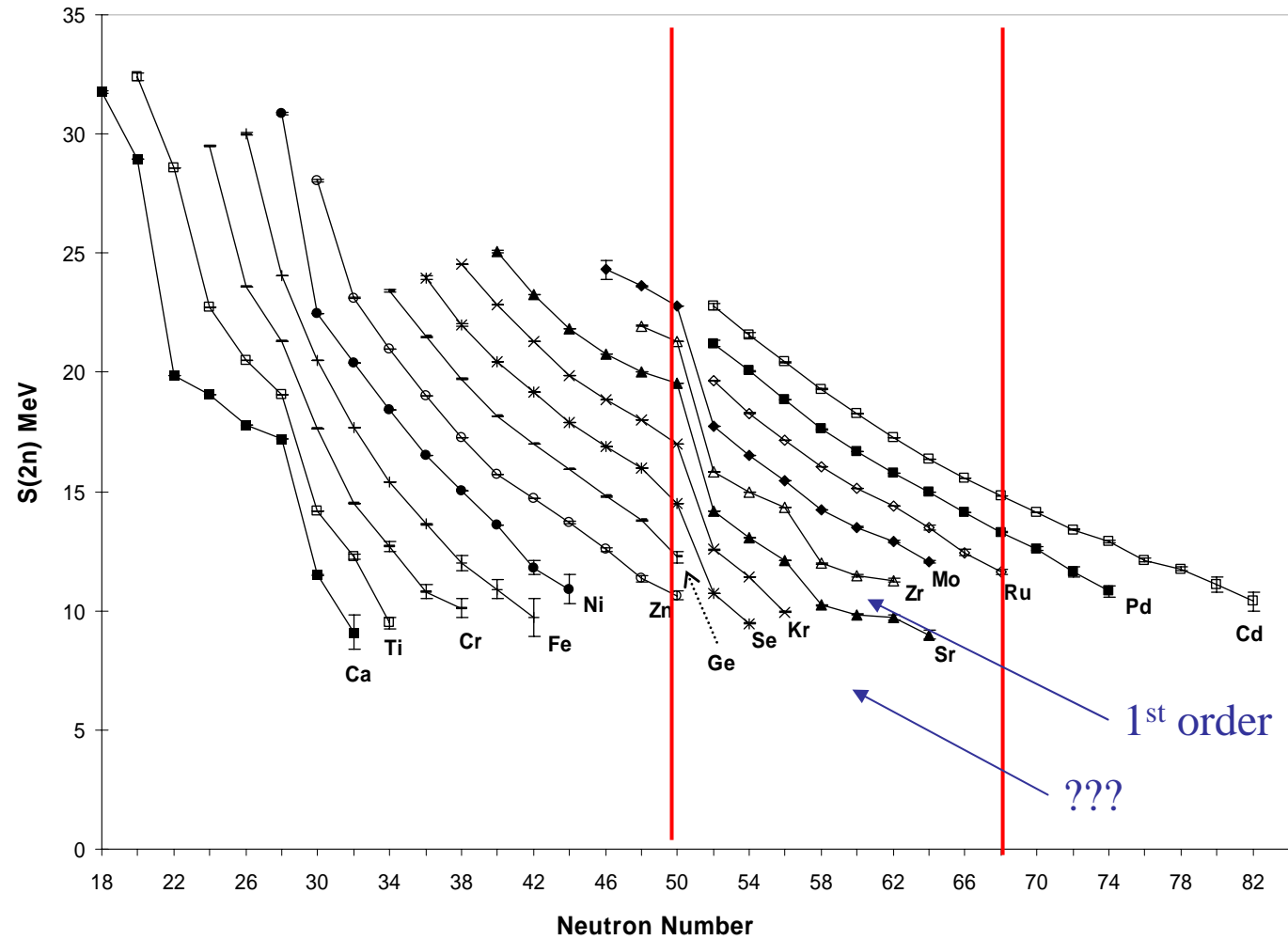
In first order transitions, deformation effects in S_{2n} are of order of **1 MeV**.
Accuracy needed to test first order transitions $\leq 100 \text{ keV}$

In second order transitions, deformation effects are of order **100 keV**.
Accuracy needed to test second order transitions $\leq 10 \text{ keV}$





Phase
transitional
region II



COMMENTS

- Explicit predictions for $S_{2n}(N)$ in regions I and II will be available in 2007.
- The analysis can be extended to **odd-even** nuclei, where new features appear. A simultaneous measurement of masses of odd-even, and even-even nuclei in regions I and II would provide information on phase transitions in odd-mass nuclei.
- A complete discussion of nuclear masses requires the explicit introduction of **proton** and **neutron** degrees of freedom and the study of two-neutron and two-proton separation energies. These are now functions of two variables

$$S_{2\pi}(N_{\pi}, N_{\nu}) = E_B(N_{\pi} + 1, N_{\nu}) - E_B(N_{\pi}, N_{\nu})$$

$$S_{2\nu}(N_{\pi}, N_{\nu}) = E_B(N_{\pi}, N_{\nu} + 1) - E_B(N_{\pi}, N_{\nu})$$

The binding energies, E_B , and two-nucleon separation energies are surfaces. The smooth contribution to S_2 is a series of parallel planes. A simultaneous measurement of isotopes and isotones would provide information on the separation energy surface and hence on the behavior of phase transitions with proton and neutron number.