Using more advanced excitation techniques – Ramsey and optimization of scan procedure

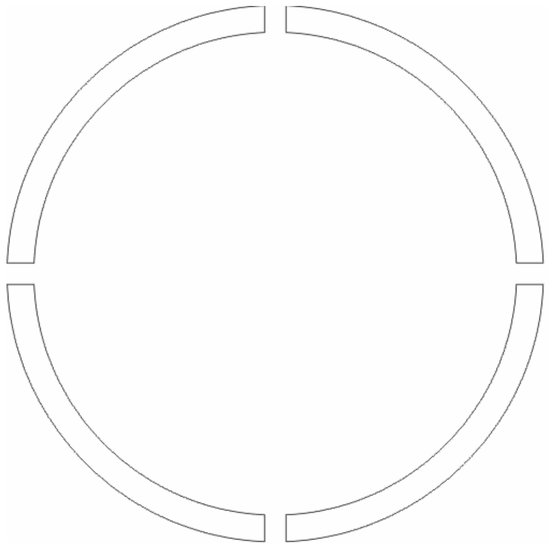
S. George, K.Blaum, M. Dworschak, M. Kretzschmar and S. Schwarz for the ISOLTRAP collaboration

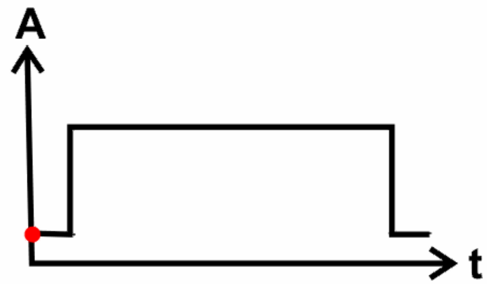
Using more advanced excitation techniques – Ramsey and optimization of scan procedure

Outline

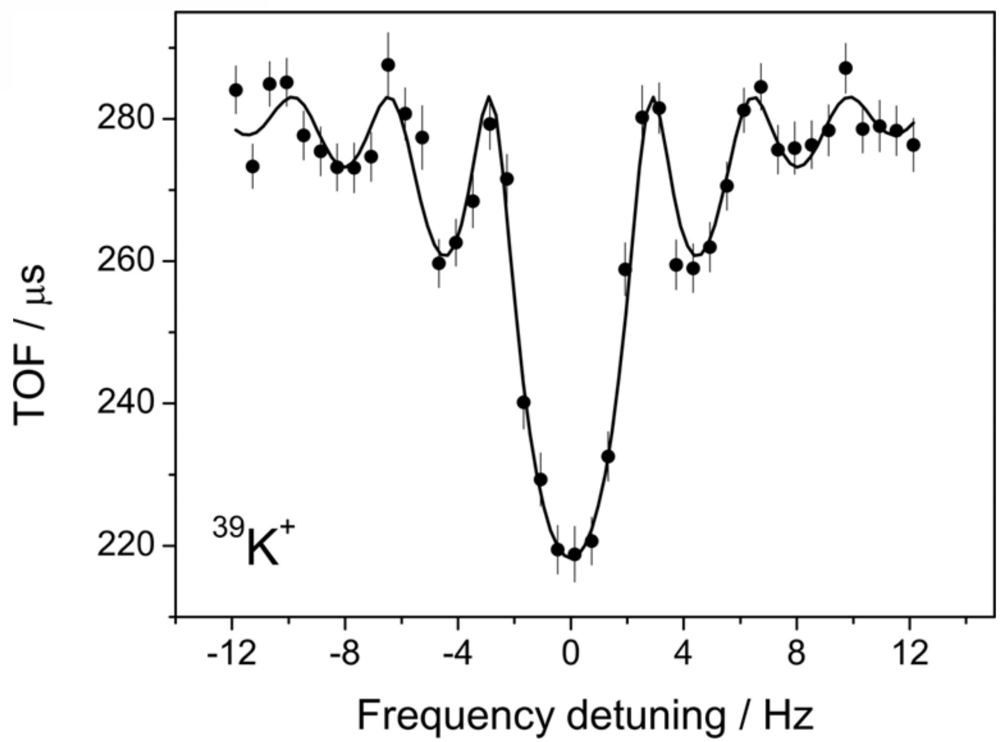
- Standard excitation in mass measurements
- Separated oscillatory fields
- Step-size optimization
- Outlook

Standard excitation in the precision trap

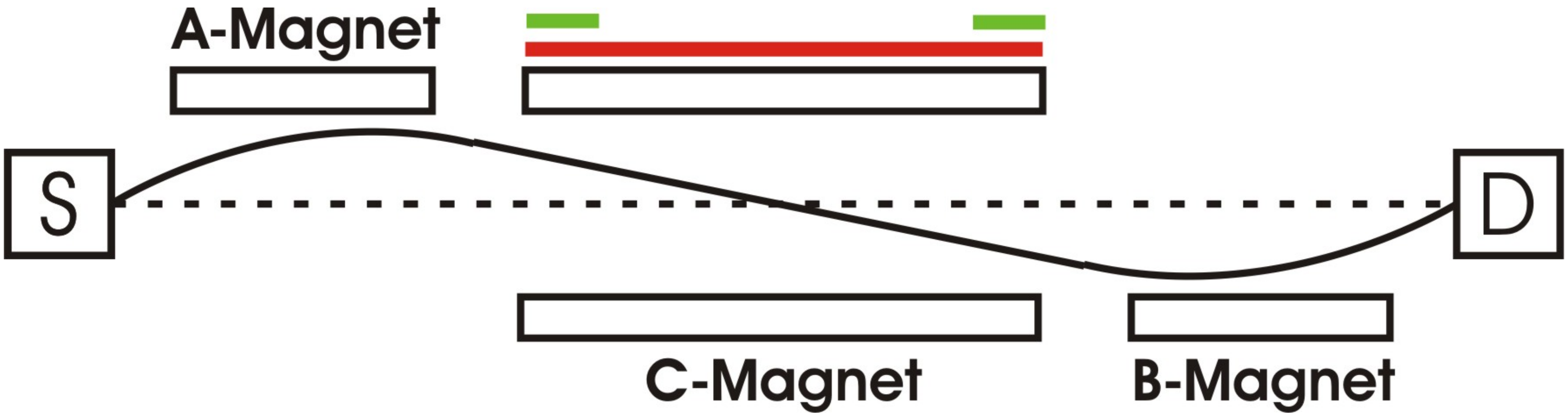




$$F_1 = \frac{4g^2}{\omega_R^2}\left[\sin\left(\frac{\omega_R \tau_1}{2}\right)\right]^2$$



Ramsey Method



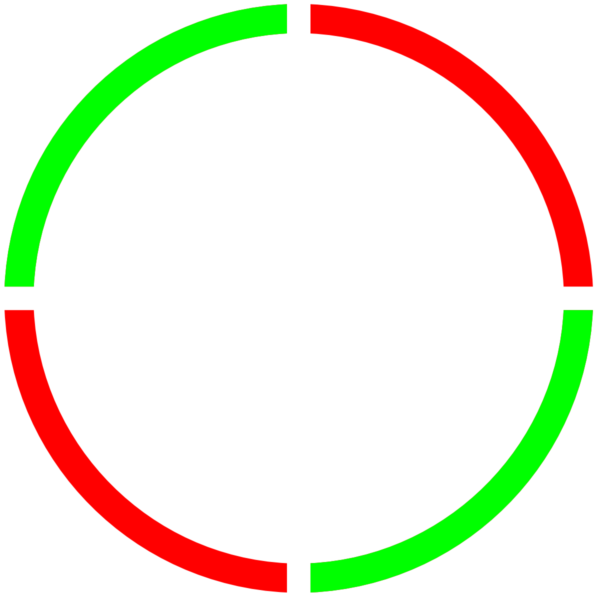
Rabi: continuous high frequency driving field (red)

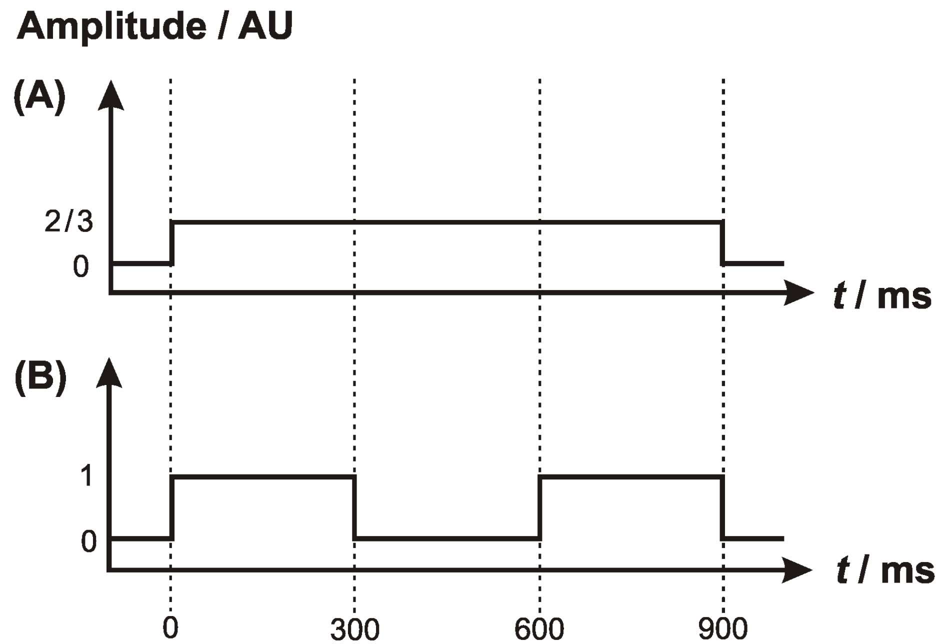
Ramsey: time separated oscillatory fields (green)

Motivation: Reduced FWHM
→ more accurate determination of the cyclotron frequency

Requirements: Complete transition
→ product of excitation time and amplitude is constant

Ramsey Excitation in a Penning Trap





Lineshape as a function of detuning δ of the driving field:

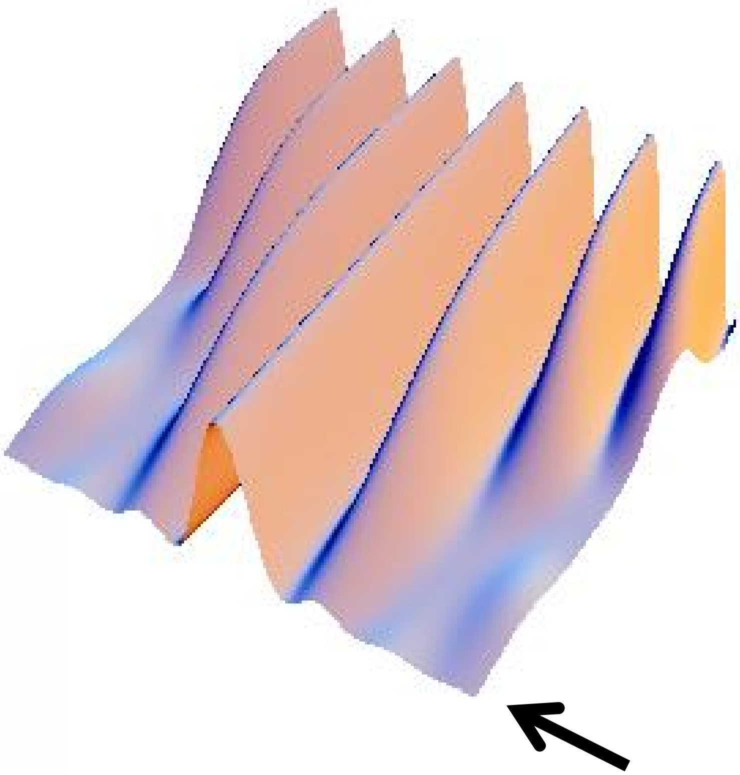
$$F_2 = \frac{4g^2}{\omega_R^2}\left[\cos\left(\frac{\delta\tau_0}{2}\right)\sin(\omega_R\tau_1) + \frac{\delta}{\omega_R}\sin\left(\frac{\delta\tau_0}{2}\right)(\cos(\omega_R\tau_1) - 1)\right]^2$$

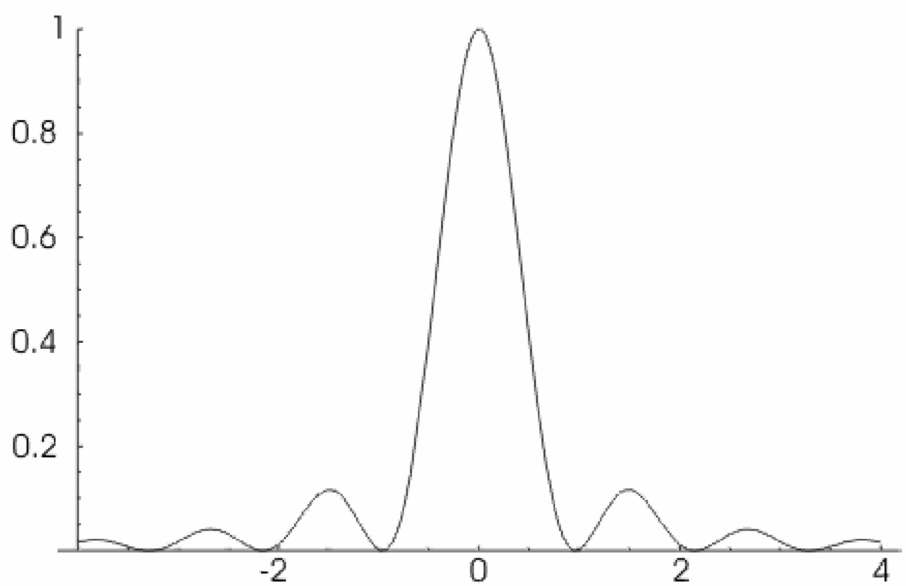
$$\omega_R = \sqrt{4g^2 + \delta^2}$$

g: strength of coupling

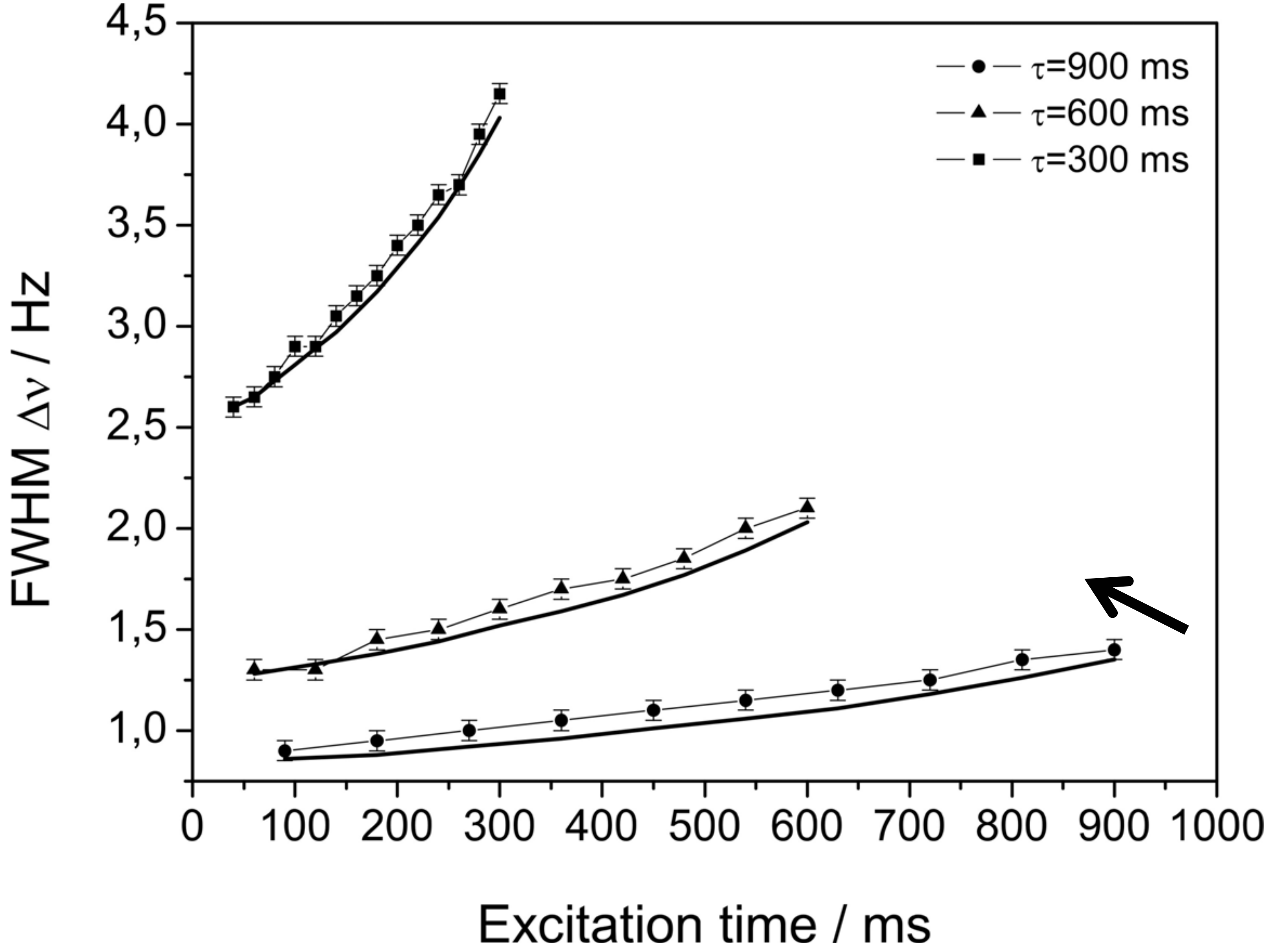
δ: frequency detuning

Two-fringe Excitation

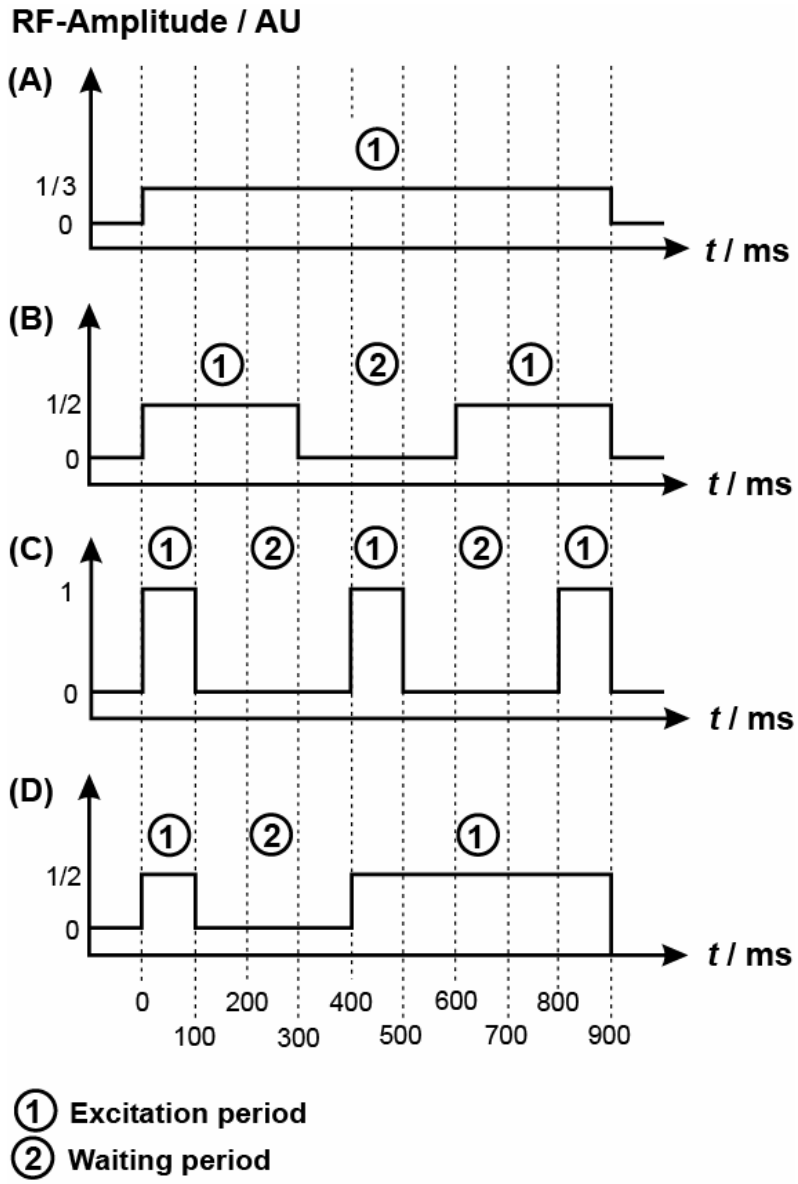


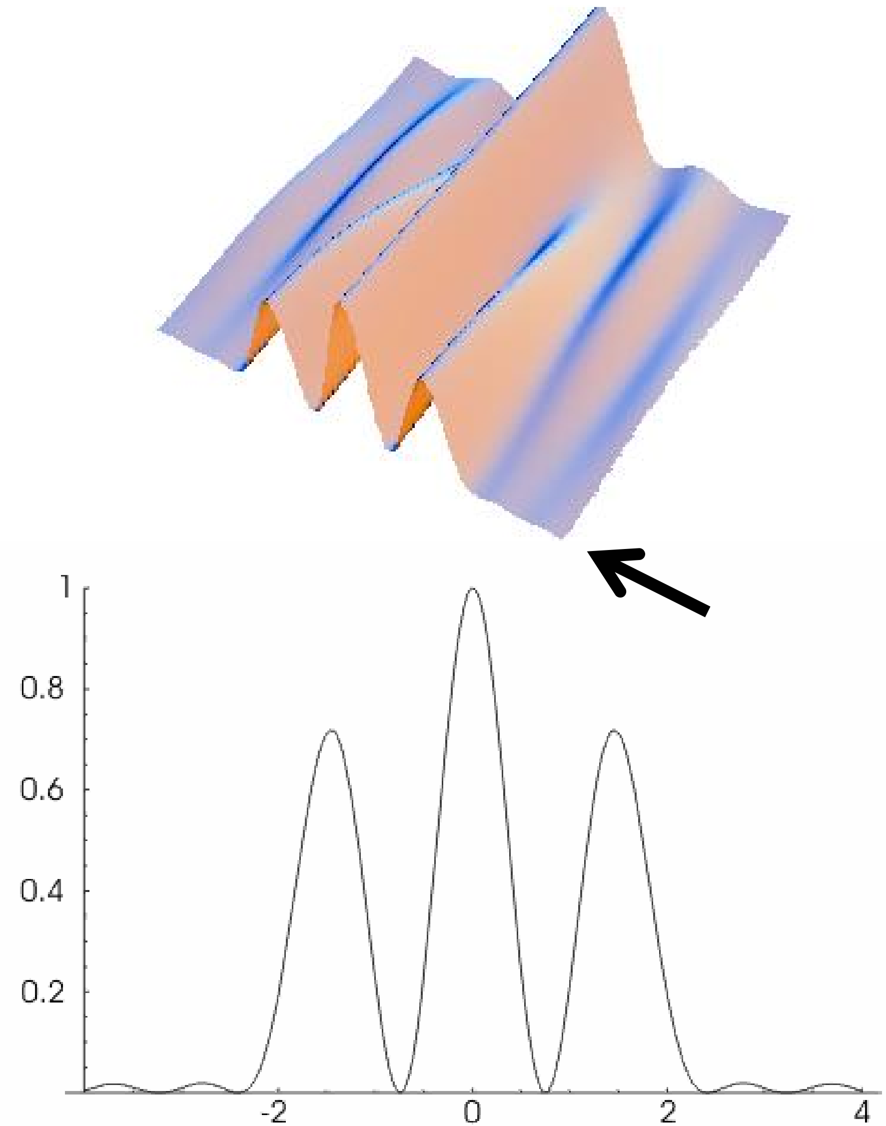


Two-fringe Excitation



Different Excitation Schemes





Theoretical Description

$$\begin{pmatrix} \alpha(t_0+\tau) \\ \beta(t_0+\tau) \end{pmatrix} = M(\tau) \begin{pmatrix} \alpha(t_0) \\ \beta(t_0) \end{pmatrix}$$

Development of the trajectories α and β

$$M(\tau_0) = \begin{pmatrix} e^{i\frac{\delta\tau_0}{2}} & 0 \\ 0 & e^{-i\frac{\delta\tau_0}{2}} \end{pmatrix}$$

Matrix without field

$$M(\tau_a)M(\tau_b)M(\tau_c) = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$M(\tau_1)M(\tau_0)M(\tau_1) = \begin{pmatrix} A & B \\ B & A^* \end{pmatrix}$$

Matrix for a symmetric two fringe excitation

$$\begin{pmatrix} \alpha(t_0+\tau) \\ \beta(t_0+\tau) \end{pmatrix} = \begin{pmatrix} A & B \\ B & A^* \end{pmatrix} \begin{pmatrix} \alpha(t_0) \\ \beta(t_0) \end{pmatrix}$$

$$\alpha(t_0+\tau) = B \cdot \beta(t_0)$$

Prepared in a single mode

Lineshapes

- The following formulas are based on the assumption that the system Penning trap + ion is prepared in a pure magnetron mode!

- The energy in the system is proportional to the square of radius and thus proportional to the square of B!

$$F_{1,1} = \frac{4g^2}{\omega_R^2} \cdot \left[\cos\left(\frac{\delta\tau_0}{2}\right) \sin\left(\frac{\omega_R(\tau_1+\tau_2)}{2}\right) + \frac{\delta}{\omega_R} \sin\left(\frac{\delta\tau_0}{2}\right) \cdot \left(\cos\left(\frac{\omega_R(\tau_1+\tau_2)}{2}\right) - \cos\left(\frac{\omega_R(\tau_1-\tau_2)}{2}\right) \right) \right]^2$$

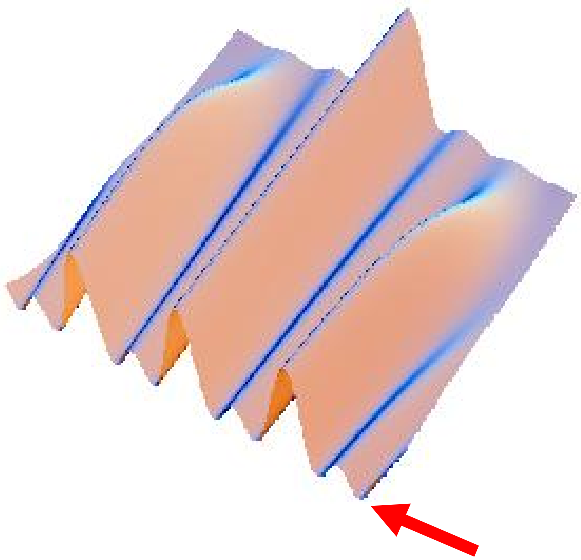
$$+ \frac{4g^2}{\omega_R^2} \cdot \left[\sin\left(\frac{\delta\tau_0}{2}\right) \right]^2 \left[\sin\left(\frac{\omega_R(\tau_1+\tau_2)}{2}\right) \right]^2$$

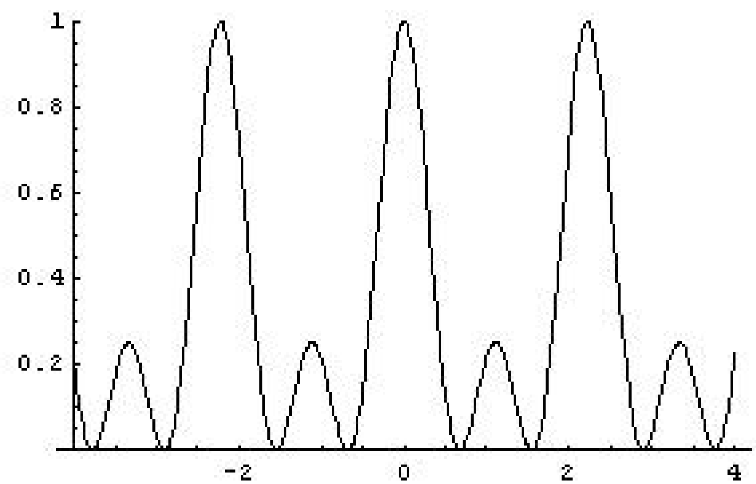
$$F_3 = \frac{4g^2}{\omega_R^2} \cdot \left[\sin\left(\frac{\omega_R\tau_1}{2}\right) \right]^2$$

$$\cdot \left[2\cos(\delta\tau_0) \left(\left[\cos\left(\frac{\omega_R\tau_1}{2}\right) \right]^2 - \frac{\delta^2}{\omega_R^2} \left[\sin\left(\frac{\omega_R\tau_1}{2}\right) \right]^2 \right) - 4\frac{\delta}{\omega_R} \sin(\delta\tau_0) \cdot \cos\left(\frac{\omega_R\tau_1}{2}\right) \right] \sin\left(\frac{\omega_R\tau_1}{2}\right) + 1 - 2 \cdot \frac{4g^2}{\omega_R^2} \cdot \left[\sin\left(\frac{\omega_R\tau_1}{2}\right) \right]^2$$

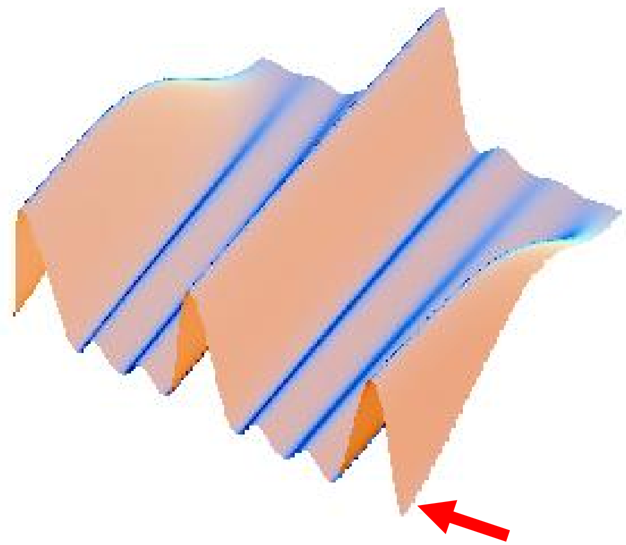
Excitation with Three and Four Fringes

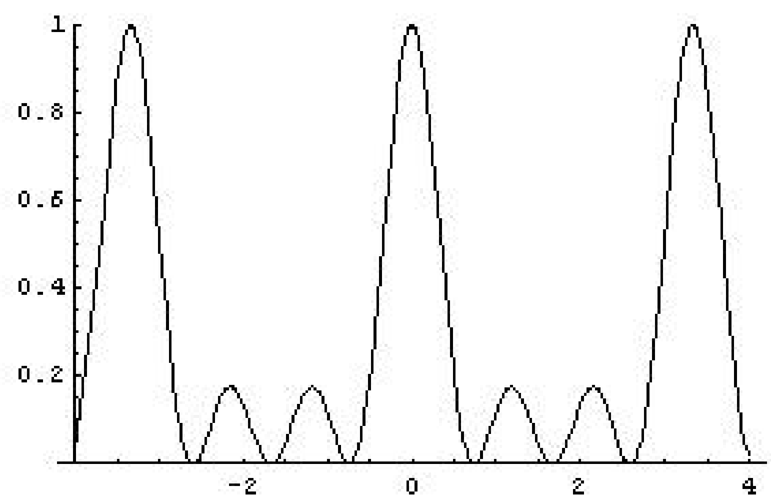
3-Fringes



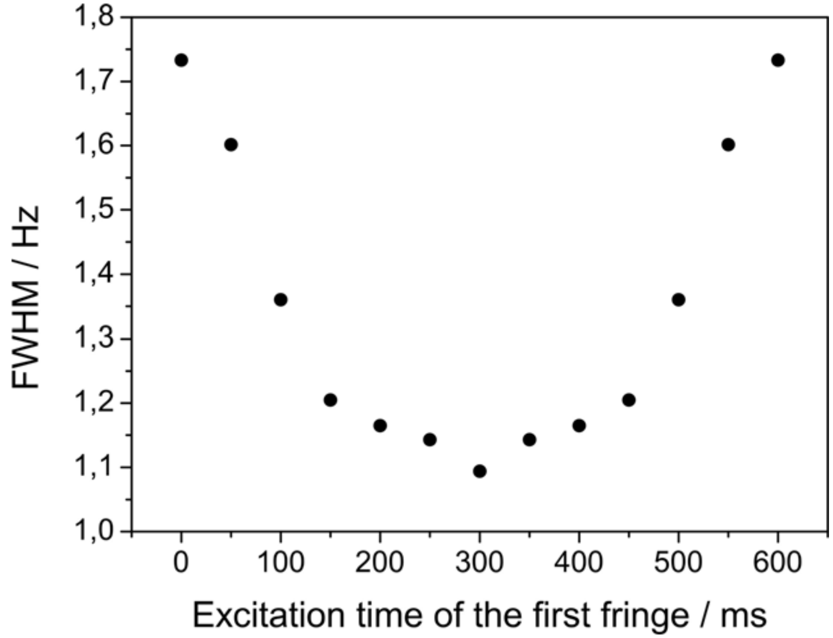


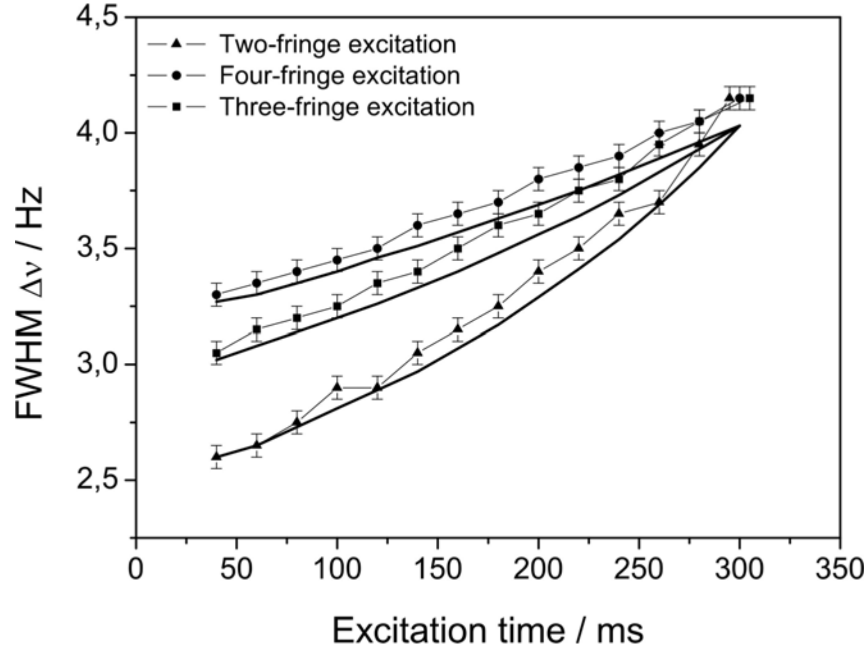
4-Fringes





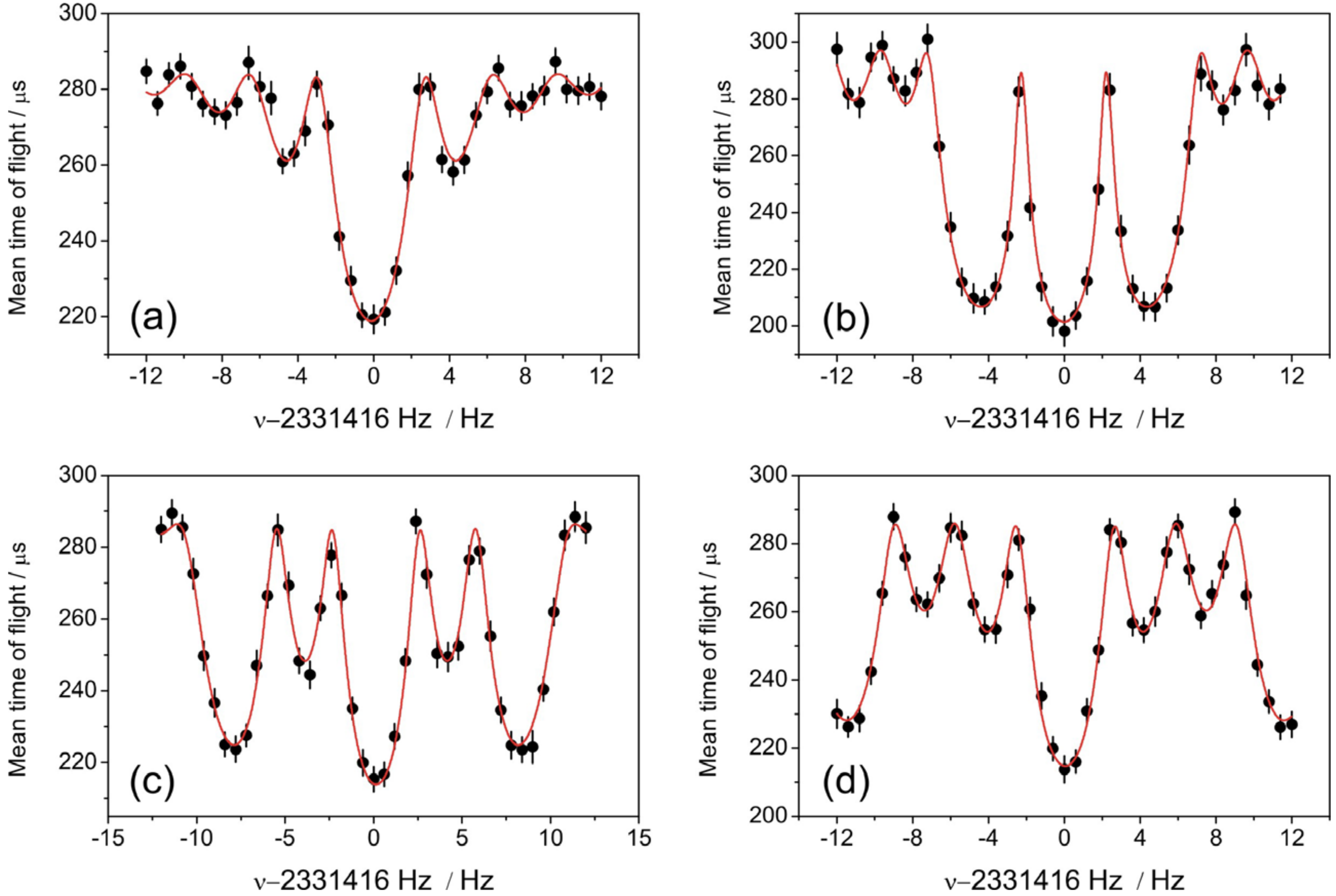
Line Width



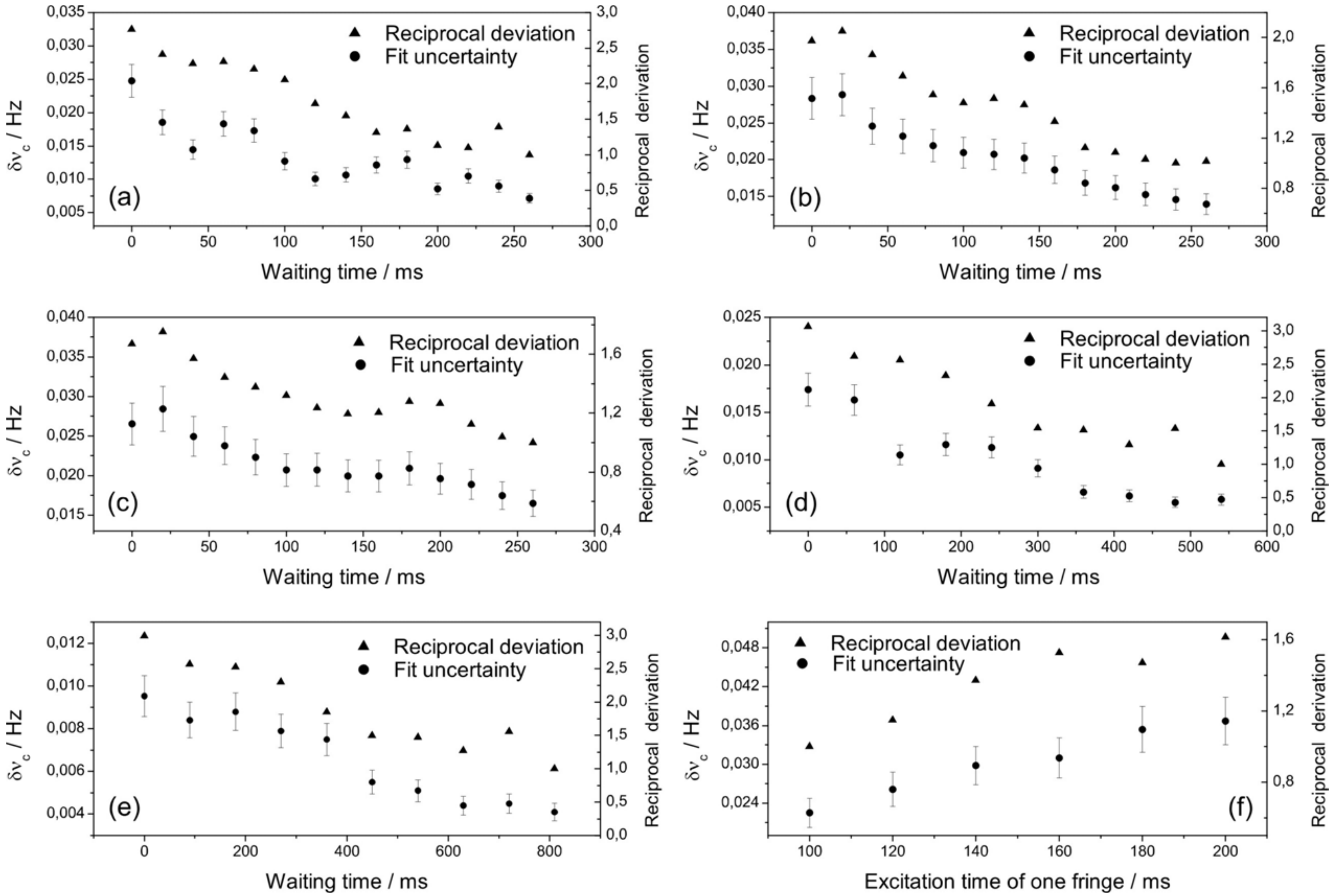


number of fringes	cycle time	max. FWHM / Hz	min. FWHM / Hz	reduction gain / %
4	300	4.1	3.3	20.5 (2.7)
3	300	4.1	3.3	26.5 (2.2)
2	300	4.1	2.6	37.3 (2.0)
2	600	2.1	1.3	38.1 (4.0)
2	900	1.4	0.9	35.7 (5.9)

Resonance curves



Uncertainty of Frequency Determination

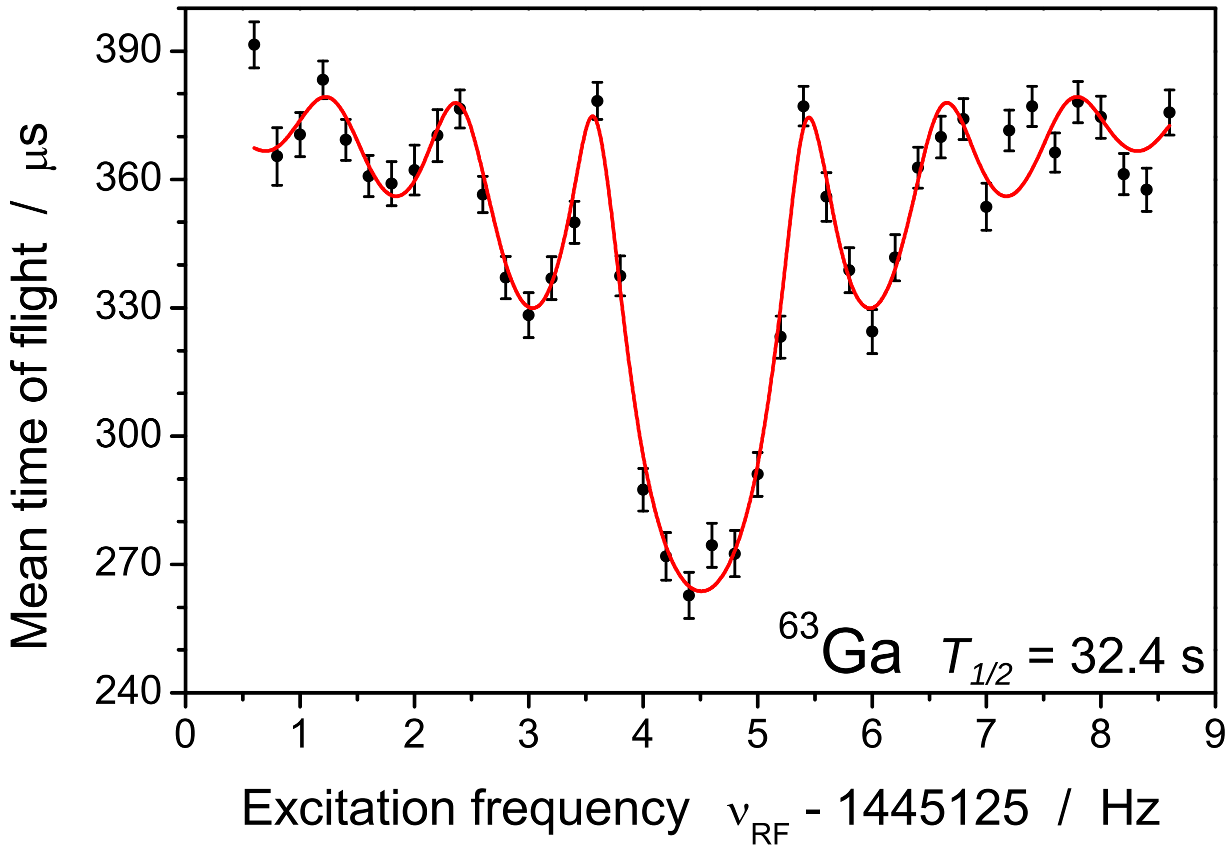


Uncertainty of Frequency Determination

number of fringes	cycle time	max. uncertainty / Hz	min. uncertainty / Hz	Improvement factor
4	300	0.027	0.017	1.6 (0.2)
3	300	0.028	0.014	2.0 (0.2)
2	300	0.025	0.007	**3.6 (0.4)**
2	600	0.017	0.006	2.8 (0.3)
2	900	0.010	0.004	2.5 (0.3)
2	unequal	0.037	0.023	1.6 (0.2)

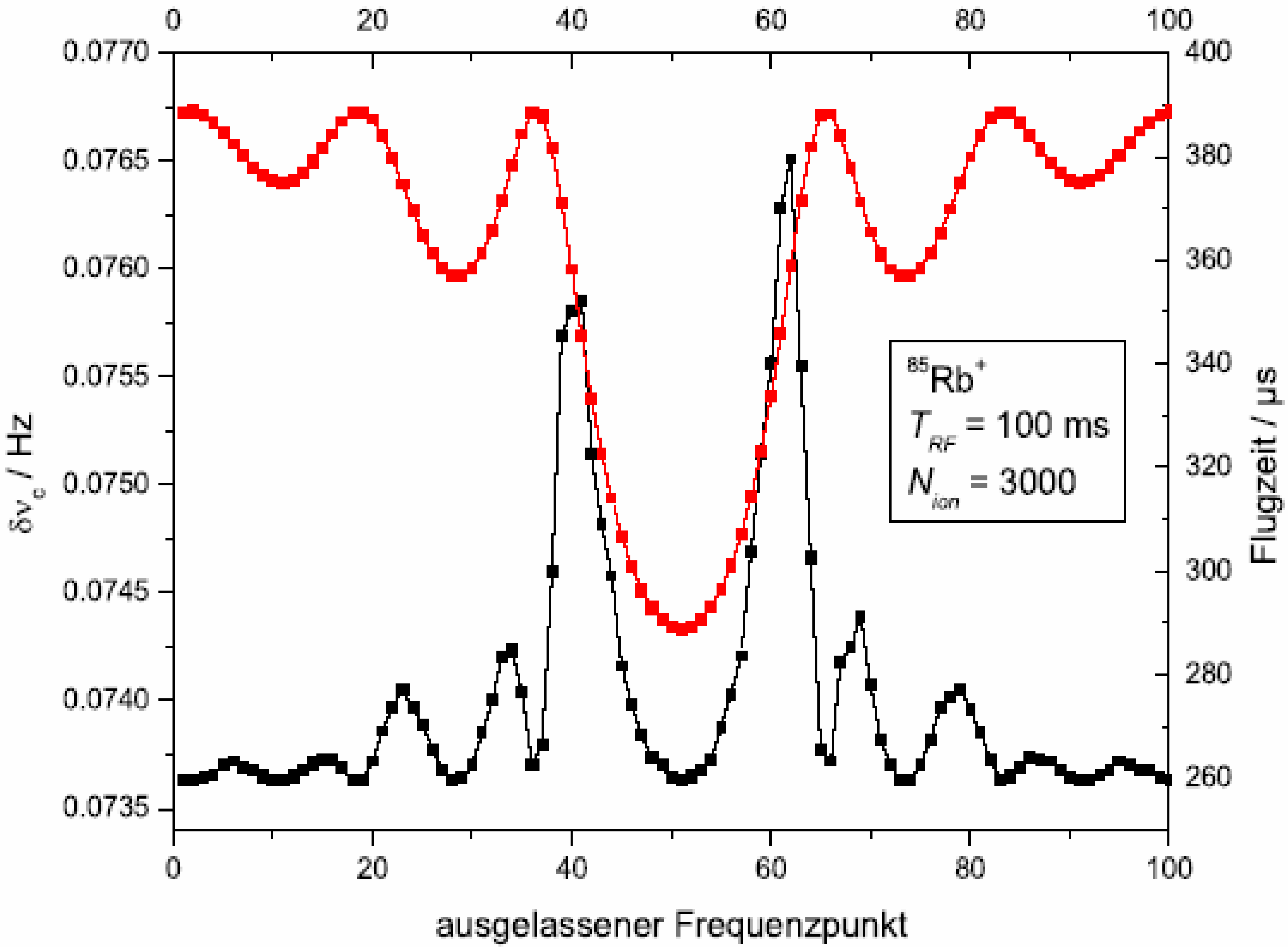
Step-Size Optimization

(Diploma thesis of Michael Dworschak)



What is the weight of each frequency point?

Step-Size Optimization: Experimental Approach



Step-Size Optimization: Theoretical Approach

TOF resonance curve: $y_i = f(c_1, c_2, \ldots, c_k; x_i)$

Central problem: Determination of the optimum horizontal shift L such that the function $f(c_1, c_2, \ldots, c_k; x_i - L)$ gives the best least squares fit to the data.

$\Rightarrow$ Minimize: $\chi^2 = \Sigma\, [f_i(L) - y_i]\, w_i$ with w_i weight of the N data points and $\Sigma w_i = 1$.

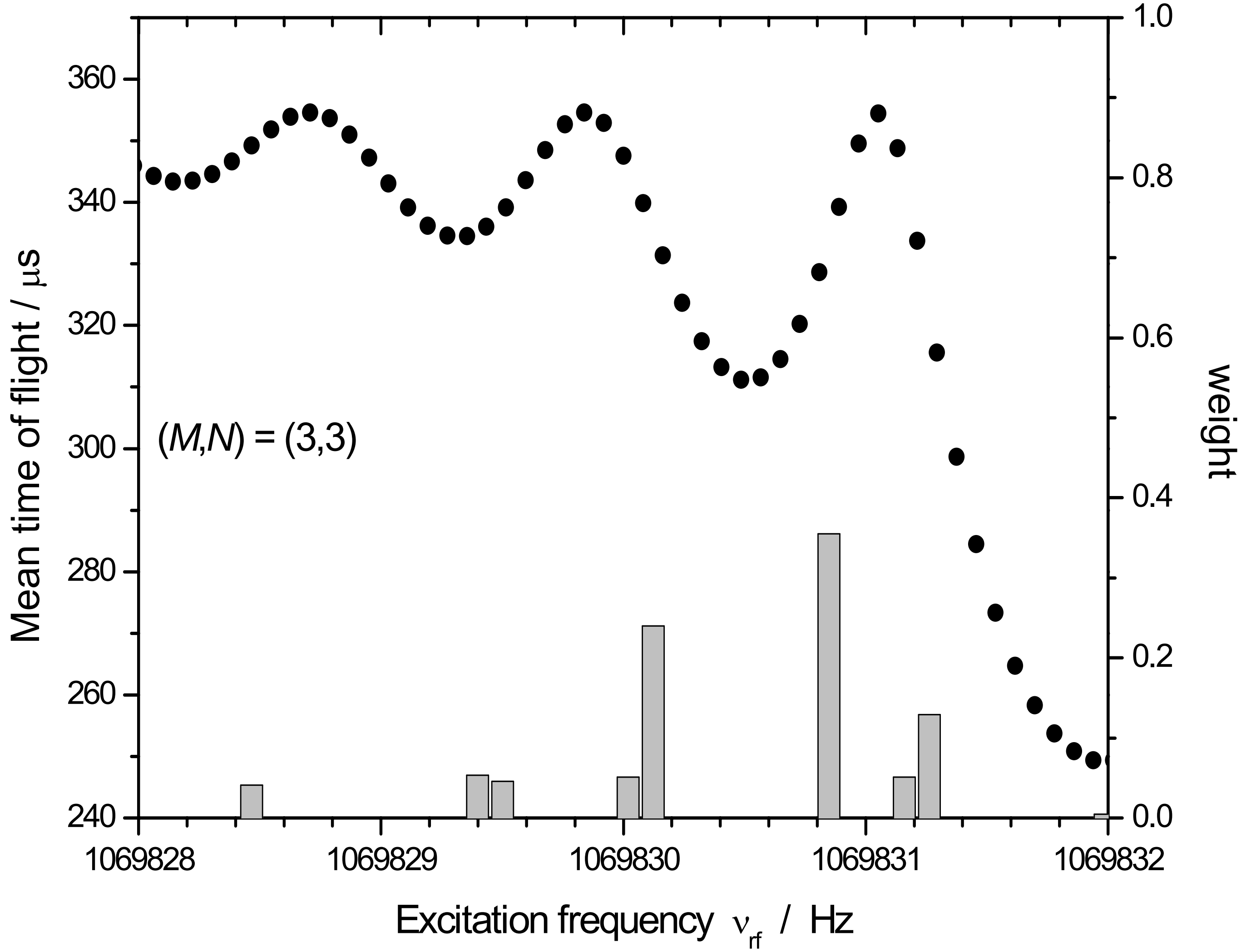
Uncertainty δL in L is given by: $(\delta L)^2 = (\Sigma\, \partial L / \partial y_i)^2\, (\delta y_i)^2$

Method of solution: *Lagrange Undetermined Multipliers*

Calculations: Assumed functional form is $y = C + \alpha f_r(x - L)$ where α is a vertical scale factor and C is a vertical shift for the known resonant function $f_r(x)$.

with $C = \Sigma\, C_i\, x^i$ and $\alpha = \Sigma\, \alpha_i\, x^i$

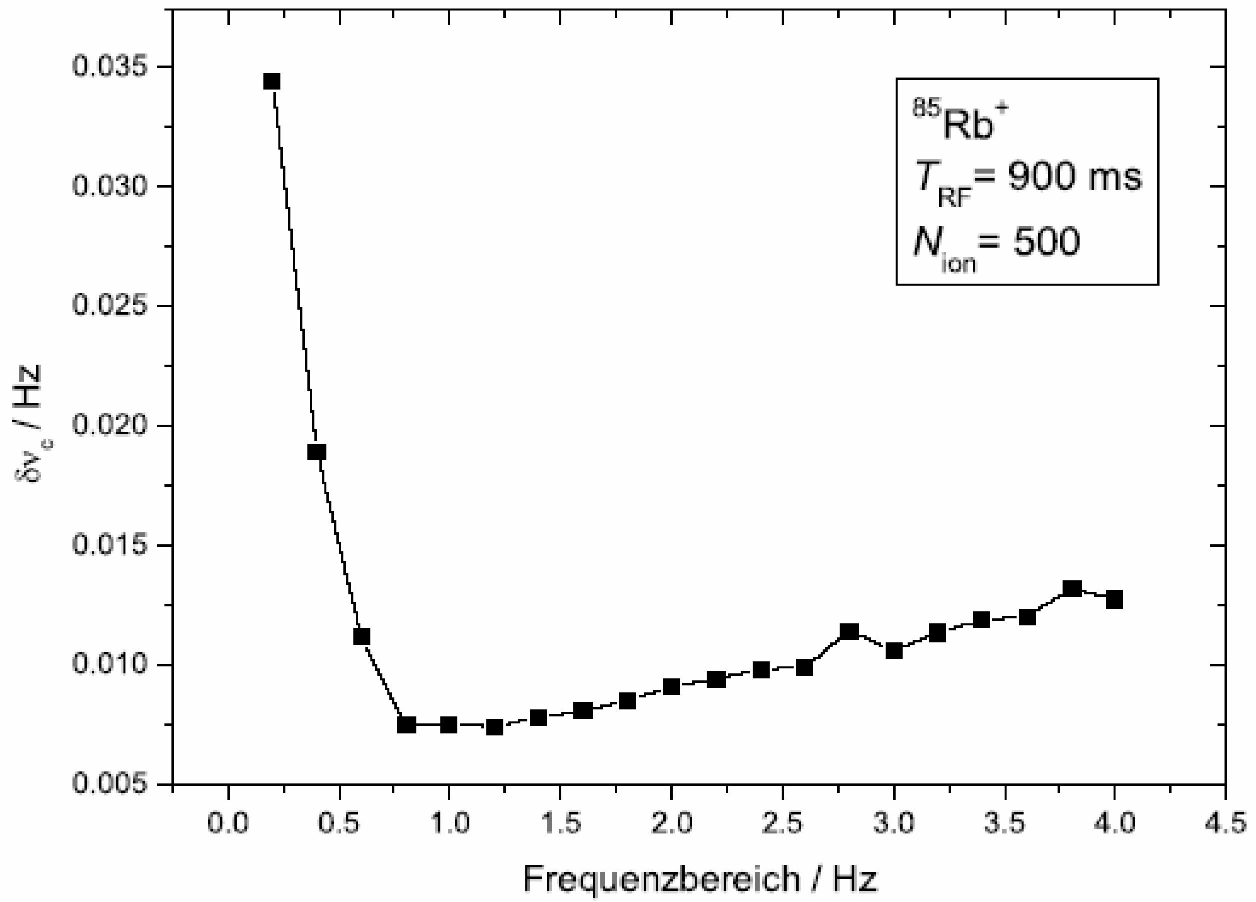
Step-Size Optimization: Prel. Theoretical Result



Factor of two improvement in the statistical uncertainty.

Scan Width Optimization

Constant number of ions (500).



About a factor of two improvement in the statistical uncertainty.

More data is available and will be soon analysed!

Conclusion and Outlook

- Both methods need well known starting conditions

- Clean probes are necessary

- Implementation of the damping in the Ramsey method

- Calculations of the weights for the step-size optimization